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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

STREAM POLLUTION IN THE OHIO RIVER BASIN
A SYMPOSIUM

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Note.—The Symposium on Stream Pollution in the Ohio River Basin was presented at the meeting of the Sanitary Engineering Division at Pittsburgh, Pa., on October 14, 1936. Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **May 15, 1938.**

FOREWORD

By George E. Barnes,[1] M. Am. Soc. C. E.

Since 1933 the construction of sanitary engineering projects has been financed largely by Federal funds allocated as loans or grants-in-aid. Federal funding agencies act in part as co-ordinating agencies also, in reviewing studies and plans for various types of projects proposed for individual population centers and local areas. In the Ohio Basin as elsewhere, such review has made more apparent than ever the desirability of more comprehensive planning for the region as a whole, as the basis for a long-range policy on conservation and proper utilization of water resources.

The Ohio Basin, from the standpoint of size, natural wealth, and population supported, is one of the most important in the United States; and it seems appropriate at the present time to focus technical and lay interest on its peculiar sanitation problems. With this object in mind, the Cleveland Section of the Society appointed a Special Committee to work in co-operation with the Sanitary Engineering Division and to organize a Symposium on Stream Pollution in the Ohio Basin. The Committee included the writer as Chairman, and George B. Gascoigne, Rollin F. MacDowell, and George B. Sowers, Members, Am. Soc. C. E. The Committee was of the opinion that, although there may be interesting and comparatively unexplored problems in sanitary engineering, particularly in the field of industrial wastes treatment, nevertheless questions of design or treatment processes might properly be subordinated in this Symposium to an appraisal of the critical problems now existing in the water-shed, the outlook for co-ordinated planning, and the procedure for initiating and enabling sanitation improvements. Emphasis in the following papers is directed toward this end.

Mr. Streeter's paper describes methods for surveying the extent of pollution and gives the results of such a survey on the Ohio River, together with suggestions for minimum standards for river condition. Mr. Ryder's paper, discussing the operation of the Pymatuning Reservoir, shows that in certain cases it may be at least as valuable to regulate dry-season flow for dilution purposes as to invest in sewage plants complete with secondary treatment. Mr. Stevenson's paper is devoted to the extraordinary duties carried by the Pennsylvania Department of Health in time of flood, and indicates from experience many factors that must be considered in the design of sanitary engineering projects and in the organization of service agencies, if these are to function during emergencies. Mr. Tisdale's paper sets forth an excellent picture of sanitary conditions in the Ohio Basin as a whole, and is supplemented with papers by Mr. Davis on the problems at Pittsburgh, Pa., and by Mr. Root on those at Cincinnati, Ohio, the two major metropolitan centers of the Ohio Basin. The concluding paper, by Mr. Wolman, is concerned with the medium through which comprehensive planning and adequate financing may be secured. The discussions of this Symposium will be no less important than the topic papers themselves, in presenting a balanced understanding of sanitation problems in the Ohio Basin.

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[1] Prof. of Hydr. and San Eng., and Head, Dept. of Civ. Eng., Case School of Applied Science, Cleveland, Ohio.

SURVEYS FOR STREAM POLLUTION CONTROL

By Harold W. Streeter,[2] M. Am. Soc. C. E.

Synopsis

The effective control of stream pollution, when viewed as an engineering problem, entails not only the design of sewage and industrial wastes treatment plants, but also the formulation of comprehensive plans for restoring and maintaining entire river systems in proper condition for various water uses, avoiding the wastefulness of over-correction in some areas and the ineffectiveness of under-correction in other areas. The base data for such plans are obtainable from field and laboratory surveys of a river and its tributary drainage area.

Field surveys of this character will embrace all major sources of pollution; vital statistics of water-borne diseases in various parts of the water-shed; the extent and kind of water uses in different areas; hydrometric data for the river and its main tributaries; and information bearing on time intervals of flow in the river between various sources of pollution. Laboratory surveys will include data on the quality of stream waters at water intakes and below sources of pollution; on progressive changes in "oxygen balance" in more highly polluted river zones, and in special cases, on turbidity, alkalinity or acidity, hardness, and certain metals detrimental to the quality of the stream waters and their different uses. Biological data will embrace major groups of pollutional and non-pollutional organisms, both in the stream proper and in the bottom sediments, and likewise organisms detrimental to water supplies.

Where pollution control is designed so as to utilize the natural purification capacity of a waterway, limiting conditions of pollution should be fixed tentatively for various water uses, as the basis of interpreting the laboratory findings. Examples of such criteria are given with respect to water supply sources, nuisance prevention, and maintenance of normal aquatic life in stream waters.

Considering the Ohio River System as presenting a fairly representative large-scale problem of stream pollution control, a brief account is given of the various surveys of this river made during recent years and the main conclusions derived from them. To illustrate the application of laboratory survey data collected in this river, results are shown of an estimate of the relative responsibility of sewered population groups draining into various zones of the river for conditions of bacterial pollution at several water intakes. These results indicate that except in the winter, when the effects of natural purification are at a minimum, the major portion of sewage bacteria surviving in the river at each intake appears to be traceable to population groups draining

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[2] **Senior San. Engr., U. S. Public Health Service, Cincinnati, Ohio.**

into the river, either directly or indirectly, within about 200 miles up stream from that point. In winter, the effect of more distant population groups is apparent. It also is shown that acid conditions in the Ohio River and its tributaries above Wheeling, W. Va., exercise a powerful repressive influence on bacterial pollution originating in this part of the river system. With the amelioration of the acid conditions resulting from mine-sealing and other corrective measures in progress, extensive treatment of the sewage from populations now draining into this zone of the river will be necessary, in order to avoid serious overburdening of water purification plants with excessive bacterial pollution originating in sewage. Study needs to be given to the degrees and methods of sewage and industrial wastes treatment best adapted to meeting water use requirements in different parts of this river system, and also to the probable effects of large volumes of treated effluents on the stream waters and their several uses.

With the growth of public interest in stream pollution as a nation-wide problem, legislators and sanitarians have been impelled to extend their views of the subject far beyond local horizons. Until recently the sole objective of "stream cleaning" was usually the removal of local "sore spots." Little, if any, thought was given to more remote down-stream conditions, which were left to those concerned to deal with as best they could. However, it is now becoming more generally recognized that river pollution, like floods, is no respecter of local boundaries, and that if control measures are to be permanently effective they must comprehend entire drainage areas, or, at least, those portions of them in which sewage and industrial wastes impose a definite burden on streams.

This general broadening of viewpoint with respect to stream pollution and its control has brought with it a marked change in the nature and complexity of the engineering problems to be solved, not so much as regards the actual treatment of wastes, as regards the formulation of workable plans for pollution control. Such plans must be designed to restore and maintain complete river systems in proper condition for various water uses, and must avoid, as far as possible, the wastefulness of over-correction in some areas and the ineffectiveness of under-correction in other areas. It is with this latter phase of the problem and more especially with the surveys and other studies essential to a well-planned system of river-pollution control that this paper is concerned.

In a broad sense, a certain engineering analogy may be said to exist between a river system under effective pollution control and a well-planned structure, such as a bridge, which must be designed so that the maximum stress imposed on every individual member does not exceed its safe working strength. In such a river system, for example, the degree of pollution, both of the main stream and of the various tributaries, must be limited to a maximum consistent with the suitability of those streams for specified water uses, and in no case may it exceed their respective capacities for dilution and natural purification.

The limitation in pollution imposed by water use may be likened to the safe working load on a structure, and the limitation imposed by dilution and self-purification corresponds to the ultimate breaking load. A well-marked distinction is to be noted, however, between ordinary structural design and stream-pollution control in that, in the first instance, the load is predetermined and the strength or capacity of the structural members can be adjusted to support such a load, whereas, in the second instance, the limiting strength of the structure is predetermined (by the capacity for dilution and natural purification) and the load, or degree of pollution, must be adjusted accordingly.

From the foregoing analogy, it is fairly evident that as it is necessary to make a thorough study of loadings and allowable stresses in designing an ordinary engineering structure, it is equally essential in planning stream-pollution control, to make a similarly careful and detailed study of the nature and extent of the pollution of a river system and also its capacity for dilution and natural purification. The basic data for such a study are usually obtained through field and laboratory surveys of the river and its drainage area, extending over a sufficiently long time to reveal the normal seasonal fluctuations in temperature, run-off, velocities, and time intervals of flow in the streams, and other seasonal variables.

General Survey Methods

A survey of sources of pollution will include rural, urban, and sewered populations, and their trends of growth or change in different parts of the river system. It also will include waste-producing industries, listing them according to character, location, and the strength and volume of wastes produced in relation to amounts of manufactured products. All water intakes along the main stream and its major tributaries should be located. In the vicinity of the larger sewered communities, all the more important sewer outlets should also be located, and detailed information should be obtained in each case on the size of the contributing population, the volume and strength of the sewage, and the nature and extent of sewage treatment. Such a survey, which would entail visiting practically every sewered community on the water-shed, might well be supplemented by collecting available statistics on the prevalence of water-borne disease in each community (particularly typhoid fever and gastro-enteritis) and on the quality of the public and private water supplies. Drainage areas of the main stream and its various tributaries should be mapped and determined by careful planimeter measurements. These several items of data would fall under the ordinary field survey, to which the hydrometric and laboratory surveys would be supplementary.

The main objective of the hydrometric survey is to obtain detailed information as to the amount of dilution water available at different points in the river system, and its variation from day to day, or week to week, throughout the period covered by the laboratory survey. Collection of these data involves the establishment of gaging stations in the main stream and on all important tributaries, the systematic collection of gage-height records, and a sufficient number of discharge measurements at each station to define accurate rating curves. Where data are available bearing on cross-sectional and surface areas of the main stream at different stages of flow, such data are of value in estimating mean velocities and time intervals of flow at these various stages. Although usually less important, similar data for some of the major tributaries may be very useful in estimating periods of flow concentration for these tributary drainage areas. Where such data are not obtainable, velocities and time intervals of flow may be estimated from float measurements or, in very small streams, through the use of strong dyes.

The purpose of the laboratory survey is to show actual conditions of pollution, with their seasonal and other variations, in the main stream and major tributaries, having in mind maximum and minimum as well as average conditions at various times and the periods of duration of the several conditions observed. This survey should be designed to show the quality of water above and below main sources of pollution and dilution (tributaries) and also the extent of natural purification occurring in longer stretches of the river comparatively free from the disturbing influence of pollution or dilution. Particular attention should be given to the quality of water at water-supply intakes and at points in the immediate vicinity of major sources of pollution, where conditions are more likely to be unfavorable to recreational uses of the stream and where the demand for such uses is ordinarily more widespread.

For purposes of sampling, fixed points in the stream at known range line intersections are preferable to random stations. In the wider stretches of channel, three points on each cross-section may be selected and river samples collected, ordinarily at mid-depth, after checking several times the variations in river-water composition as between surface, mid-depth, and bottom. Under usual circumstances, the quality of water in a stream varies more widely at a given time in a horizontal direction on a cross-section than it does vertically at any point in such a section, although where marked sludge deposits are present, exceptions to this rule may be noted.

The number and character of laboratory determinations made in connection with a survey may vary widely with the size and complexity of the problem. However, for ordinary stream-pollution control, certain minimum requirements should be met. A detailed study should be made of the numbers of sewage bacteria (the *coli-aerogenes* group) and of their variations from day to day, or week to week, at different points in the main river and tributaries, particularly at sources of public water supplies and in the local areas affecting them. These data preferably should be supplemented by parallel observations of bacterial plate counts at the same points and in the same samples, in order to provide a check on the trends of the *coli-aerogenes* observations. The methods used in these and other laboratory determinations should follow the standard procedures described by the American Public Health Association.

In addition to the bacteriological tests, which are the most sensitive indices of significant changes in the degree of pollution of a river, systematic observations should be made of the dissolved oxygen and bio-chemical oxygen demand (B.O.D.) of the stream water at every major sampling station, in order that progressive changes in the "oxygen balance" (that is, the relation of available oxygen to oxygen demand) may be measured. The oxygen balance serves as an index of the adequacy of dilution and natural purification to dispose of more immediate pollution and also as a record of the fitness of the stream for normal aquatic life. The river water temperature should always be recorded at the time of collecting each dissolved oxygen sample, in order to provide a basis for estimating the degree of oxygen saturation, which varies with the temperature. The B.O.D. test as now made ordinarily is carried out by incubating sealed samples of the river water (undiluted, as far as possible) at a temperature of 20° C for 5 days, and measuring the loss of dissolved oxygen by the sample under these standardized conditions. Under some conditions, such as of heavy pollution or sludge deposits, B.O.D. determinations on identical samples, with incubation periods of 1, 5, 10, and 20 days, may be desirable occasionally in order to check the course of the oxidation curve through its first or carbonaceous stage. In other cases, notably where evidences of nitrification in the river are apparent, extension of the test period to 40 days may be desirable, in order to show the course of oxidation through both primary and secondary (nitrification) phases.

The addition of other laboratory tests to the routine schedule of the survey will depend largely on circumstances and on the facilities available at field laboratories. As indices of advanced oxidation in a river, the determinations of nitrite and nitrate are essential, as they show the final state of the oxygen used up in the stabilization of nitrogeneous organic matter. Determinations of turbidity and alkalinity are useful in measuring, respectively, the effects of sedimentation and of dilution in the river, the latter mainly through tributaries.

Certain other laboratory tests may be of value where special conditions or requirements are to be met. If the hardness or incrustant content of a river is an object of special interest, these determinations, together with those of calcium and magnesium, should be made regularly. Occasionally, the presence or absence of phenols in rivers from which water supplies are taken may be an important consideration. Similarly, iron and manganese are frequently matters of concern. In the streams of New England and the Southeastern States, the presence of dissolved coloring matter is often significant. In some rivers of the Western States draining lead and copper mining regions, the search for quantitative evidence of these toxic metals may be a matter of predominating importance in a laboratory survey. In every particular case, due consideration must be given to the uses made of the river water and to the presence of undesirable substances in the water. This applies particularly to the presence of substances originating in certain industrial wastes. As indices of ordinary sewage pollution, the bacterial and oxygen balance determinations are of primary importance.

The third main division of the laboratory survey is that which has to do with the biology of a river. In this connection it is not essential to stream-pollution control that an extensive or detailed biological survey of a river system be undertaken, although it is highly desirable that a sufficiently thorough examination be made of the supernatant stream and also of the bottom deposits to establish a clear record of the relative prevalence of pollutional and of nonpollutional organisms in different river zones. As these biological forms are an essential link in the chain of natural purification and as they doubtless will undergo marked changes in kind and relative numbers following the institution of sewage and industrial waste treatment, it is particularly important that these changes be recorded as a significant portion of the history of a river during and following the period of active stream sanitation measures. A fairly thorough biological survey before these measures are instituted and another after they have been completed will be well worth the time and expense required. In some instances, it will be advisable to maintain a more or less continuous check on biological conditions in a stream below major sources of pollution during the progress of initial corrective measures, in order to reveal any marked changes occurring in the balance of aquatic life and, where water supplies are involved, to furnish evidence of any tendencies toward the development of filter-clogging or taste- and odor-producing organisms which may impose an additional burden on water purification systems.

CRITERIA FOR PERMISSIBLE LIMITS OF POLLUTION

Although field and laboratory surveys will show the river zones in which conditions are either definitely satisfactory or otherwise for various water uses, they usually fail to reveal the limiting boundaries which lie between these two conditions; that is, in the rare instances in which these boundary conditions may prevail, the indications with respect to suitability or unsuitability for various water uses may be somewhat indefinite. If full utilization is to be made of the dilution and natural purification capacities of a given river system in undertaking a systematic program of stream pollution control, it is essential that some definite limiting criterion with respect to allowable density of pollution be adopted, tentatively at least, in order to provide a guide-post for correctional measures.

In general, the particular criterion, or set of criteria, adopted will depend on the more important uses made of a river in a particular zone, such as water supply, recreation, commercial fishing, industrial requirements, etc. In general, the suitability of a river as the source of public water supply is the predominant interest to be served. In many river zones from which water supplies are not ordinarily taken (as in the zones immediately below sewered communities), the rapidly growing pressure for enlarged recreational uses has brought to the forefront many problems of local sanitation affecting the use of streams for boating, fishing, camping, and even bathing. In relatively clean rivers now devoted principally to recreational uses, the question of preventing future impairment for such uses is a vital one in many States, affecting not only the interests of riparian dwellers, but also those of an increasing number of people from all parts of the country, whose annual migrations to these water centers in quest of rest and change are rapidly assuming the proportions of a national movement.

The results of an extensive study carried out by the United States Public Health Service in 1923–1927, led to the conclusion that where the sewage pollution of sources of public water supplies does not exceed that which is represented by a *coli-aerogenes* bacterial number of 5 000 per 100 cu cm as a rounded figure, purified water supplies conforming in quality to the primary requirements of the Treasury Department drinking water standard should be consistently attainable with the aid of efficient modern water purification systems. This figure would provide a fairly simple criterion for limiting river or lake pollution at intakes from which public water supplies are withdrawn. If taken as a yearly average, this figure would be scaled downward to some extent, depending on the character and variability of pollution at the particular water source considered, in order to ensure conformance of the purified water supply to both primary and secondary requirements of the Treasury Department *B. coli* standard. Thus, it has been noted by the writer in a previous paper,[3] that an annual average *coli-aerogenes* index of about 3 000 per 100 cu cm would represent a safer upper limit of pollution for sources of water supply located along the Ohio River, and probably also along its major tributaries.

TABLE 1.—YEARLY AVERAGE *B. coli* NUMBERS IN RAW WATERS OF OHIO RIVER FILTRATION PLANTS, 1926–1935.
(From Filtration Plant Records)

Place	AVERAGE *B. coli* INDEX PER 100 CUBIC CENTIMETERS										
	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	Mean
East Liverpool, Ohio	24 900	15 500	13 300	4 610	10 100	11 700	10 400	1 840	1 220	8 750	9 420
Steubenville, Ohio	6 230	1 640	6 690	2 820	6 430	555	1 860	338	2 060	1 370	3 000
Wheeling, W. Va.	—	2 330	6 360	3 250	4 730	1 570	1 290	388	50	854	2 080
Marietta, Ohio	1 160	1 080	1 880	916	22 500	999	2 230	3 950	1 660	1 560	3 790
Huntington, W. Va.	446	327	128	72	32	33	63	2 430	1 310	2 170	584
Ashland, Ohio	24 900	28 000	19 400	20 000	16 600	26 200	21 900	27 500	21 900	19 300	22 600
Ironton, Ohio	27 700	20 900	18 500	232	6 740	6 310	11 700	13 600	8 860	12 300	12 700
Portsmouth, Ohio	4 880	5 310	6 160	5 420	5 100	3 560	2 360	3 400	3 340	3 650	4 320
Cincinnati, Ohio	3 720	3 990	3 380	3 010	411	2 040	3 710	3 540	4 120	13 600	4 150
Louisville, Ky.	3 690	3 860	1 970	2 090	558	1 220	2 500	3 680	1 350	3 620	2 450

It is of interest to note in Table 1 the yearly average numbers of *coli-aerogenes* bacteria recorded during the 10-yr period, 1926–1935, in the raw water supplies of ten municipal filtration plants along the Ohio River.[4] If judged by the aforementioned criterion, the raw waters at East Liverpool, Marietta, Ashland, Ironton, Portsmouth, and Cincinnati, in Ohio, are shown to be definitely over-polluted, and the water at Steubenville, Ohio, has reached approximately the safe upper limit of average pollution. During the extreme drought year of 1930, conditions at all the cities named are shown to have been considerably aggravated, except at Cincinnati, where they were improved to a marked extent, owing to the effects of long storage and sedimentation in the relatively unpolluted and canalized 100-mile stretch of river above this point. Consistently, gross over-pollution is indicated at Ashland and Ironton, where the effects of heavy local pollution in zones immediately up stream are particularly manifest.

In setting the permissible limits of pollution in river zones from which public water supplies are not taken, the problem is largely one of fixing conditions satisfactory for maintaining freedom from local nuisance, supporting normal aquatic life, and, in some instances, keeping certain areas fit for bathing and camping purposes. For prevention of local nuisances, it is primarily desirable to eliminate organic sludge deposits, whether they originate in sewage or in certain kinds of industrial wastes. It also is necessary to maintain dissolved oxygen in the stream proper at all times, to prevent septic action with ensuing odors. In order to support normal aquatic life, which has a vital function in maintaining natural purification and in the general fitness of waters for recreational uses, a minimum oxygen content sufficient to support respiration in the higher aquatic animals, notably fish, is necessary. Some authorities recommend an oxygen minimum ranging from 2.5 to 5 ppm, with the additional requirement that the reserve oxygen supply present in a stream should always exceed the bio-chemical oxygen demand. A fair average for this dissolved oxygen minimum, under present general requirements, appears to be about 4 ppm. Organic sludge deposits should be eliminated not only because they interfere with the normal spawning process of fish, but also because these deposits impose a heavy oxygen demand on the overlying stream.

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[3] "Limiting Standards of Bacterial Quality for Sources of Purified Water Supplies," by Harold W. Streeter, *Journal*, Am. Water Works Assoc., Vol. 27, September, 1935, pp. 1110–1119.

[4] For convenient access to these data, the writer is indebted to H. R. Crohurst, Senior Sanitary Engineer of the U. S. Public Health Service.

As the prevention of nuisance and the maintenance of normal aquatic life in streams are closely inter-related, it would appear reasonable to assume that a single requirement sufficient to meet both conditions should be imposed in all river zones where the provision of recreational facilities for the general public and the preservation of other "amenities," such as the use of adjoining shore areas for residential purposes, may be involved. With the exception of bathing, which would require a relatively high bacterial standard of water quality, the simple requirement that the dissolved oxygen content should never be less than the oxygen demand, nor less than a minimum of about 4 ppm, should be sufficient to maintain favorable conditions, with the added proviso that all organic sludge deposits should be eliminated from the stream channel. It also may be noted in this connection that a stream conforming in all zones to a reasonable requirement for protection of sources of water supplies, such as that stated previously in this paper, would be brought more or less automatically within a favorable oxygen balance requirement such as the one just suggested. Neither of these two requirements would be adequately rigid, however, to meet even the more lenient of the various bathing water standards now being followed in different parts of the United States. It is, indeed, a debatable question as to whether under present circumstances the greatly added expense of restoring a highly polluted river to a condition safe for bathing throughout its entire course would be economically justified, although the institution of general corrective measures designed to protect sources of public water supply probably would result in the reclamation for these purposes of some areas remote from immediate sources of pollution.

From the foregoing discussion it thus appears that, in general, limiting pollution requirements may be fairly simple, consisting of the following: (1) Fixing maximum permissible numbers of *coli-aerogenes* bacteria in stream waters at public water supply intakes; (2) setting minimum oxygen and oxygen balance criteria in local areas below sources of pollution; and (3) working toward the practical elimination of all sludge deposits originating in sewage and in putrescible industrial wastes. In addition to these requirements, there would be certain special ones, applicable only in certain cases, such as the elimination of toxic, highly acid or alkaline, and taste- and odor-producing substances, not ordinarily removable by water purification processes. In the

Ohio River, acid wastes from mines and steel mills and phenolic wastes from by-product coke plants have presented major problems in this respect, although systematic efforts are being made to eliminate these sources of pollution, through mine-sealing operations and the diversion of steel and coke wastes at individual plants.

Surveys of the Ohio River

Owing to the size and importance of the Ohio River System and to the fact that it constitutes both a drainage channel and a source of public water supply for a large population, this river and its main tributaries have long been considered as presenting one of the most complex and difficult problems of stream pollution control in the entire country. Recognizing this fact, the U. S. Public Health Service, under authority of an Act of Congress passed in 1912, conducted in the years 1914–1916 a survey of pollution along the Ohio constituting probably the most detailed and comprehensive study of its kind ever undertaken, either in the United States or abroad. In connection with this investigation, a thorough hydrometric and laboratory survey was made at a series of sampling stations covering nearly the entire course of the river from Pittsburgh, Pa., to its mouth. During the summers of 1914 and 1915, a detailed sanitary survey was made of every incorporated community of more than 8 000 population in the entire water-shed.

The results of this survey, which were published in three reports,[5] revealed the main sources of pollution of the river, including both sewage and industrial wastes, the normal ranges of flow in the main stream and major tributaries, and certain definite zones of retrogression and of recovery in the density of pollution of the river. Zones of retrogression included the densely populated Pittsburgh-Wheeling Section, and stretches immediately below Cincinnati and Louisville. Zones of recovery, owing to dilution and natural purification, occur below Wheeling and in the long and relatively unpolluted stretches extending from Portsmouth to Cincinnati, from Cincinnati to Louisville, and from Louisville to the mouth of the river. The effects of acid pollution, derived from coal-mine and acid-iron wastes, were clearly apparent in the Pittsburgh-Wheeling Section, in stimulating precipitation of suspended material in the river and in exerting a well-marked bactericidal effect on sewage and other wastes discharged in that section.

A later survey of municipal water purification systems along the Ohio River, which was carried out by the Public Health Service in 1923 and 1924, indicated that the most highly polluted zone of the river from which public water supplies are taken extends from a point below Huntington to one just above Portsmouth, a distance of about 35 miles, in which the effects of sewage from Huntington are augmented by those of the combined wastes of Ashland and Ironton. Next in importance from this standpoint was the zone extending from below the mouth of the Beaver River, about 25 miles below Pittsburgh, to Steubenville, 35 miles farther down stream. In this section are the water intakes of East Liverpool and Steubenville, where increasing difficulties in producing satisfactory purified water supplies have been experienced during recent years.

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[5] *Public Health Bulletins Nos. 131, 143,* and *146,* U. S. Public Health Service, Washington, D. C.

In general, this water purification survey[6] indicated that: (1) Water filtration plants in the Huntington-Portsmouth Zone are clearly over-burdened by sewage pollution; (2) the plants at East Liverpool and Steubenville are likewise over-burdened during considerable periods and also have difficulty at all times in producing palatable effluents; and (3) those at Cincinnati and Louisville, although not seriously over-burdened, are in a marginal zone between safety and danger in this respect. Without the aid of effective and continuous chlorination, no water filtration plant located on the Ohio River, or on a large number of its major tributaries, would be able, under present conditions of pollution, to produce effluents safe for human consumption or domestic use.

In the 15-yr interval between 1915 and 1930, the completion of Government dams in the Ohio River brought that river to a state of full canalization during periods of low water. In order to ascertain the effects of this canalization, and of the natural increase in tributary population, on the state of pollution of the river, a re-survey of its mid-section (from above Cincinnati to below Louisville) was carried out in 1930–1931 by the Public Health Service.[7] This survey showed that with full canalization in effect during summer periods of low water, conditions at water intakes located in or immediately below long and unpolluted stretches of the river were improved, but that during periods of freshets, when accumulated deposits of sludge are flushed out of the channel, conditions were markedly worse than would have been expected under continuous open-channel flow. In river stretches extending below large centers of pollution, the main effect of canalization during low-water periods appeared to have been to move the critical zone of pollution-density up stream toward these centers, with a corresponding temporary improvement in conditions farther down stream.

In addition to the foregoing studies, several more localized pollution surveys confined to limited areas have been made in the Upper Ohio River, including a survey of the Weirton-Wheeling Section by the West Virginia Department of Health, in July and August, 1932; a survey of the Ohio, Allegheny, and Monongahela Rivers in and near the Pittsburgh District, by the Pennsylvania Department of Health (September, 1932, to February, 1933); and a more recent study in the Pittsburgh District, in 1935, known as the Metropolitan Main Drainage Survey.

As a basis of stream-pollution control, the data now available on the main river probably need extension in a few respects, and more detailed information is required on conditions in some of the more highly polluted tributaries, notably those draining into the upper section of the Ohio River above Cincinnati. In general, the following supplementary data would appear to be desirable:

(1) Revised estimates of total, urban, and sewered populations in various sub-divisions of the water-shed, brought up to date from the 1930 Census.

(2) Complete data on industrial wastes and on the present extent of sewage and industrial waste treatment.

(3) Revised information on present velocities and time intervals of flow in the river under low-water conditions with full canalization in effect.

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[6] *Public Health Bulletin No. 172*, U. S. Public Health Service, Washington, D. C.

[7] *Loc. cit., No. 204*, U. S. Public Health Service, 1933.

(4) Extension of local main drainage surveys along lines similar to those of the recent Pittsburgh survey.

(5) A complete survey of water surface areas and rates of atmospheric re-aeration in zones of the river below major sources of pollution and, more generally, in sections of the river in which the maintenance of a favorable oxygen balance is an important objective.

(6) A survey of the location, extent, and nature of organic sludge deposits in the river, particularly during periods of canalization.

The practical significance of Items (5) and (6) has been emphasized by the results of studies[8] made by the Public Health Service at its stream pollution research station at Cincinnati. It has been found that in stream waters polluted by sewage, or by underlying deposits of sewage sludge, the surface rate of atmospheric re-aeration (which is the most important limiting factor in determining the speed of natural purification) may be retarded measurably with sewage concentrations as low as 2 to 5%, which is within the range of local pollution density in many streams, including certain sections of the Ohio River. With higher concentrations of sewage (up to 25%), this retardation may reach 60% or more of the normal re-aeration rate. It also has been ascertained from the study that this retarding effect tends to be diminished by natural purification and also by artificial oxidation processes of sewage treatment, although it apparently is unmodified by simple chlorination. It has also been observed that the rate of oxidation of sewage sludge deposits under stream-flow conditions tends to be considerably lower than the corresponding rate for sewage matters in free suspension and solution.

The practical inference to be drawn from these results is that sewage pollution imposes a double burden on streams, by increasing the oxygen demand and retarding natural oxygen recovery; and that it is necessary to take into account both these elements in estimating the benefits to be attained by sewage and industrial-waste treatment in a particular river zone. The studies also point to the importance of sludge deposits as a burden on polluted streams and to the necessity of locating and eliminating these deposits at their sources, if a given stream is to be restored to a healthy condition.

Application of Survey Data

The applications made of survey data in formulating practical measures of stream-pollution control are so numerous and varied that any comprehensive discussion of them would extend beyond the limited scope of this paper. For the present purpose, a single example will serve to illustrate how such data may be utilized in studying a specific problem.

One of the important problems connected with the pollution of a large river system in which adequate protection must be given primarily to sources of public water supply, is the relative effects which various population groups discharging sewage into the river, either directly or indirectly, may have on pollution at down-stream points, notably those at which public water supplies are taken. In a river of the size and length of the Ohio, dilution and natural purification are dominant factors in the situation, although in this particular river the effect of acidity, chiefly from mine wastes, on conditions both in the upper river and at points down stream, is an important element to be considered.

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[8] Publication pending in *Sewage Works Journal.*

From the *B. coli* data given in Table 1, it is apparent that the density of bacterial pollution of the Ohio varies widely at different water intakes and that all the factors previously noted exert their influence. The data reveal little, except by inference, as to the relative responsibility of various sources of pollution for the conditions observed.

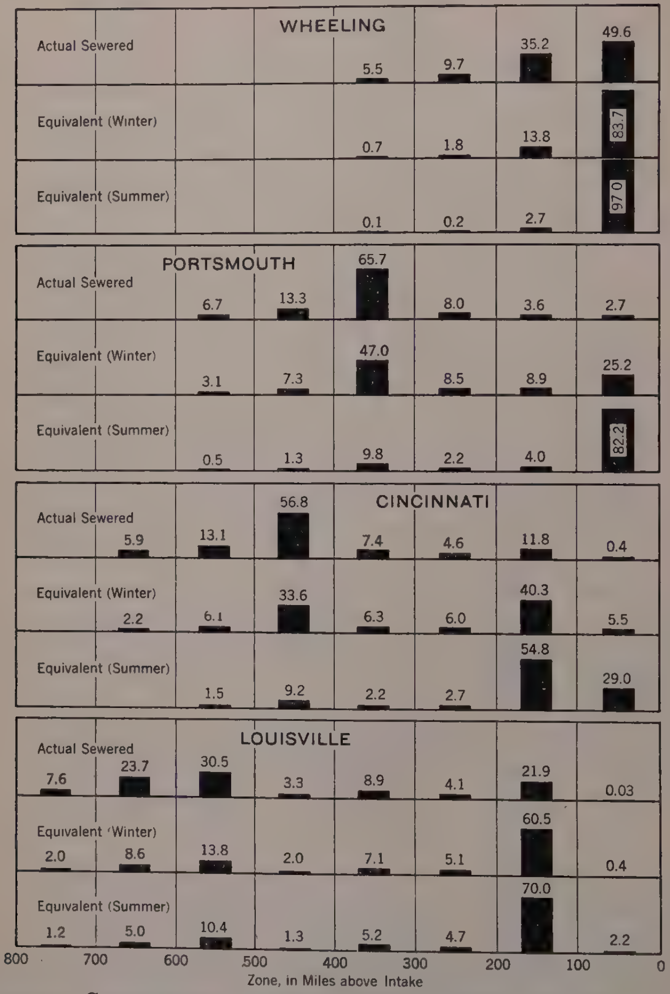

FIG. 1.—PERCENTAGE CONTRIBUTION OF UP-STREAM POPULATION GROUPS TO BACTERIAL POLLUTION AT FOUR WATER-SUPPLY INTAKES ON THE OHIO RIVER

In a paper given before the Society in 1927,[9] a method was described for determining the extent of such relative responsibility by "stepping down" the polluting effect of population groups located in successive river zones above a certain point in accordance with the observed mean time of flow from each zone to that point, and assuming that the reduction in the effect of each group would be measured along a time curve (varying with season and temperature) showing the rate of bacterial decrease in the river below sources of pollution. Curves of this kind were derived from extensive observations made by the U. S. Public Health Service in its 1914–1916 survey of the Ohio River, and were included in the paper mentioned.[9] By summing up the "stepped-down" population equivalents for all the river zones above a given point, a total equivalent population figure is obtained representing the population which, if it discharged sewage immediately above the point, would produce the same pollution effect as the actual population distributed in the various successive zones.

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[9] "Sewage-Polluted Surface Waters as a Source of Water Supply," by H. W. Streeter. (Presented at the Spring Meeting of the Society, Asheville, N. C., April 20, 1927. Not published by the Society.) Public Health Reports, June 15, 1928, pp. 1498–1522; Reprint No. 1232, U. S. Public Health Service.

In the present instance, a study has been made of the relative effects of sewered population groups draining into the river in successive 100-mile zones above the intakes at Wheeling, Portsmouth, Cincinnati, and Louisville. In Fig. 1 are shown the estimated actual sewered populations (as of 1930) in successive zones above each intake, expressed as percentages of the total sewered population above the intake. Directly beneath them are shown, for average winter and summer conditions, the relative effects of these population groups at each intake, expressed as percentages of the total "stepped-down" population-equivalent originating in the successive zones.

It will be noted that at Wheeling, a large proportion of the bacterial pollution is indicated as being due to the effect of the sewered population, aggregating about one-half the total above this point, draining into the first 100-mile zone up stream. Within this zone is included the population of the Pittsburgh District and the dense population draining into the Lower Beaver River. Passing down stream, the effect of this large population group is apparent at Portsmouth (300 to 400 miles above), at Cincinnati (400 to 500 miles), and at Louisville (500 to 700 miles), although in progressively diminishing proportion. At these three lower intakes, however, the predominant factor in the pollution of the river is shown to be the sewered population located within a river distance of about 200 miles up stream. Above Portsmouth and Cincinnati, the actual sizes of these groups are relatively small in comparison to those of the total populations draining above these points, being only 6.3% of the total at Portsmouth, and 12.2% at Cincinnati. Above Louisville, however, the effect of the Cincinnati District, located in the 100 to 200-mile zone up stream, is clearly apparent in the percentage of the total actual population in this zone (21.9%) and in its very large relative effect (60.5% of the total in winter and 70.0% in summer).

In order to obtain a rough idea as to the extent to which acid conditions in the Upper Ohio River above Wheeling might tend to modify the relative pollutional effects shown in Fig. 1, the same calculations were repeated, assuming reduction factors for all sewered population groups draining into the river above Wheeling. The factors assumed were based on the comparative results of measurements made by the Public Health Service at Pittsburgh and Wheeling in 1914, and at Cincinnati, Louisville, Peoria, Ill., and Chicago, Ill., in 1914–1916 and 1921–1922.[10] These measurements showed that because of acid conditions in the river, the measured effect of sewage from Pittsburgh and Wheeling on the *B. coli* content of the river was much less in proportion to contributing population than at the other cities noted, which were not thus affected and were considered as normal in this respect. The per capita contribution of sewage bacteria to the river in zones above Wheeling averaged 25.6% of the normal under winter conditions and 0.9% under summer conditions, these percentages being the actual ratios of the measured contribution at Pittsburgh to the mean of those observed at Cincinnati, Louisville, Peoria, and Chicago. In Fig. 2 are shown the results of this recalculation in comparison with those first illustrated in Fig. 1.

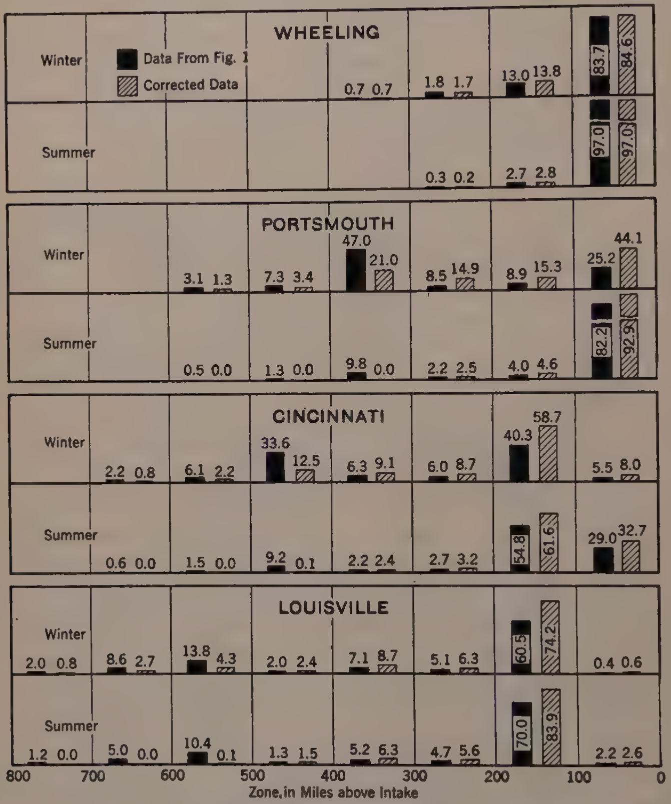

FIG. 2.—COMPARISON OF DATA IN FIG. 1 WITH CORRESPONDING DATA CORRECTED FOR EFFECT OF ACID WASTES DISCHARGED ABOVE WHEELING, W. VA.

Although the actual density of pollution in the river in terms of equivalent sewage-contributing population, was considerably lower when allowance was made for the influence of acidity above Wheeling, the relative effects of the several population groups were not modified to any large extent, except for a measurable reduction in the effect of the Pittsburgh-Wheeling Zone population group at Portsmouth, Cincinnati, and Louisville, as indicated in Fig. 2 by the smaller percentage of the total pollution originating in the 300 to 400-mile zone above Portsmouth, the 400 to 500-mile zone above Cincinnati, and the 500 to 700-mile zone above Louisville.

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[10] *Public Health Bulletin No. 143;* also, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), pp. 1366–1370.

In order to show quantitatively the estimated reductions in the total pollutional effects of the sewered populations draining into the river above the four intakes, Table 2 has been prepared, giving these totals for three assumed seasonal and flow conditions, both uncorrected and corrected for the influence of acid wastes above Wheeling.

TABLE 2.—ESTIMATED TOTAL SEWERED POPULATIONS IN 1930 ABOVE FOUR OHIO RIVER INTAKES AND THEIR CALCULATED EQUIVALENTS AT EACH INTAKE, ALLOWING FOR NATURAL PURIFICATION UP STREAM.

(Uncorrected and corrected for effects of acid wastes above Wheeling, W. Va.)

Intake	Sewered population above intake (estimated for 1930)	RESIDUAL POPULATION-EQUIVALENT AT INTAKE					
		Uncorrected for Acid Wastes			Corrected for Acid Wastes		
		Winter	Summer (high stages)	Summer (low stages)	Winter	Summer (high stages)	Summer (low stages)
Wheeling, W. Va.	2 186 500	569 150	391 280	167 080	155 800	3 680	1 465
Portsmouth, Ohio	2 846 500	150 790	34 960	18 630	86 680	31 700	16 480
Cincinnati, Ohio	3 152 400	148 850	36 010	12 040	102 320	33 430	10 700
Louisville, Ky.	3 814 900	178 820	19 200	6 420	145 670	17 640	5 370

As would be expected, the most marked effect of the acid wastes is shown at Wheeling under summer low-stage conditions, although it is also apparent at all four intakes under winter conditions. The enormous decrease in the total sewered population-equivalent at Wheeling resulting from the correction for acid effect indicates the large extent to which the discharge of acid wastes above this point reduces the bacterial pollution of the river originating in sewage discharged at Pittsburgh and other points up stream. Under summer conditions the effect of acidity in the upper river would appear, from the comparative values, to exert relatively little influence on bacterial pollution of the river at Portsmouth and below this point. Under winter conditions, some effect is apparent, although of relatively small magnitude.

In Table 3 the population equivalents given in Table 2 have been converted to their corresponding numbers of *B. coli* in the river, using average volumes of flow obtained from the 1914–1916 survey by the Public Health Service and assuming per capita contributions of *B. coli* as derived by Mr. J. K. Hoskins,[11] from observations made by the Public Health Service at the cities previously enumerated. For comparison with these computed values are given the winter and summer average numbers of *B. coli* observed at each intake during the years 1926–1929, inclusive, when river conditions were more stabilized than during the drought years beginning with 1930. With a few exceptions, the calculated *B. coli* densities in the river are shown to be of about the same order of magnitude as those actually observed although somewhat higher, on the average, than the latter, possibly because the calculation has been based on average flow conditions prevailing before the river was fully canalized.

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[11] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1367.

TABLE 3.—CALCULATED NUMBERS OF *B. coli* IN OHIO RIVER WATER AT FOUR INTAKES, BASED ON POPULATION EQUIVALENTS IN TABLE 2, COMPARED WITH SUMMER AND WINTER AVERAGE NUMBERS OBSERVED DURING THE YEARS 1926–1929.

Intake	CALCULATED *B. coli* INDEX PER 100 CUBIC CENTIMETERS						OBSERVED *B. coli* INDEX AVERAGE, 1926–1929	
	Uncorrected for Acid Wastes			Corrected for Acid Wastes				
	Winter	Summer (high stages)	Summer (low stages)	Winter	Summer (high stages)	Summer (low stages)	Winter (December–March)	Summer (May–November)
Wheeling, W. Va.	49 900	238 000	400 000	13 700	2 240	3 760	2 960	4 470
Portsmouth, Ohio	6 380	10 300	19 400	3 680	9 350	17 200	5 630	5 580
Cincinnati, Ohio	5 710	9 580	10 800	4 220	8 900	9 700	3 130	4 040
Louisville, Ky.	5 260	4 440	3 480	4 290	3 200	3 710	2 180	3 150

Making due allowance for this and other possible sources of error, the quantitative relationships shown in Tables 2 and 3 and Figs. 1 and 2 appear to be fairly representative of those which can be expected under present conditions of flow and distribution of the sewered population along the Ohio River proper and its main tributaries. Among their more striking indications are the following:

(1) Under both summer and winter conditions of temperature and flow, a large proportion of the total bacterial pollution of the river at the various water intakes probably originates in wastes discharged within about 200 river-miles above these intakes.

(2) Pollution originating at more distant points up stream is a considerably larger factor in the condition of the river at these intakes during the winter than in summer, although it appears to be outweighed at all times by pollution from more immediate sources.

(3) Acid wastes in the upper river do not appear to exert much influence on the bacterial condition of the river at Portsmouth or farther down stream, but are a powerful factor within and immediately below the Pittsburgh-Wheeling Zone. In that region, in the absence of sewage treatment, the present acid condition of the river undoubtedly has prevented excessive bacterial pollution of raw-water supplies and a consequent intolerable over-burdening of water purification systems.

From these indications it might be inferred that efforts to reduce over-pollution of the river at various water intakes, or to prevent such over-pollution where it does not now exist, would logically be aimed primarily at sources of wastes discharged within about 200 river-miles above these points and, secondarily, at more important sources, such as the Pittsburgh-Wheeling District, farther up stream. The present benefit received from the presence of acid wastes in the upper river, although it appears to be confined largely to that section of the river and points immediately below it, is of such magnitude that the removal of these wastes, or their substantial reduction, doubtless will bring about a serious over-burdening of water purification plants both in this upper section and possibly at some points below Wheeling, unless provision is made for extensive treatment of sewage, now discharged into the river, both directly and indirectly, at points above Wheeling.

The space limitations of this paper do not permit a similar analysis of survey data on oxygen balance requirements, although it may be pointed out that such an analysis probably would be somewhat more localized in its character and would deal more especially with summer low-water conditions in river stretches extending within and immediately below major zones of pollution. Although a considerable amount of basic data for such an analysis is available from the two surveys made by the Public Health Service in 1914–1916 and 1930–1931, and from more recent local surveys, these data probably need considerable extension along the lines previously indicated.

In conclusion, it may be desirable to add that before any final program of corrective measures in the Ohio River Basin is undertaken, thorough study should be given to the question of adapting degrees and methods of sewage and industrial waste treatment to the various requirements to be met in relieving over-pollution of the river in different zones. These methods, in so far as sewage is concerned, may range from sedimentation and simple chlorination to complete biological oxidation, with chemical treatment possibly falling into an intermediate position. Consideration also needs to be given to the possible effects of various kinds of treated effluents on the river below their points of discharge, particularly in pooled sections under summer conditions. These and a few other similar questions are of general significance and fall more especially in the field of research, which in the long run must keep pace with that of the sanitary survey, as the two are virtually inseparable in meeting engineering problems of stream-pollution control.

LOW-FLOW REGULATION BY PYMATUNING RESERVOIR

BY CHARLES E. RYDER,[12] ESQ.

SYNOPSIS

Pymatuning Reservoir, with a capacity of 8 365 000 000 cu ft, has been built in Northwestern Pennsylvania at a cost of approximately $3 700 000 for the primary purpose of regulating the flow of the Shenango and Beaver Rivers. A part of the project area is used as a Game Refuge, and there are opportunities for hunting, fishing, and other forms of recreation. These uses require careful evaluation of conflicting interests to determine how the project may be operated to serve best the needs of the region. Industries in the valley below require large quantities of water, especially for cooling purposes. The load on water-works filtration plants had been gradually increasing, and the State Department of Health was insisting upon the installation of sewage treatment plants to provide for the removal of settleable solids, oxidation, and chemical disinfection. Since the adoption of the reservoir plans, this high degree of treatment has not been required. Recreation interests demand that the reservoir be maintained at the highest possible elevation. Releases were made during the filling of the reservoir sufficient to satisfy the minimum industrial requirements. Since filling, tests under varying conditions of reservoir releases have been, and will continue to be, made at sampling stations established at a number of locations in the Beaver and Shenango Basins. The tests apparently show that the requirements for sanitation and public water supplies will be the criterion for releases, and a tentative operating schedule was adopted and used in 1937. Provision is made for flood regulation by the installation of flash-boards in the main spillway, and by maintaining the reservoir level below spillway elevation during the winter months. The reservoir was of material assistance in reducing flood heights during the floods of 1936 and 1937.

Pymatuning Reservoir (Fig. 3) is in the Beaver River Basin, at the headwaters of the Shenango River, in Crawford County, Pennsylvania, about 40 miles south of Lake Erie, 90 miles north of Pittsburgh, Pa., and 60 miles east of Cleveland, Ohio. When full, the reservoir covers an area of 26.5 sq miles, of which the greater part is the region formerly known as Pymatuning Swamp. It is 18 miles long, has a shore line of 70 miles, a maximum width of 2.2 miles, and an average width of about 1.5 miles. The maximum depth of water is about 35 ft and the capacity is 8 365 000 000 cu ft at the spillway elevation, 1 008 ft above mean tide at Sandy Hook, N. J.

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[12] Chf. Engr., Pennsylvania State Dept. of Forests and Waters, Harrisburg, Pa.

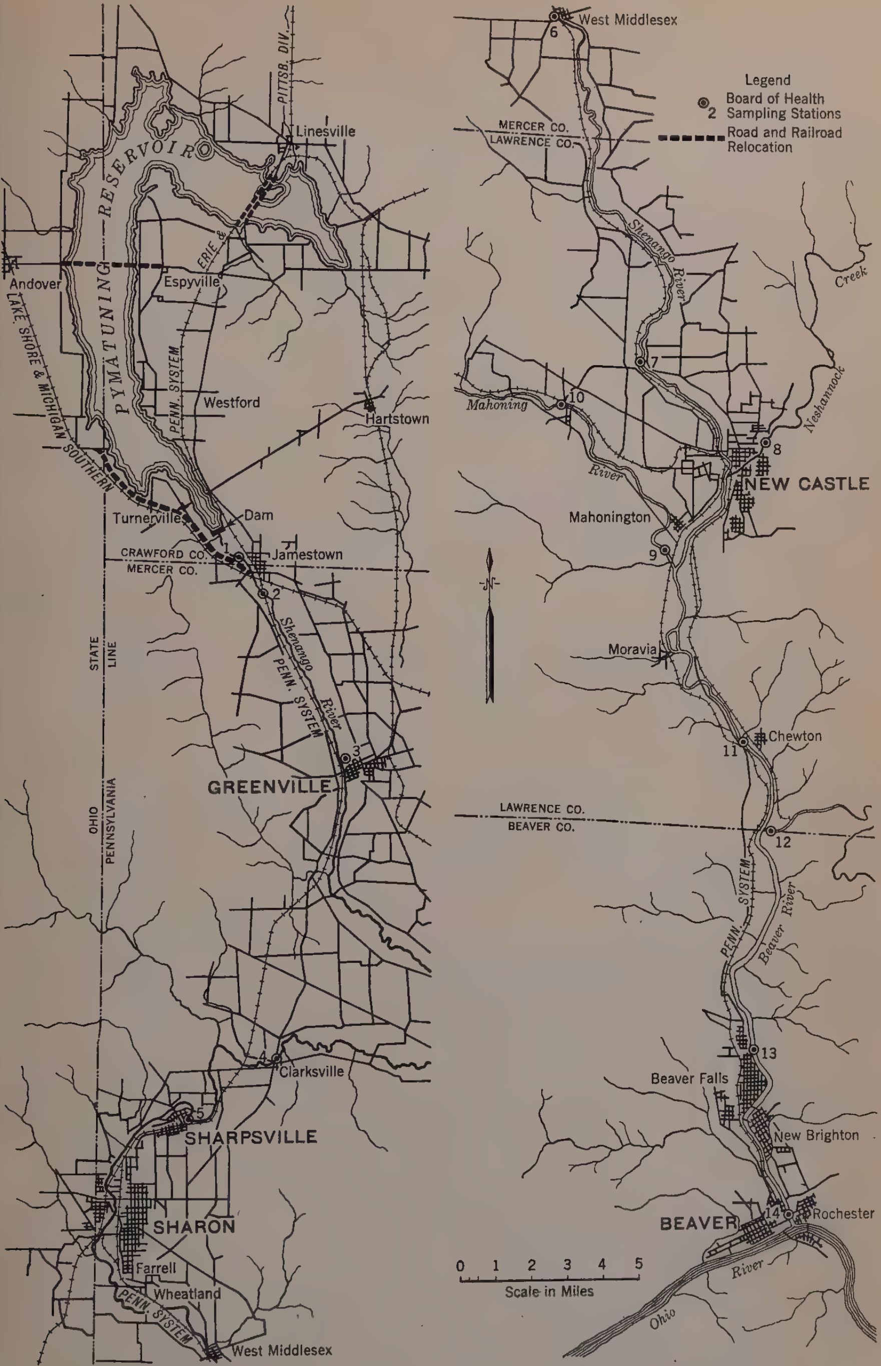

FIG. 3.—PYMATUNING RESERVOIR AND PARTS OF SHENANGO AND BEAVER RIVER VALLEYS

History of the Project

The completion of the dam in 1934 culminated a long period of interest in Pymatuning Swamp. In 1868 the General Assembly of Pennsylvania ordered a detailed study of the practicability and expense of draining the area and reclaiming land for agricultural purposes. A survey was made and a favorable report submitted, but nothing further was done at that time. In 1907 the Pennsylvania State Highway Department was authorized to make a survey and study for draining the swamp and improving the highways. The resulting plans were submitted to the Water Supply Commission of Pennsylvania for approval, and hearings were held at which objections were raised on the ground that the swamp was a valuable water-storage area which helped to maintain the dry-weather flow of the Shenango and Beaver Rivers. The hearings quickened public interest in a proposal to convert the swamp into an artificial storage reservoir for increasing the dry-weather flow of these rivers, and the Commission finally refused to approve plans for draining the swamp. The Legislature in 1911 passed an Act appropriating $10 000 to the Water Supply Commission, directing it to examine the feasibility of a reservoir project. The Commission reported to the Legislature of 1913 that the construction of a storage reservoir was feasible, and that such a project would result in accrued benefits to the State at large, as well as to the communities along the stream. The estimated cost, including the dam and spillway, highway and railroad changes, purchase of land and buildings, clearing the reservoir site, and improving the shores, was $1 556 402.

Following this report, the Legislature in 1913 passed the "Pymatuning Dam Act," which directed the Water Supply Commission to erect a dam at the outlet of Pymatuning Swamp for the purpose of establishing a reservoir to conserve the water entering the swamp and to regulate the flow in the Shenango and Beaver Rivers. This Act carried an appropriation of $100 000. During subsequent sessions of the Legislature, various laws relating to the project were enacted, most of them carrying appropriations in varying amounts but frequently containing provisions which made it impossible to use the money. The first land purchases were made in 1921, but actual construction work was not started until 1931, when the Legislature appropriated $1 500 000 for the construction of the Pymatuning Dam and Spillway.

There were three major construction items, each let under separate contract; (1) The dam and spillway; (2) an embankment to carry a highway across the middle of the lake; and (3) an embankment to carry the Pennsylvania Railroad and a highway across the lake near its upper end. This latter embankment was built to form a secondary dam, thereby creating an upper lake, 2 500 acres in extent, which is maintained at an approximately constant elevation of 1 010 ft above sea level, or 2 ft higher than the spillway of the main dam. This water area and the adjoining State-owned land have been set aside as a wild life sanctuary 3 670 acres in extent.

An area of more than 20 000 acres of land was purchased in Pennsylvania for the project, and more than 5 000 acres in Ohio—the latter by private interests, principally steel companies of the Shenango Valley. About one-fourth of the lake extends into the State of Ohio.

Cost of Project

The total cost of the project, as completed in 1934, was $3 717 739. In addition to the various State appropriations, $170 000 was advanced from Federal Reconstruction Finance Funds for use as a part of the clearing allotment which was completed with relief labor, and $415 000 was raised by the Pymatuning Land Company for the acquisition of Ohio lands.

Appropriations and donations were expended under the following general sub-divisions:

Surveys for feasibility of project		$ 10 000
Surveys, engineering investigations, and land appraisals		90 859
Land acquisition		1 101 756
Clearing Reservoir Site:		
State appropriations	$618 201	
Federal Reconstruction Finance Funds	170 000	788 201
Construction of dam		368 139
Construction of highway across reservoir		419 547
Relocation of railroad and highway		304 899
Relocation of State Highway Route No. 206		27 060
Reconstruction of Blair Road		1 894
Relocation of telephone lines		3 445
Reforestation of land around lake		5 745
Construction of measuring weirs on Sugar Creek and Shenango River		3 799
Improvements to buildings and grounds		6 538
Administration, supervision, and incidental expenses		170 857
Acquisition of Ohio lands by Pymatuning Land Company		415 000
Total		$3 717 739

Present Status

On December 5, 1933, the gates were closed in the upper dam and on January 23, 1934, the four regulating gates were closed in the main dam. The upper basin, with a storage of 448 000 000 cu ft at Elevation 1 010, was filled and overflowing on March 4, 1934.

The main regulating gates in the lower dam were operated to some extent during the summer months of 1934, 1935, and 1936. During 1937 the reservoir has been operated to regulate floods and to increase low-water flows in accordance with a tentative schedule. The quantities of water released during 1934 and 1935, while the reservoir was filling, were based upon the actual quantities necessary for the operation of the industrial plants at Sharon, Pa., then operating at less than full capacity, and were much below the amounts which will be required to insure the minimum flow necessary in normal times for water supply and for sanitation purposes.

By March 24, 1936, the water reached the spillway level for the first time, and seven days later the lake attained its maximum stage at Elevation 1 008.81, due to rapid run-off from rains and melting snow. On March 28, 1936, the gates were opened to release the flood waters and to provide additional flood-storage capacity. They were closed again from May 13 to May 25, since which date they have been operated continuously until November 2 for the purpose of increasing the flow in the Shenango and Beaver Rivers. In the early part of 1937, the reservoir was operated to control floods, and from June to November to maintain a low-water flow of at least 200 cu ft per sec at Sharon, in accordance with a schedule of operations tentatively adopted.

PURPOSE OF THE PYMATUNING RESERVOIR

The primary purpose of the reservoir, as stated in the original Pymatuning Act of July 25, 1913, and in later amendments, is to conserve the water entering the Pymatuning Swamp, and regulate the flow therefrom so as to maintain, throughout the year, as regular a flow of water as possible in the Shenango and Beaver Rivers. It was further specified that the reservoir, and the land surrounding it acquired by the Commonwealth, might be developed and used for fishing, hunting, game refuge, recreation, park, or other purposes; provided that such uses would not, in the opinion of the Water and Power Resources Board, materially interfere with the primary purpose.

The Water and Power Resources Board was vested with complete and final authority concerning the use and development of the land and water comprised within the project boundaries, and the maintenance and operation of the project. Later, it was provided that the Department of Forests and Waters, with the approval of the Water and Power Resources Board, might enter into agreements with other departments, boards, or commissions of the Commonwealth of Pennsylvania and the State of Ohio relative to the use of the reservoir and surrounding lands for fishing, hunting, game refuges, and other purposes.

It is clear that the mandate of the Legislature with respect to the operation of the reservoir was to store and conserve flood waters during the winter and spring months, and at other times when the natural flow was in excess of the needs in the Shenango and Beaver Valleys, so that such stored waters could be released to make the flow as uniform as possible during the summer and fall months, and at other times of deficient flow from the tributary drainage area. In other words, the Pymatuning Reservoir must be operated so as best to meet the needs of the valley below it for public water supply, sanitation, industrial water supply, navigation, water power, recreation, and esthetic purposes. In addition to such low-water control, it is equally certain that the Act provides for the regulation of the flow from the standpoint of the reduction of floods in the valley below the dam.

The Water and Power Resources Board must accordingly evaluate conflicting interests and determine how the project may be operated to best serve the particular needs of the valley. For example, to secure maximum flood-control benefits, sufficient storage capacity must be preserved in the reservoir to collect flood waters originating above the dam during the winter and spring seasons, and hold them back until the river down stream has receded to below flood stage. On the other hand, such dry storage capacity must necessarily reduce the total quantity of stored water which otherwise might be made available for release during dry seasons. It, therefore, reduces, to a greater or less degree, the efficiency of the reservoir as a regulator of the stream for water supply, sanitation, and other purposes. Fortunately, the water-surface area of the Pymatuning Reservoir is so great at spillway elevation that a storage depth of about 2 ft will afford complete control of the inflow under the worst known flood conditions. Hence, by providing such storage above spillway elevation the reservoir may be depended upon, with only a comparatively small loss in efficiency, to regulate the dry-weather flow.

Another very important phase of the project as a whole must be given consideration, namely, the use of the reservoir for fishing, boating, hunting, and other forms of recreation. For this purpose the lake level should be kept as nearly constant as possible. This requirement naturally is incompatible with releases during the summer season; hence, the reservoir should not be drawn down by releases more than is necessary to afford adequate stream-flow regulation. Although the Legislature was most careful to define the primary use of the Pymatuning Reservoir, nevertheless it must be borne in mind that the principal interest of the project to a large majority of the people in Western Pennsylvania and Eastern Ohio is the recreational value it will afford.

There are then three more or less conflicting purposes to consider in working out a schedule of reservoir operation. Fortunately, it appears from the studies made so far that the requirements for all three purposes can be met satisfactorily.

NEED FOR LOW-WATER CONTROL

There has long been a need for augmenting the low-water flow of the Shenango and Beaver Rivers. Prior to the completion of the reservoir the Sharon-Farrell District, 35 miles below the dam, with a population of about 45 000, had continuously been subject to shortages of water for domestic and industrial uses during dry seasons. The City of New Castle, Pa., about 20 miles below Sharon, with a population of approximately 50 000, and numerous other smaller intervening localities were likewise affected. The shortage was felt also in the Beaver Valley, where the cities and towns for a distance of 20 miles are rather closely grouped, with an aggregate population exceeding 50 000. The steel and other mills along both rivers require adequate water for economic and efficient operation. Periods of deficiency were a yearly occurrence and always curtailed operations at some of the mills. During dry seasons, water was re-used several times and as a result its temperature reached as high as 140° F. In the summer of 1933, the water was so low at Sharon that even by re-using it, there was not enough for operating purposes. The steel mills were considering closing down, when the small quantity of water that had been stored in the reservoir during construction was released and proved sufficient to tide them over the critical period.

Water-works taking their supplies from the Shenango and Beaver Rivers serve about 175 000 people, with intakes and filtration plants at Sharon, New Castle, West Pittsburgh, Beaver Falls, Pa., and New Brighton, Pa. Prior to the construction of the Pymatuning Reservoir, the Department of Health reported that the load on the water filters was gradually increasing. This condition was recognized as a menace to public water supplies, and was creating a nuisance in the Shenango and Beaver Rivers. At Sharon, flows as low as 8 cu ft per sec were experienced. Flows of less than 30 cu ft per sec were almost a yearly occurrence, and extended over periods of several months at least three times since 1930. The natural flow at New Castle was scarcely 20% greater than that at Sharon during drought seasons.

SEWAGE TREATMENT WORKS

Prior to the passage of the Pymatuning Act the Pennsylvania Department of Health properly insisted upon the installation of sewage treatment works by all municipalities discharging sewage into the Beaver and Shenango Rivers, or any of their tributaries, above the intakes of public water-works. The treatment plants were required to provide for the removal of settleable solids, oxidation, and chemical disinfection. The Department recognized, however, that with the Pymatuning Reservoir in operation, the releases during dry seasons would naturally improve the condition of the river water so that sewage-plant effluent could be safely assimilated with a lesser degree of treatment. Upon the basis of information furnished by the Water and Power Resources Board as to the amount of regulated flow which could be secured through the operation of the reservoir, it was decided that oxidation would not be required. Plans for sewage treatment works in various municipalities in the valley have accordingly been approved and works have been constructed that do not involve the high degree of treatment that otherwise would have been necessary.

LIMIT OF LOW-FLOW REGULATION

The original studies made to determine the maximum amount of regulation which could be secured through the operation of the Pymatuning Reservoir covered the period from 1908 to 1930, just prior to the actual construction of the dam. They did not include the series of dry years beginning in 1930. Within this period the most critical years were found to be 1922 and 1923, and during these years it was found that a minimum flow of 400 cu ft per sec could be maintained in the Shenango River, at Sharon, throughout the year without completely emptying the reservoir. However, to maintain this flow throughout the entire year would have meant a waste of water, as the full quantity was not necessary during the winter months. Accordingly, the calculations were revised to provide a minimum flow of 400 cu ft per sec at Sharon for the months of July, August, and September, 350 cu ft per sec through June and October, and 300 cu ft per sec for the remainder of the year.

Then came the drought years, beginning in 1930, and it was found that the reservoir, if operated on this schedule, would not have been fully replenished in 1930, 1931, or 1932, and that by November, 1933, the storage would have been exhausted.

As a result, a new analysis was made of the dependable yield which could have been secured from the reservoir had it been in operation for the critical period from 1929 to 1935. This investigation was based upon releases to maintain a minimum flow at Sharon of 200 cu ft per sec during the months of June, July, August, and September; 150 cu ft per sec during May and October; and 100 cu ft per sec during the remaining months of the year. With these releases the reservoir would have filled each year, and the maximum draw-down would have amounted to 3.9 ft. Corresponding computations covering the years 1908 to 1929 have not yet been made, but it seems probable that in normal years the fluctuation in the reservoir would hardly exceed 2 ft.

It is scarcely necessary to explain that the assumptions of minimum rates of flow to be maintained at Sharon are not to be construed as the actual flows which would have occurred throughout the period. The actual flow, particularly during the winter and spring months, would have greatly exceeded the minimum figures indicated, due to the higher rates of run-off from the tributary drainage area below the dam, and to overflow from the reservoir itself.

Tests to Determine Water Requirements

When the reservoir filled for the first time, in the spring of 1936, it became possible to release varying quantities of water and to study the resulting river conditions. Establishment of a definite program of reservoir operation has been delayed pending the completion of these tests, which are being conducted by the Engineering Bureau of the Pennsylvania State Department of Health. However, a tentative program was adopted in July, 1937.

Sampling stations were established at various locations in the Beaver and Shenango River Basins, as shown in Fig. 3. Then a series of test runs was made, with water released from the Pymatuning Dam to maintain as closely as possible a given specified flow at Sharon. The first four runs were as follow:

Dates of analytical traverses	Flow, at Sharon, in cubic feet per second	Release from dam, in cubic feet per second
June 3, 1936	111	88
June 15, 1936	215	140
July 17, 1936	240	200
August 27, 1936	490	93

The tests of the water made by the Department of Health included:

Alkalinity and pH: To determine the reaction of the stream before and after receiving industrial wastes, particularly pickling acid.
Dissolved Oxygen: To determine oxygen available to prevent nuisance by assimilating decomposing organic matter; also to determine whether stream was capable of supporting fish life.
Bio-Chemical Oxygen Demand: To determine the amount of decomposable organic matter present, and, by comparison with the results of the dissolved oxygen test, to determine whether the pollution load is greater or less than can be assimilated by the dissolved oxygen and still leave a safe oxygen balance.
Color: To determine effects of colored swamp water from Pymatuning Reservoir upon water-works.
Iron: To determine effect, other than oxygen depletion, of metallurgical industrial wastes.

The information given in Table 4, was furnished by the Department of Health with respect to sewage load and present methods of treatment in the larger towns in the Shenango and Beaver Valleys.

The studies indicated that the stream discharges tested were not large enough to provide sufficient oxygen in the diluting water to put the river in satisfactory sanitary condition below the sewage and industrial waste discharge from the Sharon-Farrell District and, at times, below New Castle and, again, at the mouth of the Beaver River; and in 1937, acting upon the advice of the Department of Health, a tentative schedule of reservoir operation was adopted to provide for a minimum flow at Sharon of at least 200 cu ft per sec, during June, July, August, and September; 150 cu ft per sec, during May and October; and 100 cu ft per sec, during the other months of the year. Further studies are to be made by the Department of Health.

TABLE 4.—PRINCIPAL SEWAGE LOADS IN SHENANGO AND BEAVER VALLEYS

Town or district	Population	Nature of effluent	Remarks
Sharon, Pa.	21 000	Treated	Removal of settleable solids
Sharon, Pa.	5 000	Raw	....
Farrell, Pa.	14 000	Raw	Primary sewage treatment works now under construction
In Ohio Near:			
Farrell-Sharon-Farrell, Pa.	3 000	Raw	....
Sharpsville, Pa.	5 000	Treated	Removal of settleable solids
Total equivalent population, raw sewage load	39 000	....	....
New Castle, Pa.	45 000	Treated	Removal of settleable solids
West Pittsburgh, Pa.	1 000	Treated	Removal of settleable solids
Elwood City, Pa.	12 000	Treated	Removal of settleable solids
Koppel, Pa.	1 000	Treated	Removal of settleable solids
Beaver Falls, Pa., and vicinity	30 000	Treated	Removal of settleable solids

At present it appears that the criterion for releases from the Pymatuning Reservoir will be the requirements of sanitation and public water supplies, and that if such needs are met, or met approximately, there will be more than sufficient water for industrial purposes. It also seems that the Sharon-Farrell District is the critical one with respect to sewage load and industrial wastes. The Department of Health is now taking steps to reduce materially both sewage and industrial pollution originating in this district.

The State has done its part in building the Pymatuning Reservoir to increase normal low flows in the Shenango River sufficiently to assimilate reasonably treated sewage and industrial wastes. The municipalities and industries should likewise be expected to co-operate in bringing about a reasonably clean stream, which will add materially to the attractiveness and economic advantage of the entire valley. It certainly should not be permissible for a small group of municipalities or industrial plants to continue to place a heavy pollution load upon the stream when, by taking reasonable measures to eliminate or reduce such loads, it would be possible to curtail greatly the releases and thus lessen the yearly fluctuation in the reservoir. There would seem to be too much interest by the public at large in the recreational use of the area for the State to countenance such gross and unwarranted pollution.

Indications point to a need, for sanitary purposes, of at least 200 cu ft per sec at Sharon and possibly more during the hot summer months. The requirements during the colder months will be much less, probably less than one-half the summer maximum. Industrial requirements for water for the Sharon District appear to be greater than for the New Castle and the Beaver Valley Districts, and these requirements during the most critical months of 1936 (June, July, and August) did not exceed 150 cu ft per sec and were closer to 100 cu ft per sec the greater part of the time. This is significant when it is recognized that those mills requiring by far the largest quantity of water were operating generally at 90% to 95% capacity during July and August.

FLOOD CONTROL

Although this paper is limited primarily to the effect of the Pymatuning Reservoir in regulating stream flow for dilution purposes, it would not be complete without some reference to the flood-control aspects of the project, as the operation of the reservoir for one purpose affects its usefulness for others.

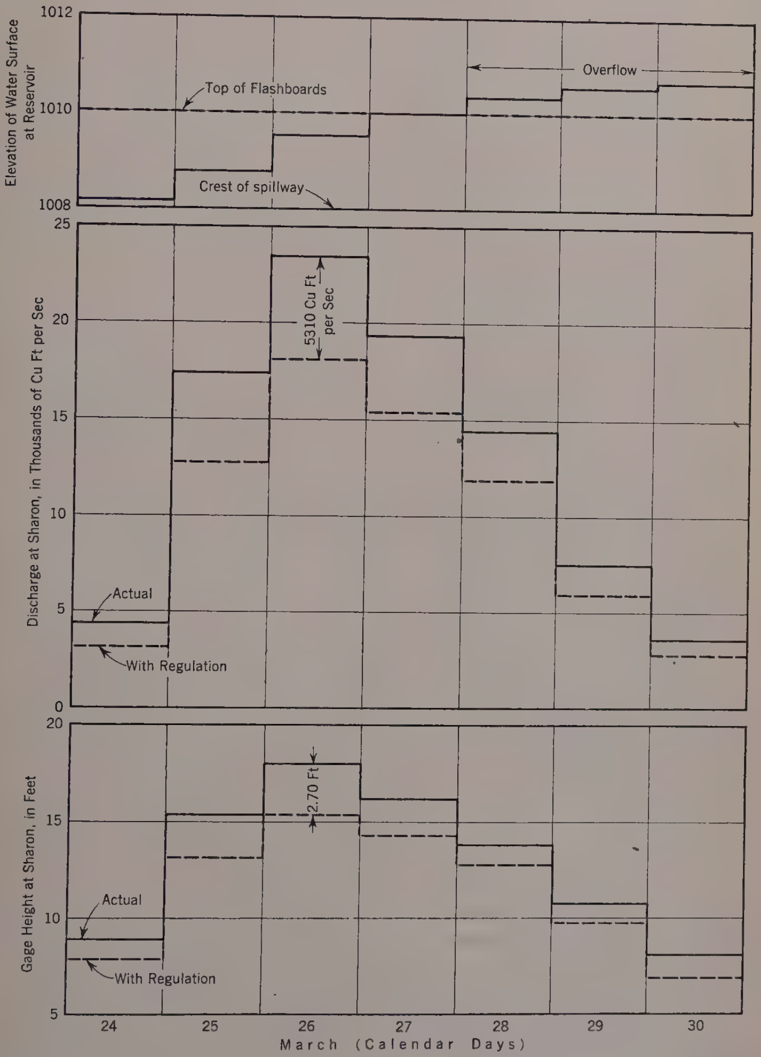

FIG. 4.—HYPOTHETICAL EFFECT OF PYMATUNING RESERVOIR ON MARCH, 1913, FLOOD AT SHARON, PA.

Thirteen disastrous floods have been experienced in the Beaver River Basin since 1806, the year of the first major flood of record. Little authentic information is available concerning them prior to 1884, although there is evidence that the flood of 1832 may have exceeded the two major floods of 1884 and 1913. This flood occurred before the valleys were densely populated and when few obstructions existed in and along the river channel.

The largest and costliest flood of recent years was that of March, 1913. Its magnitude and effect are comparable to that of the 1936 flood in other sections of Pennsylvania, and every city and town bordering the river was either completely or partly submerged. Bridges were swept away, railroad service was disrupted by numerous washouts, and dwellings, manufacturing establishments, and business places were inundated. Water-works were forced to shut down for several days, and on resuming operation pumped unfiltered water into the mains. Water entered gas mains and cut off the supply for fuel and light. The total reported loss exceeded $2 000 000.

The flood of March 17, 1936, did not extend to the Shenango and Beaver River Valleys, although other streams in the Ohio River Basin established new high-water records. Reports emanating from the valley below the dam credited the reservoir with preventing a disastrous flood at this time, but this was not true as the precipitation over most of the water-shed was in the form of snow, rather than rain. However, on March 25, a moderate flood occurred due to a warm rain and melting snow. The reservoir level was then 0.7 ft below spillway crest, and the storage capacity was sufficient to hold back the waters from the drainage area above the dam until the crest of the flood had passed Sharon. After the flood had subsided there was a small quantity of water passing over the spillway which, however, did not add to the flood height; later, the lake was drawn down gradually to provide additional storage in case another flood should follow. The reservoir reduced the discharge at Sharon approximately 25% and prevented overflow and flood damage in the business section of the city.

Studies show that 2 ft of storage above the spillway elevation of Pymatuning Dam would hold back flood waters from a storm equal in intensity to that of March, 1913 (the highest of record). Provision for such dry storage has now been made by installing a 2-ft weir in the spillway. The reservoir will be maintained at spillway level, or somewhat lower during the winter and spring months, by releases through the outlet gates. During flood flows the gates will be closed and the reservoir will then function to retard the inflow until the crest of the flood has passed out of the valley below.

The additional storage made available by the weir amounts to 1 297 000 000 cu ft, and would reduce the crest of a high flood at Sharon by 2.5 to 3 ft. Actually, the reservoir might be maintained about a foot or two below spillway crest during the flood season, and thus provide an additional factor of safety, without detriment to the project as a regulator of low-water flow.

Fig. 4 illustrates the effect the Pymatuning Reservoir would have had in reducing the March, 1913, flood, at Sharon, with a 2-ft weir in the spillway of the dam.

Benefits of Reservoir in 1936

During 1936, the first year of experimental operation, benefits derived from the reservoir at least met expectations as to the value and usefulness of the project. Tests of both its flood-control and low-flow replenishment features came quickly.

During the summer of 1936, the Beaver River and, especially the Shenango River Basin, experienced a very dry season. Although the drought was not so severe as that of 1930, nevertheless, it threatened to approach that condition and was, in effect, comparable to those of 1932 and 1934. Starting with a full reservoir, the gates were operated continuously from May 25 to November 2 to meet demands, primarily, in the Shenango Valley. During this period the average release was 100 cu ft per sec, and at no time was it less than 50 cu ft per sec. The average resultant flow maintained at Sharon was 175 cu ft per sec, and the lowest daily flow was about 100 cu ft per sec. Had there been no storage to help, the flow at Sharon could not have exceeded 30 to 40 cu ft per sec during the greater part of the time and would probably have been less over short periods.

Flood-control operations of March, 1936, have already been mentioned. Although the March run-off in that year failed to approach a record high stage, it was high enough to produce a secondary flood along many stretches of the Shenango River. Without the reduction in stage effected by the Pymatuning Reservoir, the damage in this valley would have been considerably greater than actually occurred.

At the end of 1936, consensus of opinion in the districts benefited could have been summed up in the expression, "We feel that the dam has already paid for itself."

Until the Engineering Bureau of the State Department of Health completes its investigations and submits its final report to the Water and Power Resources Board, no definite plan will be established for rates and periods of release from the Pymatuning Reservoir. Even after a plan of operation is adopted, it will probably be necessary to modify it as actual experience is gained from year to year, and as conditions change in the valley below with respect to public and industrial water supply, sanitation, etc. It is certainly to be expected that, under efficient regulation by the Sanitary Water Board, the pollution load will become no greater than it is at present, despite probable increases in population and industry.

SANITATION PROBLEMS INCIDENTAL TO FLOODS

By W. L. Stevenson,[13] M. Am. Soc. C. E.

Synopsis

In time of flood, public health authorities and municipal and water-works officials are confronted with a variety of emergency sanitation problems that demand immediate attention. It is important to be prepared in advance. This paper outlines the major factors in such planning, and makes suggestions for the conduct of activities during floods and as the waters recede. Experience in Pennsylvania in 1936 is drawn upon for many of the details.

Although flood-control reservoirs can reduce maximum flood stages, and dikes can safeguard more or less limited areas against inundation, it requires large sums of money and long periods of time to develop such projects. In fact, it is inconceivable that all rivers can ever be controlled so that flood stages will not occur at times. Therefore, preparation to meet the sanitation problems incidental to flood disasters must still be made.

For the purpose of this paper, the word, sanitation, simply means the keeping clean of the environment of Man or making it clean; and, in this sense, cleanliness means the absence of dirt resulting from Man's life and work.

Rivers draining developed areas must receive and carry away sewage, industrial wastes, and the rain-water run-off from towns and cultivated fields. Where sewage and some industrial wastes are inadequately treated, and the receiving stream at normal or low stages is too sluggish to be self-cleansing, then sludge deposits form on the bed of the river. Consequently, flood waters are always dirty, and more or less befoul inundated communities and water-works, thus creating a serious emergency sanitation problem.

The principles of sanitation are well known and in daily use. What creates the acute problem in sanitation incidental to floods and makes difficult the application of these principles, is the need for immediate action and the difficulties of transportation and communication. Seemingly everything must be done at once and everywhere at the same time, but floods prevent normal transportation and communication. Men and materials immediately needed in the stricken areas have to be taken over long detours because of inundated highways and railroads and washed-out bridges. The enormous load of business upon telephone and telegraph wires caused by flood conditions swamps their reduced capacity and makes it difficult even to ascertain the sanitary needs of the flooded areas.

The fact that floods inevitably produce these conditions makes apparent the importance of preparing and planning for preventive and corrective sanitary measures before the next floods actually arrive. This paper considers the problem in three steps: (1) Present planning and work; (2) things to be done during floods; and (3) things to be done as the waters recede. The suggested course of action is predicated upon the doctrine of home rule; that is, the primary responsibility should rest upon local authorities, with State agencies guiding and correlating activities and helping as far as possible, especially in the rural areas and smaller municipalities.

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[13] Chf. Engr., Pennsylvania Dept. of Health, Harrisburg, Pa.

PRESENT PLANNING AND WORK

Water-Works.—In recent years, the trend in public water supply projects has been toward the development of impounded, relatively clean, upland sources delivered by gravity, in preference to the use of polluted river water that requires a high degree of purification, and that must be pumped as well. Those in charge of water-works subject to danger from floods should study the practicability and advisability of obtaining upland water, so as to eliminate the flood menace to low-lying water-works, and simultaneously secure a superior water supply. Even with an upland source, however, care should be exercised to safeguard all the mains at places where they may be endangered by floods; otherwise, the areas served by such mains will be no better off than if they were served by a low-level source of supply.

In Pennsylvania, during the floods of 1936, most of the communities with upland gravity supplies not only had water during the flood, but also had enough water in storage for clean-up purposes afterward. The opposite condition occurred at many of the low-level water-works, which were flooded.

Adequate storage capacity for purified water is always needed to meet drought conditions. For this purpose the storage facilities of many water-works have been enlarged since the droughts of 1930 and following years. Nevertheless, during the March, 1936, floods in Pennsylvania, many storage reservoirs were exhausted before the emergency was over, despite the fact that they were well filled before the flood, and that restrictions were enforced to prevent waste while the water-works were shut down.

Officials of such water-works should now give attention to their storage facilities, and if they find them to be inadequate, they should take the needful steps to enlarge them as a safeguard against future drought or flood.

Of the 106 water-works adversely affected by the floods of 1936 in Pennsylvania, the following parts were put out of operation: Pumps in 61 plants; chlorination apparatus in 35; clear wells in 23; filters in 13; and chemical feed apparatus in 13. Inundated motors had to be baked, which caused delay in resuming operation. Flooded switch-boards took time to clean. The flooding of filters or clear wells was most dangerous.

Water-works officials should study their plants to determine whether the vital parts can be raised above maximum flood height. If raising, in whole or in part, is found to be impracticable, then consideration may well be given to providing substantial flood guards for doors and windows of low-lying pumping stations, which could be put in place when floods occur. The building of dikes might also be considered. However, if the top of a dike is not surely above maximum flood height, then after the flood the dike will retain the flood waters and cause further delay in getting the water-works back into service. A sluice-gate set at a low elevation in such a dike would be a help in draining impounded water.

Other matters to be considered are auxiliary sources of power, duplicate and well-protected water mains under rivers (with proper valves at each end to prevent loss of water in case of pipe failure), adequately valving the distribution system to provide flexibility of operation, and emergency connection with the mains of a near-by water system.

The doctrine of home rule applies to water-works. It is the duty of water purveyors to have competent works operators, capable on short notice to take charge of emergency work, and to handle their plants successfully when disaster threatens.

Municipalities.—In some municipalities with public water supplies the residents of areas subject to flooding still obtain their drinking water from private wells. In such cases, consideration should be given to extending the public water mains into the low-lying parts of the town site. Flooded private wells are a grave menace to health, and the best way to eliminate the hazard is to replace them with a safe, piped, public water supply. Meanwhile, the local board of health should prepare a map showing the location of all private wells in areas subject to flooding, so as to be prepared, when floods come, to direct their cleansing and sterilizing without delay.

No privies or cesspools should be permitted on properties to which a public sewer is accessible, for they are an actual or potential health menace at all times, and especially when flood waters carry the night soil into water wells and into houses.

In Pennsylvania, the clean-up of flooded dwellings in 1936 was markedly expedited by help from Federal Work Relief agencies, such as the Works Progress Administration and the Civilian Conservation Corps. It may be that when severe floods next occur, these valuable aids may not be available. Therefore, municipal officials would do well to make general plans for flood clean-up work and file them away, so as to be prepared to handle the job with their own facilities and labor. Then when the emergency comes they will be ready to proceed with a minimum of State supervision or aid. In cities such plans might include the establishment of refugee centers, first-aid stations, and emergency hospital facilities. It is not to be expected that the small municipalities can handle such a problem without help.

The State.—There are some services which the State Government can and should render in flood disasters. First, it should supply forecasts of the flood stage and time of cresting, in order that local authorities may be duly forewarned. Further, the National Guard can augment local police and completely take over police duties in the rural districts to prevent looting. The State Health Department can correlate measures for the prevention of disease and take charge of public health work in the rural districts and weaker municipalities.

Considerable planning should now be done along these lines, in order to be generally prepared for future floods. This preparatory work may include urging municipal and water-works officials to make suitable and practicable plans for meeting flood disasters in their own communities.

Hydrography.—After the March, 1936, floods in Pennsylvania, U. S. Army Engineers and the Pennsylvania Department of Forests and Waters in cooperation with the U. S. Geological Survey collected as much information as possible along certain main rivers as to the maximum height reached by flood waters and the day and hour of cresting. Levels were then run to ascertain the elevation of these marks. From these data maximum flood profiles were plotted and the time of transit of the crest height from place to place was determined.

Such information is so valuable in planning for the future that where it has not been undertaken as thoroughly as in Pennsylvania it should be begun at once. (Plans should also be in readiness, in case of future high floods, for establishing high-water marks on stable structures as soon as possible after the crest has passed.) In connection with this work it would be helpful to locate the horizontal limits which the flood waters have reached in built-up communities. With such data available, municipal and water-works engineers can make plans intelligently for changing existing structures or designing new ones, as far as floods are concerned.

The maintenance of stream gaging stations at strategic points is important. At stations that are difficult of access at flood stage, auxiliary gages should be established well back from the normal channel of the river in order to assure positively continuance of the record during the period when it is most needed for flood forecasting.

The value of accurate flood forecasting to health authorities, to water-works operators, and to the public at large, is so great as to warrant the development of a nation-wide service organized to the maximum possible degree of efficiency and dependability.

When the flood emergency comes, it is vitally essential that gage-height readings be reported promptly to the central authority. This indicates the wisdom of the special arrangements that have already been made with telephone companies in some States, to maintain telephone service as long as possible during floods between the gage readers and the central office of the hydrographers. Consideration is also being given to the use of radio for this purpose.

Health Departments.—State Health Departments should prepare for future floods by making plans for establishing, equipping, and manning first-aid stations in flooded districts, and for the distribution of needed biological products. Mobile bacteriological laboratories have proved to be valuable in the ordinary conduct of the business of State Health Departments, and they are especially valuable in flood-emergency work. Therefore, it would be well for State Health Departments not now provided with such facilities to endeavor to obtain them.

The number and location of Highway Department and National Guard mobile water tanks naturally change from year to year. It is futile, therefore, to prepare an inventory of such equipment and make definite plans, long in advance, for their use in hauling drinking water to stricken communities during a severe flood. However, where arrangements do not already exist between State Health Departments and such other branches of State Government for the emergency use of such equipment, it appears wise to make such arrangements in a general way, chiefly for the purpose of having immediate co-operation when the emergency comes.

Health Departments should also arrange with telephone companies for the maintenance, as long as possible during floods, of telephone service between the central office of the Department and the headquarters of important field agents, such as district engineers, medical officers, etc.

Things to Be Done During Floods

When it appears that floods are likely soon to occur, the State Health Department should notify all local Boards of Health as to their emergency duties, and give them advice on such matters as cleansing flooded water wells, privies, cesspools, and inundated dwellings and their surroundings. Particular emphasis should be laid on sanitation of milk stations, and warning the public to boil all water to be used for drinking purposes. Similar instructions should be issued to field medical officers of the Health Department as to their duties in rural districts where there are no local Boards of Health.

The Health Department Engineers should warn officials of water-works to store in reservoirs as much purified water as possible and take emergency measures to prevent, as far as possible, submergence of low-lying pumping stations and filter plants; also, where practicable, to make arrangements for temporary connections with the mains of near-by water systems less likely to be affected by the coming flood.

Industrial concerns are often large consumers of the public water supply, and if conditions justify, service to them should be temporarily discontinued. Householders should also be requested to save water, but hospitals and hospital laundries should not be restricted, as water is essential for care of the sick.

As soon as the State Hydrographers receive reports of rising river stages, the data should promptly be transmitted to the main office of the Bureau of Engineering of the State Health Department, whence it should be relayed to the Department's engineers in the field: First, for their information as to which rivers are rising toward flood stage; and second, in order that they may notify the low-lying water-works and communities of the coming danger. At the same time these field engineers should gather as much information as possible on river stages, from water-works or other sources, and report it back to the central office for prompt transmission to the hydrographers to augment their available data. As river stages approach flood magnitude the State Hydrographers should furnish to the Health Department Engineers their forecasts of flood-crest height and the time of cresting at various places. This procedure was used in Pennsylvania during the March, 1936, floods and proved to be of great value to the Health Department in preventing epidemics of disease.

When floods are imminent, but always before inundation occurs, every possible effort should be made by the State Health Department, by local Boards of Health, and by municipal and water-works officials to warn the public to boil all water to be used for drinking purposes. This can be done by means of newspapers, radio, printed notices posted in conspicuous places, and projected on the screen in motion-picture houses, and in all other ways that will receive the attention of the public. In foreign-language communities the warning should be given in the language understood by the citizens.

In recommending a universal warning to boil all water to be used for drinking, it is recognized that a seeming injustice is done to those public water supplies that are able to continue to furnish an entirely safe water during the flood. However, to qualify the "boil water" warnings by specifically including certain public supplies feared or known to be unsafe, and excluding other public supplies believed to be safe, would bewilder the public and lead to confusion thrice confounded. The universal warning was used in Pennsylvania in 1936 and is believed to have played a considerable part in the prevention of disease. There were no epidemics reported from the flooded area.

As soon as it can be determined which water-works are likely to be adversely affected by the coming flood, an inventory should quickly be made of available motorized water tanks for use in hauling pure drinking water to communities whose public water supply fails. These tanks can usually be obtained from State and municipal highway departments, the National Guard, and possibly from milk companies, and gasoline companies.

Where water must be hauled long distances, railroad tank cars may be used, but probably the actual distribution of such drinking water in the stricken town would have to be made by motorized or horse-drawn tank wagons. Every tank to be so used should be thoroughly cleansed and then disinfected with a chlorine solution or by steam under pressure. As a safeguard, it is wise to add a proper amount of a solution of chloride of lime to the drinking water in these tanks at the time they are filled, in order that the germicide may have adequate time to act before the water is served to the public.

Before the floods actually come, officials of municipalities and water-works should make arrangements with vendors of lime and chlorinated lime, so that when the emergency arises telegraphic orders may be placed for the immediate delivery of these disinfectants. The State Health Department should do likewise in order to insure quick delivery of such supplies to flooded rural districts and small municipalities.

Where properly trained persons are available, high-test chlorinated compounds may be used to advantage because of their more definitely known strength, lesser bulk, and better keeping qualities. However, if the chlorine compounds are to be handled by untrained persons (who are likely to use far more than is necessary), then the ordinary bleaching powder is preferable.

As the floods develop, and as soon as a reasonable forecast can be made as to the areas likely to be affected, the State Health Departments should station their mobile laboratories at strategic locations for the prompt bacteriological examination of public and private water supplies.

Things to Be Done as the Waters Recede

After a flood there are hundreds of things to be done, and all of them should be begun simultaneously. More men, materials, trucks, and tools are needed than can be furnished simultaneously throughout the flooded area. Then comes the test of previous careful planning to meet the emergency.

Officials of municipalities, water-works, and the State should, therefore, begin to plan for the clean-up just as soon as it can be forecast what places are likely to be inundated. Then rehabilitation can begin as soon as the flood crest is past. The major sanitary problems at that time are chiefly related to rehabilitating public water-works, cleansing milk stations and private wells, and cleaning up town sites.

Measures to Be Taken by Public Water-Works Officials.—The cleansing and rehabilitation of public water-works is the most important sanitary measure, for widespread disease will surely follow the drinking of a contaminated public water supply. It is not necessary to describe here the innumerable things that must be done to put flooded pumping stations back in service. In particular, however, it may be noted that filters and clear wells must not only be cleansed but sterilized—generally by means of a solution containing free chlorine. Chemical feed and chlorine apparatus which have been under water will need careful attention if they are to function properly when the remainder of the plant is ready for operation. When all these things have been done and the filter plant has been started, the water should be extra heavily chlorinated, and wasted for a while through fire plugs or blow-offs, to insure a good chlorine residual when it is again sent through the distributing system to the consumers. Meanwhile, field crews should be at work repairing broken mains, fire plugs, and house connections.

Speed in the resumption of service must not take priority over safety of the quality of water delivered to consumers. The warning to boil all drinking water should be continued after the flood until a sufficient number of samples of the water throughout the system, upon analysis, show the quality to be perfectly safe.

Cleansing Private Water Wells.—All mud, muck, and débris should be removed from the top of the well and its surroundings. It should then be pumped to waste and a strong chlorine solution applied and allowed to stand not less than 24 hours and preferably 48 hours. Where practicable, and especially where the well cover is not tight, the inside walls of the well should be scrubbed with a chlorine solution. After these things have been done, the well should again be pumped to waste until the ortho-tolidin test shows that chlorine is no longer present. A sample should then be taken for bacteriological analysis.

In this work mobile bacteriological laboratories are of great value. Following the 1936 floods in Pennsylvania, 10 000 samples of public and private water supplies were analyzed in the three mobile laboratories of the State Health Department and 2 000 in the Philadelphia Laboratory. This is about the same number as are normally analyzed during an entire year in the Department's Laboratory.

In cleansing private water wells it will be found that many of them were actually unsafe prior to the flood. After the flood is the psychological time to secure improvements in those found to be faulty in construction.

Milk Supplies.—No raw milk affected by the flood should be served raw. If a pasteurizing plant is flooded, the milk normally delivered to it should be diverted to some near-by unaffected plant the capacity of which can be increased by operating longer hours than usual. Often the employees of the flooded plant can work temporarily in the other one.

Flooded milk stations and pasteurizing plants must be thoroughly cleansed and all milk equipment, bottles, etc., sterilized. The plants should not be put back in operation until all the equipment is in good mechanical working order and all sanitary features have been checked by health authorities having jurisdiction over milk supplies.

The distribution of milk from feeding stations in the flood zone should be restricted to bottled milk.

Cleaning Up Towns.—Men, trucks, shovels, lime, chlorinated lime, and an orderly plan of procedure are needed to remove and dispose of the silt, mud, and débris left by the flood waters in inundated communities. Fortunate is the town whose water-supply reservoirs are full at this time so that an abundance of water under pressure is available for flushing.

Cellars must be pumped out, rubbish removed, and quick-lime applied. The devastation and filth in the rooms of flooded houses are appalling. Here cleansing had best be followed by the use of a solution of chlorine made from concentrated compounds or ordinary chlorinated lime.

In unsewered communities the flood waters will have destroyed privies and washed their night soil on to the surrounding ground. All exposed night soil should be sprinkled with quick-lime or chlorinated lime, according to local circumstances, and buried as soon as practicable at a location which will not menace any water supply. The prompt providing of temporary Army-type latrines is essential to prevent promiscuous defecation until permanent sanitary privies are rebuilt.

Suitable sites must be selected for dumps for the filth and débris hauled away from the flooded town. There is a strong temptation to dispose of such materials in the easiest way by dumping them into the nearest stream that may still be at high stage. If possible, this should not be done, as it only tends to befoul some place down stream, or leave the river banks dirty and littered when the stream returns to normal stages.

CONCLUSION

Flood emergencies try Men's souls and exhaust their bodies. Speed, sympathy, and service are essential. Above all else is needed co-operation and *esprit de corps*. The stricken people in the flooded areas must have confidence in those who are laboring to help and protect them. Of what avail is it for health authorities to warn that drinking water should be boiled and contaminated foods should not be eaten, if the public does not co-operate by heeding the warning?

With skill, zeal, and hard work on the part of those in authority, and co-operation on the part of the public, sanitation work in flood emergencies has prevented and again can prevent epidemics of disease which otherwise are sure to occur. Before the next flood strikes is the time to prepare to meet the sanitation problems it will produce.

PROGRESS IN POLLUTION CONTROL IN THE OHIO RIVER BASIN

BY E. S. TISDALE,[14] ESQ.

SYNOPSIS

Progress in the control of stream pollution in the Ohio River Basin is summarized from four angles: (1) Plans for sewage treatment; (2) handling of detrimental industrial wastes; (3) development of public opinion favoring water conservation; and (4) Federal laws and compacts.

Under Angle (1), a warning is sounded that something must be done, for in critical drought periods the sewage pollution is becoming too great a burden for even modern water purification plants to cope with. Records show that $12 000 000 have been expended under Public Works Administration and Works Progress Administration programs to improve sewage disposal along the main stem of the Ohio River.

It is pointed out under Angle (2) that co-operative programs between the United States Public Health Service and the States have overcome detrimental phenol pollution and are now reducing acid mine drainage from abandoned bituminous coal mines by air-sealing. The total daily acid mine drainage is estimated to be approximately 15 000 000 lb.

Under Angles (3) and (4) the gradually awakening public opinion is touched upon. The program started as regional water planning in the Cincinnati area. Compact agreements were proposed, and the consent of Congress was obtained for States to work together to reduce stream pollution.

The development of public opinion led to the formulation of definite legislation of a national character, upon which hearings were held. This legislation did not pass in 1936, but is being actively considered by the Congress in 1937.

To obtain a tangible concept of the canalized Ohio River, one may imagine a series of steps or boxes filled with water (Fig. 5). The difference in elevation between the top step, or box, located at Pittsburgh, Pa., and the lowest one, at Cairo, Ill., is 413 ft. In length, this series of fifty-three boxes, with a Federally operated dam in the lower end of each, extends more than 900 miles. Into this canalized river the tributary streams, large and small, empty their waters. The entire drainage area is 200 000 sq miles. It embraces parts of eleven States, having a combined population of about 17 000 000. Great industrialized cities have developed along the river and its tributaries. At the head-waters lie the richest bituminous coal fields in the world and from them comes an enormous pollution load of acid mine drainage.

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[14] Director, Div. of San. Eng., West Virginia State Dept. of Health, Charleston, W. Va.

The progress in pollution control in the Ohio River Basin may be gaged by the answers to four questions:

(1) What progress is being made in the reduction of pollution caused by human sewage from cities?

(2) What progress has been achieved in the control of industrial wastes?

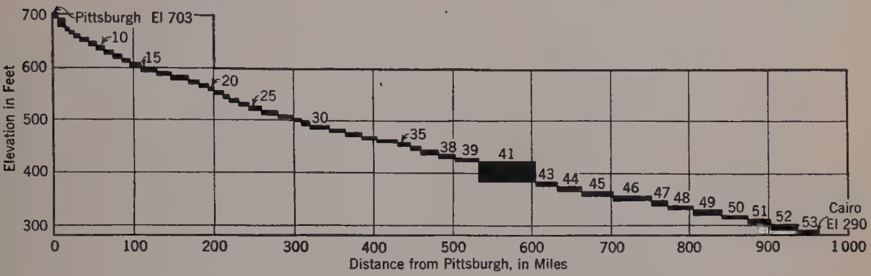

FIG. 5.—SLACK-WATER IMPROVEMENT OF THE OHIO RIVER

(3) What progress can be shown in the development of a "water-shed consciousness," involving co-operation of Federal, State, and local agencies, in conserving the waters of the basin for their varied and most useful purposes?

(4) What progress has been made which will permit co-operative effort of States and local governments in line with comprehensive Federal and interstate plans of pollution control?

PROGRESS IN SEWAGE POLLUTION CONTROL

The first of these "What Progress" questions deals with sewage pollution. It is the most important of them all, because more than 5 000 000 people obtain their drinking water from the water resources of the Ohio River Basin and nearly 2 000 000 from the Ohio River itself. The practice of the past has been for the cities on the Ohio River and its tributaries to conduct their sewage wastes directly to the stream without purification, and it is still the prevailing method in 1936. The cities have considered it their right to utilize the dilution of the large stream as a satisfactory method of disposal.

Although pollution from this source has been increasing, the science of water purification has gone forward, and thus far sanitary engineering has triumphed. Due to the design, construction, and skilled operation of water filtration systems, the public has been protected against water-borne diseases, notwithstanding the pollution of the river. Neverthcless, that there are limits beyond which streams cannot assimilate organic wastes, particularly human sewage, was amply demonstrated during the critical drought periods of 1930–1931, when it became impossible to produce a satisfactory drinking water supply from the putrefying waters contained in some of the basins between Pittsburgh and Cairo.

In January, 1936, the National Resources Committee, interested in developing a comprehensive policy to reduce pollution of the Ohio Basin, requested the Ohio River Board of Engineers (a co-operating group of State Health Department engineers who had been working on pollution control since 1926) to prepare a statement showing the number of persons using the main stem of the Ohio River as a source of drinking water supply, the status of sewerage and sewage disposal in the municipalities along the Ohio River, and a plan for the future betterment of present pollution. The engineers' report showed that the Ohio River itself was the source of water supply for public water-works serving 1 690 000 consumers, and this did not include Pittsburgh. An encouraging note in the sewerage summary in the report was the fact that, since 1932, in five States (Pennsylvania, Ohio, West Virginia, Indiana, and Kentucky) there had been approximately $12 000 000 expended toward construction, with Federal aid, of sewage collection and sewage disposal improvements. An estimate showed that approximately $50 000 000 more would be necessary to provide primary treatment for sewage from all municipalities on the main stem of the river.

There is certainly an element of progress visible in this $12 000 000 expenditure. Since Federal financial assistance has been available for building sewers on the loan-and-grant plan, 20% of the work necessary to afford primary treatment to sewage discharged to the main Ohio River has been finished. The policy of the Health Department in at least one of the States along the river is to give no more permits for extensions to city sewerage systems until the city in question agrees to develop a comprehensive plan for sewage collection and treatment. This is a step in the right direction and if all the States on the water-shed adopt such a policy, the program of reducing sewage pollution will be speeded up.

Among the general remedial measures on sewage pollution control set out in the Board of Engineers' report of January, 1936, was this statement:

> "Every sewered municipality of reasonable size should have a comprehensive plan of sewerage approved by the state health department and should diligently proceed within the limitations of financial ability to construct the intercepting sewers. The larger communities and those situate within harmful influence of waterworks intakes should be the first to install treatment works to effect at least efficient removal of settleable matter. Thereafter, but as soon as practicable, the smaller municipalities should do likewise."

The State health authorities in States along the Ohio River feel that it is becoming increasingly difficult to produce safe, palatable water under critical conditions even with a water purification plant of modern design and skilled operation. Critical periods have occurred in 1930, 1931, 1934, and 1936. Cases of intestinal disorders were noted in the basin in the fall and winter of 1930–1931, even though an acceptable bacteriological standard for water quality was attained. This experience should be a warning that the pollution load from sewage must be reduced, and that a remedial program similar to that carried out since 1932, through the assistance of Federal funds, should be accelerated.

Progress in Industrial Wastes Control

"What progress" with respect to improved practice in industrial wastes disposal is the question which immediately follows, for it is inextricably tied in with sewage disposal. Industries located in cities commonly discharge their wastes into the city sewers. Industrial wastes as well as drainage produced by the development of mineral resources, such as bituminous coal, must of necessity be discharged into the neighboring streams.

Constructive progress in dealing with the disposal of industrial wastes in the Ohio River Basin began about 1924, when the State Health Departments of Pennsylvania, Ohio, and West Virginia met in Pittsburgh under the leadership of the late Director of Health of Ohio, Dr. John E. Monger, and agreed to work co-operatively in handling industrial waste problems where they affected public water supplies. Under the Ohio River Interstate Stream Conservation Agreement, similar policies were adopted by the State Health Departments of these three States in dealing with "phenol," a tar-acid product of the by-product coke industry which was formerly discharged into the streams, causing offensive tastes and odors in public water supplies down stream.

In 1923, H. R. Crohurst, Sanitary Engineer with the U. S. Public Health Service, made a comprehensive study of the damaging effect of phenol on public water supplies throughout the Ohio Basin. Remedial action was taken by the steel industry in one State after another as a common policy was pursued, with the result that to-day in many plants, phenol is reclaimed by the steel industry; it has become a valuable by-product rather than a trade waste. Two cardinal principles of administrative practice in industrial pollution control are illustrated by this experience: (1) The establishment of a common policy over an entire water-shed area in dealing with a detrimental trade waste; and (2) the practice of having the technical man in industry meet the technical engineer in Government service to work out the practical way of eliminating or properly treating the harmful substances discharged into streams.

The experience of State co-operation in dealing with the phenol pollution served as a guide when the question arose some years later as how best to combat the tremendous acid load which was coming into the Ohio and its tributary streams from the bituminous coal mines in Pennsylvania, Ohio, West Virginia, and Kentucky. The Board of Health Commissioners of the Ohio River Basin, a voluntary co-operating group of State Health Officials, authorized the Ohio River Board of Engineers to prepare a comprehensive plan for reducing this drainage. A report was accordingly prepared by the Board of Engineers, at Huntington, W. Va., in October, 1933, and served as a basis for the program of sealing abandoned coal mines, which was inaugurated in December, 1933, and was under the direction of the U. S. Public Health Service. The workers on this project, started under the Civil Works Administration, were unemployed miners and mining engineers. With intermittent spurts and lags, the program has gone on, until now (1937) it is well organized under the Works Progress Administration. The supervisory group of mining engineers in each State is in the employ of the U. S. Public Health Service.

The public health engineers of the States involved co-operate closely with those in the Federal service. The work in each State is under the Sanitary Engineering Division of the State Health Department, for this is primarily a program affecting public water supplies and stream pollution.

The following summary of accomplishments in West Virginia up to July 1, 1936, is typical of the project as a whole:

Total number of coal mines in State	2 212
Active mines	613
Marginal mines	241
Abandoned mines	1 358
Abandoned mines sealed to July 1, 1936	405
Number of openings sealed	3 644
Average openings per mine	9

The unit costs were as follows:

Average total cost: Construction per opening	$ 89
Average total cost: Maintenance per opening	16
Total expenditure per opening to July 1, 1936	$ 105

From analyses of more than 2 000 mine-water samples, C. L. Chapman, Field Director for West Virginia, computes that by July 1, 1936, the acid drainage originally coming from the 405 air-sealed mines had been reduced by 51.1 per cent. The original daily acid production of these mines was 467 835 lb.

E. W. Lyon, Regional Director of the Mine Sealing Program, with headquarters at Pittsburgh, estimates that the total acid pollution from all mines (active, idle, and abandoned) in the Ohio Basin amounts to more than 15 000 000 lb per day. The acid load from about two-thirds of all mines—that is, the "abandoned" group—can be substantially reduced by a continuation of the present program. Again it should be pointed out that this work has been made possible by co-operative effort of the States and the Federal Government, and by Federal money apportioned on a regional basis. It is believed that the mine-sealing project is deserving of consideration in any program for water conservation in this basin.

Progress in Developing a "Water-Shed Consciousness"

The third of the "what progress" questions involves something more or less intangible—the development of a "water-shed consciousness." Progress along such lines, of course, cannot be defined quantitatively. It can be stated, however, that "water-shed pollution consciousness" in the Ohio Basin was first aroused in 1924 by the water conservation agreement between the Health Departments of Pennsylvania, Ohio, and West Virginia; and mention may be made of two more recent developments along similar lines.

The Ohio Valley Improvement Association, organized in 1893, has been interested primarily in navigation, but in 1935 it took for the theme of its Annual Meeting the control of pollution and floods on the Ohio River. This organization has been successful in securing the canalization of the Ohio River for navigation, and it is hopeful of being equally successful as it turns its energies to pollution and flood-control measures. Its members, scattered far and wide over Pennsylvania, Ohio, West Virginia, and Kentucky, form the nucleus for an appeal to the citizens and the legislatures of the several States to co-operate in a program of pollution abatement along the Ohio River.

In addition to this organization is a new one, inaugurated in Cincinnati, Ohio, on May 12, 1935, through the untiring efforts of Alfred Bettman, Chairman of the Fifth District, National Resources Committee, and Hudson Biery, Chairman of the Stream Pollution Committee of the Cincinnati Chamber of Commerce. During the winter of 1935–1936 Mr. Bettman called together representatives from the State Planning Boards of Pennsylvania, Ohio, West Virginia, Kentucky, and Indiana to effect, if possible, an Ohio Valley Planning Commission, whose purpose would be to plan for the best uses of the waters of the Ohio Basin for the future. His inspiration came from the findings and recommendations of the report of the Mississippi Valley Committee. A permanent organization was effected on March 12, 1936, with Mr. Bettman as Chairman, with headquarters in Cincinnati. As Chairman of the Stream Pollution Committee of the Cincinnati Chamber of Commerce, Mr. Hudson Biery has been successful in bringing a large number of influential civic organizations in Cincinnati to work in a united effort for an Ohio Valley Planning Commission and, for the enactment of national legislation by the Congress to permit groups of States on a water-shed to co-operate and to formulate a comprehensive plan for dealing with detrimental stream pollution.

Progress in Legislation

The fourth "what progress" inquiry deals with legislation: What progress has been made toward increased Federal and State co-operation in stream-pollution control in the Ohio Basin?

The Cincinnati group realized that a compact must be prepared, adopted by the States, and approved by the Congress before the States could work together effectively on a remedial program. Authorization of a compact by Ohio River States to work together on stream pollution control was given by the Congress in May, 1936. Thus, the ground-work has been prepared for the States to appoint commissioners to negotiate a compact to be adopted by the several State Legislatures. This is no easy task, but it is the beginning of the actual program.

The Ohio Valley is not the only water-shed or basin in the United States where interest in stream pollution is high. During 1936, from New York State and Connecticut, as well as from the Ohio Valley, measures were introduced in the Congress providing for Federal control of stream pollution. Some of the bills in Congress, particularly the Lonergan Bill, were of a drastic nature, but those prepared by the Committee of the Cincinnati Chamber of Commerce were of a more moderate type. One bill provided for a Division of Stream Pollution Control in the U. S. Public Health Service, setting up the machinery for investigating water pollution and preparing a national plan by water-sheds, also for assisting State health departments to establish proper bureaus for stream-pollution control. Another bill provided for control of stream pollution by the United States Army Engineers. Hearings were held on all these bills during March, April, and May, 1936, but none of the measures providing for national control of stream pollution was enacted by the Congress.

The Cincinnati-Louisville group associated with Mr. Bettman has furnished the motivating force to bring to a focus before Congress all the happenings in the Ohio Valley with respect to stream pollution in the past decade. The hearings on the Vinson-Barclay Bill make out a strong case for a moderate type of control, placing in the U. S. Public Health Service the functions of investigation, planning, and recommending action but retaining to the States the active control of stream pollution. This appears to be the wisest type of procedure at the present time.

Other Factors Bearing on Pollution Control

Mention has been made of the Board of Health Commissioners, Ohio River Basin, which is made up of the State Health Commissioners of States in the Basin. Under authority of this voluntary co-operating group of public health officials, the Ohio River Board of Engineers has thus far carried out the technical work and studies incidental to reducing sewage and industrial pollution within the Ohio water-shed. Certain points were stressed in the engineering report dealing with sanitary conservation of the water resources of the Ohio River Basin made to Mr. Bettman on January 18, 1936, which are worthy of mention. The engineers listed the priority of uses of the Ohio River in general as follows: (1) Public health; (2) drainage; (3) navigation; (4) industry; and (5) recreation.

Among general remedial measures, two important points are stressed: "Interlocking factors" and "low-water control." The following quotation is from that report:

> "*Interlocking Factors:* Improving the sanitary condition of the Ohio River in a way best serving the public interests is not confined merely to the treatment of pollution, which, in fact, is only one of the elements of a comprehensive plan. In the past such problems were approached from a single viewpoint. But the Water Resources Board have clearly shown in their reports the interrelation of other factors 'to secure the greatest total benefit from the natural resource,' and, on the other hand, to make the cost of each component benefit less than it otherwise would be.
>
> "As to sanitation, the chief factors are (1) adequate purification of public water supplies, (2) the interception and treatment of sewage and industrial wastes, (3) reduction of acidity, (4) low water control, (5) construction and operation of navigation works, and (6) the malign or beneficial influence of tributaries.
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> "*Low Water Control:* Streams in a natural state are subject to a wide range both in stage and discharge because of the wide variation in rainfall. So far as sanitation is concerned the low flows of summer are the critical times for stream pollution of organic origin, because of lack of diluting water and high temperatures fostering decomposition. Ofttimes quite expensive treatment plants are required to prevent harmful pollution during such critical times. Therefore, means for augmenting these natural low-flows make possible economies in the treatment of pollution.
>
> "In accordance with Document 308, U. S. Army Engineers, and other authorizations of the Sixty-Ninth Congress, many studies have been made for flood control reservoirs and some of these have been designed for low water control, that is to say, for the retention in the reservoir of sufficient water to be released for augmenting the low flows of the stream below, which will be a material benefit to the sanitary condition of the streams. For example, the reservoir now being built by the Federal Government on the Tygart River in West Virginia is said to be intended to release three hundred and forty cubic feet per second of water from July to December. During extreme drought the unregulated Monongahela River below this reservoir has reached a condition of almost no flow. Where, in the public interest, it is practical so to do, flood control reservoirs in the Ohio River Basin should be designed to provide for low water control. One reason is the actual economies in the reduced cost of treating sewage and industrial wastes, and the other, and possibly higher but intangible

benefits, through the resulting protection to the public health. Low water control is an important factor in sanitary improvement of the Ohio River and its tributaries."

SUMMARY

Progress in stream-pollution control in the Ohio River Basin may be summarized as follows:

(1) Beginning with the co-operative movement between State Health Departments in the Basin in 1924, a "water-shed consciousness" was awakened. The most recent development is the organization of the Ohio Valley Planning Commission, which is working under the auspices of the National Resources Committee.

(2) New sewerage systems and sewage disposal plants to the value of $12 000 000 have been built with funds jointly provided by the Federal and City Governments in the period, 1933–1936. This represents about 20% of the total expenditure necessary to afford primary treatment to all sewage along the river.

(3) The U. S. Public Health Service and various States in the Basin are carrying on a joint program to reduce the tremendous pollution caused by acid drainage from abandoned mines. A reduction of more than 50% in acid drainage is reported from the mines already sealed. The present program is financed through the Works Progress Administration.

(4) Legislation was enacted by Congress in 1936 providing for a compact between States on the water-shed, with respect to pollution control and other water conservation measures. It is hoped that ultimately an interstate compact will be drawn up and ratified. Hearings have been held in Congress on proposed bills providing for Federal control of stream pollution, and it appears likely that a moderate bill, involving co-operation between State and Federal authorities, may be passed in the near future.

PLANNING FOR POLLUTION CONTROL AT PITTSBURGH, PENNSYLVANIA

BY D. E. DAVIS,[15] M. AM. SOC. C. E.

SYNOPSIS

Pittsburgh's rugged topography creates a complicated sewage disposal problem. The heavy acid load in the rivers, largely from mining operations, inhibits bacterial growth, prevents a major nuisance, and postpones the attack on the disposal problem. Primary treatment plants are indicated as meeting the requirements for a considerable period; and sites are available which will avoid the expense of costly interceptors and tunnels. River transportation can be used for conveying sludge to a central plant for final disposition.

Decided economies in first cost and in operation can be achieved if the 116 interested municipalities join forces. Public sentiment would probably support a "clean-up" program; but enabling legislation will be required.

Competent authorities have stated that the rugged topography of Greater Pittsburgh, or Allegheny County, Pennsylvania, and its multitude of governmental jurisdictions, create one of the most complicated sewage disposal problems in the United States. The Allegheny River flows from the north 30 miles, and the Monongahela River from the south 35 miles through the rugged and precipitous hills of Allegheny County to form the Ohio River, which flows westerly another 15 miles before leaving the County. Along these 160 miles of shore line are 116 separate municipalities with a total sewered population of approximately 1 200 000, representing one of the most highly industrialized sections in the country, sometimes called the "Ruhr of America."

Although most of these communities are sewered, there is comparatively little treatment of the sewage, most of which enters the stream in its raw state. To this untreated sewage are added industrial wastes with a population equivalent estimated by the U. S. Public Health Service to be nearly as great as the actual human population of the area. This pollution, at times, exercises harmful effects down stream, such that for considerable periods the bacterial counts at East Liverpool, Ohio, Steubenville, Ohio, and Wheeling, W. Va. (and in the Ironton-Portsmouth District, in Ohio, possibly affected in part by the sewage of Pittsburgh) are among the highest in the country. Commenting on this situation, H. R. Crohurst, Senior Sanitary Engineer in Charge, U. S. Public Health Service at Cincinnati, has stated:[16]

"Only by constant and efficient supervision of plant operation has it been possible to procure a water of the desired bacterial quality with the rapid changes in raw water. While it has been possible up to the present time to produce satisfactory water supplies from the upper part of the Ohio River in spite of the excessive pollution, for the reasons as outlined urban population and sewage pollution will in a comparatively short time increase to the point where it will be impossible, with all present known methods of water treatment, to produce at all times a water safe for domestic use in these downstream communities."

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[15] Engr. (The Chester Engrs.), Pittsburgh, Pa.

[16] Unpublished report.

Estimates indicate that a flow of approximately 9 000 tons of acid (as pure sulfuric acid) passes Pittsburgh daily, which causes economic loss to steel barges, pipe lines, concrete work, and other structures subject to acid attack. The Monongahela River is highly acid most of the time and the Allegheny River (up to Freeport, Pa.) much of the time during the summer.

It would appear, therefore, that there is a condition requiring early and urgent consideration; and that if steps are to be taken to clean up the Ohio River and its tributaries, Greater Pittsburgh, at the head of the river, should be one of the first communities to place its filth under control.

Early Pollution Studies

Although there is evidence that the general public is becoming increasingly interested in a general clean-up program, it has not yet evidenced its concerted intention to act. However, its representatives in the various State and National Health Departments have been alive to the problem, and as early as 1910 the Pennsylvania Health Department ordered Pittsburgh to present a report on a plan for disposing of its sewage. This report was prepared and presented in 1912, and was followed in 1916 by the classic report of the U. S. Public Health Service on "A Study of the Pollution and Natural Purification of the Ohio River." Again, in 1931–1932, this Service co-operated with the Health Departments of the various States along the river in the conduct of a similar co-operative study of pollution, the results of which are not yet published. In February, 1936, the Ohio River Board of Engineers enunciated the following policy:

> "Every sewered municipality of reasonable size should have a comprehensive plan of sewerage approved by the State Health Department and should diligently proceed, within limitations of financial ability, to construct the intercepting sewers. The larger communities, and those situated within harmful influence of water works' intakes, should be the first to install treatment works to effect at least efficient removal of settleable matter. Thereafter, but as soon as possible, the smaller municipalities should do likewise."

When, in 1933, Federal funds were being made available for various local projects, W. L. Stevenson, M. Am. Soc. C. E., Chief Engineer of the Pennsylvania State Department of Health, urged on the officials of the City of Pittsburgh the desirability of a study of its sewage disposal problem. As a result, the Metropolitan Drainage Survey was initiated under the general direction of Charles M. Reppert, M. Am. Soc. C. E., Chief Engineer of the City of Pittsburgh, and continued under his successor, Henry D. Johnson, Jr., Assoc. M. Am. Soc. C. E. Although the population embraced within the City of Pittsburgh proper represented about one-half that in Allegheny County (or Greater Pittsburgh), it was realized that it would be inexpedient for the City to attack its problem independently, without knowledge of its effect on the

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county-wide situation; and that in any case it should have a picture of the problem in its entirety and the City's relation thereto. Consequently, the study covered the entire County or Metropolitan Area.

The report of 1912 had not been wholly fortunate in its reception, since it had left the impression that the costs of sewage disposal were excessive. Its study was based on "considering only those methods which will be reasonably efficient and certain in stopping the passage of infectious matter." Estimates were prepared for separate sanitary sewerage systems for all river communities within Allegheny County (above Neville Island, Pa.), and for long interceptors and tunnels terminating in a pumping station delivering to a central complete treatment plant on Neville Island, about six miles below the "Golden Triangle." The total cost for Pittsburgh was estimated at $37 300 000, and for the County as a whole (above the Island) at $57 000 000, which, in terms of present-day prices, would be at least doubled. Naturally, this huge cost imposed an effective brake on affirmative action.

The Metropolitan Drainage Survey

Since 1912 material changes have altered the aspect of the problem. The use of combined sewers in handling dry-weather domestic sewage flows is better understood and more commonly practiced; the rivers are more completely canalized; Neville Island is now largely occupied by industry, and hence not available as a sewage plant site; acid waste discharge has increased; and the Health Department Engineers now consider simple primary treatment of sewage as being generally adequate, at least as the first step in the clean-up program. These changed conditions counseled a completely fresh approach to the problem.

In assembling the fundamental design data for the new study, it was decided to adopt drainage basins as the basic units without respect to their relationship to administrative areas. The population within these basins was rather accurately determined from the small Census Districts, and from the past trend in each basin the population in 1970 was predicted; a correlation between water pumpages and sewer weirings in typical sewers supplied information as to probable sewage flows; and field surveys were made of possible plant sites and intercepting sewer locations.

Influence of Acid on Natural Purification of Streams

Concurrently, studies of the bacteriological and chemical conditions in the rivers were undertaken to determine the degree of treatment which might be required at different times of the year, and under varying conditions; as well as to develop a standard by which the degree of improvement provided by future plants might be measured. A problem special to this area was posed by the relatively large acid content of the streams and its marked effect on the process of natural purification in the rivers. It was found that the acid appeared to inhibit and postpone the normal processes. Although this action was locally beneficial, minimizing nuisances that might otherwise be expected to occur, it did not eliminate the offensive organic matter, but merely transferred it down stream for the resumption of normal processes in the reaches where alkaline conditions were restored in the river.

Naturally any change in the acid content by dilution water from the new flood-control reservoir on the Tygart River, or from others to be constructed later, or by acid elimination through mine sealing, might have a marked effect on nuisance conditions at Pittsburgh. A comparison between the Pittsburgh and the Cincinnati-Louisville observations, for 1914 and 1930–1932, yielded valuable information as to the effect of acid on the inhibition of self-purification, and of the degree with various concentrations. Except for the amount of acid and the quantity of flow, the conditions in these two districts are substantially similar, since the rivers are fully canalized and the effects of sedimentation in these enormous settling basins are somewhat alike. Table 5 indicates the marked differences in the condition of the river water in the two districts for low-flow periods.

TABLE 5.—Comparison of River Conditions Immediately Below Pittsburgh, Pa. (1932) and Cincinnati, Ohio (1930)

(Results expressed as monthly averages)

Month	Stream Flow, in Second-Feet		Methyl Orange Alkalinity, in Parts per Million		Five-Day Bio-Chemical Oxygen Demand, in Parts per Million			Dissolved Oxygen, in Parts per Million		*B. Coli* Index per 100 Cubic Centimeters	
	Pittsburgh*	Cincinnati	Pittsburgh†	Cincinnati	Pittsburgh†		Cincinnati	Pittsburgh†	Cincinnati	Pittsburgh†	Cincinnati
					Calculated	Actual					
September	1 610	4 830	3.4	66	35	0.2	3.5	0.8	2.2	345	160 000
October	2 940	3 360	4.5	78	19	1.2	4.0	4.9	2.94	237	415 000
November	18 900	5 570	10.7	72	3	3.1	5.5	10.9	7.58	8 970	1 441 000

* Seven miles below the confluence of Monongahela and Allegheny Rivers.

† Three miles below the confluence of the Monongahela and Allegheny Rivers.

The following quotation from a report[17] by Mr. Crohurst indicates what might be expected if the acid were largely eliminated so that the rivers at Pittsburgh would become alkaline:

> "With possibly twice the equivalent sewered population at Pittsburgh and much lower flows than at Cincinnati, it is believed that the dissolved oxygen would (in Sept. to Nov. 1932) have been entirely depleted in and below Pittsburgh for a considerable distance, if normal river conditions unmasked by excessive acidity had existed."

Under such conditions it is probable that the stench would be so vile that the public would then demand sewage disposal for relief. It seems fair to conclude that if acid is materially reduced during low-flow periods, nuisance conditions will unquestionably develop.

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[17] Unpublished report.

Effect of Sludge on Down-Stream Water Supplies

The U. S. Public Health Service report also emphasized the fact that the critical period for down-stream water plants is that immediately following a rise in the river with velocity sufficient to scour out accumulated sludge banks. At such times there is a precipitous increase in bacterial counts, and the pollution load on the down-stream plants becomes excessive. The minimum effective program of sewage treatment would be that afforded by primary sedimentation plants, which would remove the settleable solids now forming the sludge banks. At the same time this would prevent local nuisances, and remove the wave or "slug" of pollution which now moves down river with freshets. The general clean-up plan was based on the construction of primary plants as being adequate for present requirements.

Physical Aspects of Proposed Treatment Plan

The development of the physical aspects of the plan was largely dictated by the unusual topography of the district. Recalling the 160 miles of shore line, it may be realized that if an effort were made to concentrate the sewage at relatively few sites, the costs of interceptors and pumping stations would be enormously expensive. Most of the communities in Allegheny County extend up the hillsides back from the rivers, and their major sewers traverse the valleys of minor streams to their outlets in the rivers. Surveys revealed that sites could be found at or near the mouths of many of these streams, and that the problem could be solved by the construction of not to exceed thirty-six plants. Further study may show that this number can be somewhat reduced.

Primary treatment plants would incorporate mechanical screens, sludge grinding, detritors, plain sedimentation with mechanism for gathering the sludge, pumps, sludge digestion tanks, and provisions for chlorination, when and if necessary. At a comparatively small increase in cost, chemical treatment could be added and peak-load conditions met, should experience indicate the need. Sludge beds would be eliminated at the individual plants in the interest of economy in construction and operation, as well as for the conservation of expensive space. The digested sludge would be delivered by gravity directly to barges (similar to those now in use for transporting gasoline) at loading docks adjacent to the plants, for delivery to storage tanks at a central sludge disposal plant equipped with vacuum drying and incineration facilities. Ample storage would be provided both at the individual plants and at the central plant, so that the barge fleet could be removed from service during two or more months in the winter when the rivers might be partly covered by ice, and to permit vacations and fleet repairs.

Economies in County-Wide Program

The necessary interceptors, diversion and control chambers, pumping stations, treatment plants, barge fleet, and central sludge disposal plant, to afford primary treatment for the sewered communities along the three main rivers and their minor tributaries in Allegheny County, can be constructed for less than $20 000 000. The possible reduction in number of plants, by re-grouping, in the interest of reduced operating expenses, might affect this estimate; but in a general way it serves to fix the scale and magnitude of the task. Decided economies are made possible by approaching the problem on a county-wide basis; for the low-cost river transportation of sludge can thereby be taken advantage of, and the single sludge disposal plant can be made available at a fraction of the cost of equivalent facilities provided by each community on its own responsibility. Moreover, the cost of a few large primary treatment plants, each serving an entire drainage basin, is less than that of a multitude of small plants, each serving a separate community. The project lends itself to unified direction and group operation, making possible expert technical supervision that, on a shared basis, will be better in quality and lower in cost than would likely be possible to individual municipalities.

The studies indicate that the proposed method of treatment would increase the oxygen content of the river at all times and would remove the danger of an oxygen deficiency at Pittsburgh. (Such a deficiency existed for a short period during the 1930 drought.) It is believed that this degree of treatment should suffice for an extended period. If secondary treatment should later be found advisable, the primary plants could continue to function as such; and a number of secondary treatment works could be provided, each of which would handle the pumped effluent from a group of primary units. There are six prospective sites, scattered over the area, which, in their present state of development, would prove acceptable locations for such secondary plants.

Such an expenditure ($20 000 000) is not an unreasonable price to pay for sewage treatment at Pittsburgh, in view of the benefits that it would confer—the removal of foul local nuisances, the reinstatement of river recreation as a relatively safe diversion, and the "good-neighbor" service of improving the quality of the raw river water for down-stream communities. By way of comparison, it may be pointed out that the estimated cost of sewage purification is about four times the cost of the George Westinghouse Bridge and its approaches, and about one-fourth the sum that would be necessary for reproducing the water-works systems in Allegheny County.

The Problem of Stimulating Public Interest

It is one thing to indicate the nature and extent of a problem and the means to its solution, but it is quite another matter to bring it to execution. The need is present, and the practical question turns on the best way to achieve the desired clean-up. It is obvious that without the whole-hearted approval of the general public, little can be accomplished. The first job is to overcome the apathy of the electorate and to carry home the realization that civic morality requires that communities dispose of their waste products without endangering health, or occasioning undue financial burdens on their neighbors below.

Time was when the drinking of unfiltered water was the general rule, and water-borne disease stalked the land. The great majority of filtration and water purification plants in the United States have been built within the easy recollection of men now in their prime. When the public became convinced that the dreaded typhoid fever was a water-borne disease, potentially affecting the health of each member of the community, it did not require much argument to develop the conviction that pure water was cheap at any price. The small increase in the water rates was much less than the ever-possible doctor and hospital bill. This was personal knowledge and elementary financial common sense.

The case for sewage purification, however, is not made out so easily. Many communities are not moved to the building of disposal plants until threatened with damage suits by lower riparian owners; then it becomes a nice calculation as to whether it is cheaper to build a plant than to pay damages. Few communities treat their wastes solely for the altruistic purpose of bettering the river for the water plants of communities below. They have projects that are much nearer their hearts and pocket-books—school houses for their own children, better streets for their personal automobiles, and bridges for streams which they themselves must cross frequently, are tangible goods whose benefits are obvious to those who pay the bills. Until these pent-up demands have been at least partly satisfied, the voter can hardly be expected to be enthusiastic about the disposition of his waste products unless they assail his nose or threaten his bank account. However, there is much evidence to the effect that these insistent needs have now been pretty generally satisfied in most communities, and if (as is predicted), the growth in population should soon slow down or even cease, the usual outlets for public moneys may dry up, and funds may be available for projects previously regarded as luxuries.

Increasing leisure for healthful outdoor pursuits should stimulate interest in cleaner streams. Fishermen are not greatly interested in cleaning up streams in some other taxpayer's county, but if they could be brought to realize that fishing could be made good in their own vicinity, they might be more willing to loosen up on their own purse strings.

The rivers here under discussion flow through a territory peculiarly devoid of lakes. However, the Ohio River and its main tributaries are fully canalized, and during low-flow periods in the summer are to all intents and purposes long inland lakes, whose waters are usually quite clear. Under Health Board control, they would lend themselves admirably to water diversions of all kinds, were the conditions more propitious. Bass could be caught at Pittsburgh less than forty years ago, and if the conviction and fear of disease were not generally prevalent, it is probable that the streams would be alive with recreational enthusiasts. Wind currents are known to carry the sewage of some communities past their own water-works intakes, thus placing an undue load on water purification processes. These are all valid arguments for affirmative action, and there is good reason to believe that the public might be appealed to on grounds of its own self-interest as to the value of clean streams. The first job then is to convince the public of its obligations, the honoring of which will yield dividends in health and recreation.

The absence of fish in the Upper Ohio River is very largely attributable to acid wastes, mostly from abandoned mines. The responsibility for its removal is obviously that of the State and National Governments. The mine-sealing program now in progress in the Ohio River Basin gives promise of a material reduction in this highly objectionable waste.

Co-operative agreements between industries and State Health Departments have been effective in setting up research programs within the affected industries, looking to the profitable reclamation of by-products which formerly polluted the streams. These programs should be expanded, but, in addition, legislation similar to the Lonergan or Barclay Bills should empower the policing of recalcitrant industries. Preferably this clean-up should be on a broad geographical basis, so as not to impose unfair competitive burdens on industries complying in good faith.

Sewage Disposal a Municipal Responsibility

The disposition of human wastes, which represent the preponderant source of pollution, is squarely up to the various communities. It is apparent that in the Pittsburgh District, the most economical and only practical way of accomplishing this result is by bringing all the communities together in a co-ordinated body, such as a sanitary district. This will require enabling legislation, since the job of coaxing voluntary co-operation among 116 separate jurisdictions is humanly out of the question. A few communities have worked out joint compacts for the construction of intercepting sewers, but the negotiations covered an extended period and the method proved unwieldy in the extreme, and would not apply to Allegheny County as a whole.

In 1927, the Pennsylvania Legislature passed a bill permitting a "Metropolitan Plan" for Greater Pittsburgh, embracing Allegheny County as a whole. Had this been adopted, it would have provided for numerous projects which would lend themselves advantageously to joint effort in their execution. However, it failed of ratification on the two occasions when it was presented to the voters. The 1935 Legislature passed a Municipal Authority Act with the intent of permitting somewhat more liberal methods of financing than those now available, so that many governmental jurisdictions could participate in improvements by accepting the financial co-operation of the Federal Government. This would have permitted the issuance of revenue bonds and the assessment of charges for service rendered (such as sewer rentals), but the provisions as to new projects terminated in 1937, without any action having been taken.

There is no reason to believe that the Legislature cannot provide means for creating the necessary machinery for advancing a unified program. The minimum provisions should incorporate joint ownership of the facilities, power of compulsory assessment (preferably of sewer rentals) for construction and operation by the central representative executive group, the right to issue revenue bonds, and preferably a provision for administration on a Civil Service basis.

Conclusion

The Pittsburgh study indicates that the rivers in its vicinity can be cleaned up at a reasonable cost for initial construction and for operation. It is believed that on a fair presentation of the problem, the public may be prepared to sanction an adequate disposal of its sewage, and that the requisite legislation may be had to effect this result.

STREAM POLLUTION PROBLEMS AT CINCINNATI, OHIO

By J. E. Root,[18] M. Am. Soc. C. E.

Synopsis

The nuisances created by sewage and industrial wastes as they have affected the condition of the Ohio River have been under observation since 1908. From time to time remedial measures have been recommended. Due to the fact that the river for the most part is the boundary line between a number of States, a correction of the pollution problem will require the joint co-operative action on the part of these States, or a number of them. Complete canalization of the Ohio River further complicates the problem, as do floods and extreme periods of low flows. The paper for the most part deals with river conditions within the Cincinnati Metropolitan Area.

Pollution of the Ohio River has received almost continuous public attention since 1908, when the subject was first investigated by a Joint Ohio River Sanitary Commission composed of representatives from Pennsylvania, West Virginia, Kentucky, Ohio, and Indiana. The general conclusion of the 1908 and 1909 investigations[19] was "* * * that the Ohio River is generally of objectionable appearance, is frequently very turbid, carries high color in the fall, and must be purified before being used for public water supplies." The river water was found to be "grossly polluted at many points along its course," and it was stated that the pollution was becoming more pronounced and more dangerous to the health of the inhabitants of Ohio River cities. A resolution, adopted at a meeting of the Joint Commission on December 1, 1909, recommended that the various municipalities along the Ohio River using the river as a source of water supply take immediate measures to discontinue the discharge of unpurified sewage into the stream.

Early Investigations

For the City of Cincinnati, J. W. Ellms, M. Am. Soc. C. E., as Superintendent of Filtration at the Water-Works, in 1910 and 1911 investigated the condition of the Ohio River water from the water-works intake up stream from the city proper to Fernbank Dam, about 20 river-miles down stream from that point. As to the effect of the sewage from the Metropolitan Cincinnati area upon the general quality of the Ohio River water and the influence of the Fernbank Dam, when in service, upon the water, Mr. Ellms summarized[20] in part:

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[18] Director, Dept. of Public Works, Cincinnati, Ohio.

[19] Rept. of the Commissioner of Health, State of Pennsylvania, 1909, Pt. II.

[20] Rept. on Investigation of Ohio River at Cincinnati, Seventy-Second Annual Rept. of City Water Works Dept., City of Cincinnati, Ohio.

"The quality of the Ohio River water entering the Cincinnati intake is subject to considerable variation, showing the greatest degree of contamination during flood periods.

"In general, the water at the intake was of better quality during the period the Fernbank Dam was in service than for the most comparable period of low water in 1911 preceding its completion.

"An exception to the above general statement is required for a brief period during which the dam was in service, because of a sudden rise in the Little Miami River, which, in discharging large volumes of water into the Ohio, retarded the flow of the latter, and even produced a back flow of water toward the intake, temporarily causing an inferior and somewhat more contaminated water to pass into the intake.

"The present contaminated condition of the Ohio River water which passes into the Cincinnati intake, whether under normal or abnormal conditions, need not be feared by consumers so long as the present purification plant is carefully and efficiently operated, since the river water can be clarified and purified and rendered a perfectly safe drinking water by the processes employed."

During July, August, September, and the first week of October, 1911, J. A. Cravan, Water Chemist of the Indiana State Board of Health, made a sanitary survey of the Ohio River between the Ohio State line and the Illinois State line, a river distance of 360 miles. In general, Mr. Cravan's investigation[21] indicated:

"(*a*) That the dissolved oxygen in the river water is not progressively depleted from Cincinnati to the Illinois State line.

"(*b*) That the bacterial content of the water between the same points is not materially higher than at the Cincinnati Water Works intake.

"(*c*) That the river water soon recovers a substantial proportion of the dissolved oxygen lost in its passage through the Cincinnati pool."

Under the direction of Henry M. Waite, M. Am. Soc. C. E., City Engineer of Cincinnati during 1912 and 1913, as part of a comprehensive plan of sewerage for the city, studies were made for sewage treatment and disposal, and a system of intercepting sewers was developed with outlets that could be adapted to a number of possible plans for sewage treatment. The problem of sewage treatment and disposal was investigated by the late Harrison P. Eddy, Past-President and Hon. M. Am. Soc. C. E. His report,[22] entitled "Report upon Necessity and Feasibility of Treating the Sewage of Cincinnati Before Its Discharge into Ohio River," was filed with the City in the latter part of 1913. Mr. Eddy based his studies of the problem upon two main considerations:

"*First.*—Consideration of the character of the water as it reaches Cincinnati, and the effect of the sewage and wastes discharged into the river above the city.

"*Second.*—The effect of the sewage discharged by the cities and towns in the immediate vicinity of Cincinnati, herein designated the Cincinnati District."

The general conclusion of the report may be summarized in the following quotation:

"In view of the fact that it does not appear that sewage treatment will be necessary at Cincinnati for a number of years, it does not seem wise to recommend any specific method of treatment or that any site be secured for a treatment plant. Methods of sewage treatment have been improved, and new methods have been devised during recent years, and much further improvement may reasonably be expected during years to come. It would, therefore, seem idle to take steps at this time which would commit the City of Cincinnati in any way to a particular method of treatment. It is necessary and wise, however, to so plan and lay out the system of intercepting sewers which may be built from time to time that they may be brought together in harmony with any method for the treatment of sewage which may be adopted; and should it later prove advisable to intercept the sewage along the Ohio River, before a sewage treatment plant is needed, the site on which such a plant is to be built should then be selected so that the intercepter may be laid out with a view to conveying sewage to that site.

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[21] Indiana State Board of Health (unpublished).

[22] Rept. on a Plan of Sewerage, City of Cincinnati, 1912–1913, p. 295.

"From these investigations, the conclusion is that Cincinnati is not at present justified in going to the expense of building and operating a sewage treatment plant, since the benefits derived therefrom would not be commensurate with the expense."

From July, 1913, until December, 1916, the Ohio River was subjected by the U. S. Public Health Service to special study as a type of large inland river presenting a complex and difficult problem in the control of sewage pollution. The purposes of this study[23] were:

"(1) To give a quantitative statement of the pollution of the river in important zones, as existing at the time of the study, with such evaluation as possible of the relative importance of individual sources or units in contributing to this pollution, and of the relation of the conditions to the public health.

"(2) To furnish the basis for estimating with reasonable precision the changes in status of pollution which may be expected in the future to result from a given change in one or more of the factors concerned; as, for example, from a given increase in the various units of population, or from a given reduction in the polluting effect of their sewage by artificial treatment; or from changes in the velocity of flow, due to the construction of additional dams, as now projected, for improvement of navigation.

"(3) To investigate the possibility of establishing definite quantitative relations between the intensity of pollution, as measured by various laboratory tests, and such obvious factors as are readily determinable by field surveys. Especially has it been the purpose, in this connection, to make a quantitative study of natural purification as related to time of flow, temperature, and other determinable factors of presumptive importance."

THE RE-SURVEY OF 1930

At the time this study was made seventeen of the proposed fifty navigation dams were completed and in operation. Dam No. 37 of the series, at Fernbank, forms the Cincinnati pool. After complete canalization of the Ohio, the U. S. Public Health Service made a re-study[24] during 1930 of that section of the stream between Dam No. 36 (above Cincinnati) and Dam No. 43 (about twenty miles below Louisville, Ky.), a river length of about 164 miles. The re-study was to determine:

"(1) The extent to which the normal increase in population and changes in industrial activities during the intervening 15 years would be reflected by changes in the sanitary condition of the stream as indicated by differences in the chemical and bacteriological findings.

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[23] "A Study of the Pollution and Natural Purification of the Ohio River," U. S. Public Health Service, *Public Health Bulletin No. 143.*

[24] *U. S. Public Health Bulletin No. 204.*

"(2) The effects of canalization during periods of high water temperature and low stream discharge when the sewage and wastes flowed through a series of pools below the points of discharge.

"(3) The present sanitary condition of the river, making possible a comparison of probable conditions in 1930 as forecast by the data collected in 1915, with conditions as they actually existed in 1930.

"(4) Possible differences in the rates of chemical and bacteriological changes indicated by the two sets of observations as a result of changes in the amount and character of the pollution and in channel conditions."

The effect of canalization, as indicated by the re-study, is summarized in the following quotation from the printed report:[24]

"Canalization, at least between Cincinnati and Louisville, has had a tendency to complicate rather than simplify the problems connected with sewage disposal, possible nuisance production, the operation of water-treatment devices, and the preservation of the public health."

Pollution Control Requires Co-Operative Action

Parts of fourteen States lie within the Ohio River drainage area. The drainage area above or to the east of Cincinnati is approximately 76 300 sq miles, or slightly less than one-third the total area of the basin. When the Ohio River leaves the State of Pennsylvania it forms the boundary line between the States of Ohio, Indiana, and Illinois on the north, and the States of West Virginia and Kentucky on the south. To correct a bad condition along this river, such as pollution, therefore, becomes an inter-state problem. Actually, all the States in the water-shed share a responsibility and are contributors toward the creation of this undesirable river condition. Since 1935 considerable public agitation has developed demanding that proper corrective measures be taken. These demands were presented to the Second Session of the Seventy-Fourth Congress, with the result that in June, 1936, a Joint Resolution was passed enabling certain States—including Pennsylvania, West Virginia, Kentucky, Indiana, Illinois, Tennessee, and Ohio, or any two or more of them—to negotiate and enter into agreements or compacts for conserving and regulating the flow, lessening flood damage, removing sources of pollution of the waters thereof, or making other public improvements on any rivers or streams whose drainage basins lie within any two or more of the specified States.

Since the passage of the Federal resolution, a number of the States have arranged for and appointed Commissioners to meet jointly with Commissioners from the other Ohio River States. At the present time (1938), therefore, there is in the making the possibility of joint State action which may eventually lead to definite plans for pollution control.

Whatever the problem of stream pollution may be for Cincinnati, the measures required for its correction cannot be accomplished by the City of Cincinnati acting by itself; for the Metropolitan District consists of a number of politically independent incorporated cities and towns—about twelve north of, and ten south of, the river. The problem calls for inter-city as well as inter-state co-operation. Conversely, what the City of Cincinnati does or fails to do which may result in the creation—or, more correctly, the re-creation—of the pollution problem in the Ohio River is the co-operative negligence of a large metropolitan district with a total population of approximately 700 000, of which number approximately 470 000, or about 70%, are within the corporate limits of the city.

Each investigation of river conditions, from the first in 1908 to the most recent in 1930, established the fact that the Ohio River on its arrival at the eastern edge of the Metropolitan District was a polluted stream. In part, therefore, the conditions at Cincinnati are the result of negligence on the part of others up stream, not only on the Ohio River proper, but on practically all its tributaries; and an equitable solution for the problem at Cincinnati must pre-suppose that equal corrective measures will be taken on the part of those responsible for the polluted condition of the water as it enters the district.

The Effect of Canalization

Canalization of the Ohio River has intensified the pollution problem. The Cincinnati pool is created by Dam No. 37, located at approximately the most westerly corporation limits of the city. This pool extends twenty miles up stream to Dam No. 36, which is approximately at the most easterly corporation limits of the city. The business or central district of the city is about equi-distant from the two dams, and on the Kentucky side, the center of the developed area is at about the same point. Maximum pollution, therefore, occurs in the river near the central section of the city as well as of the Metropolitan District.

For navigation purposes the crest of Dam No. 37 is at such elevation that a minimum stream depth of 9 ft obtains at the easterly end of the pool. A rough estimate of the quantity of water in this pool at a time of zero velocity, is 16 000 000 000 gal. During periods of continued dry weather the minimum quantity of stream flow at times may be less than 3 400 cu ft per sec. For five months—July to November, 1930—during an investigation made by the U. S. Public Health Service, the average monthly stream flow was 5 000 cu ft per sec. The time required for water to pass through the Cincinnati pool, at periods of minimum discharge in 1930, was on the average 56.6 hr, or more than $2\frac{1}{3}$ days. For the extreme minimum monthly average stream discharge (October, 1930) it was estimated that the time required was in excess of 100 hr, or more than $4\frac{1}{6}$ days. At times of low flow, therefore, the pool acts as a receiving and settling tank for the sewage and the industrial and commercial liquid wastes from the Metropolitan District; and decomposition soon takes place and offensive odors are produced. There are extended periods during the summer and early fall months when the pollution from the Metropolitan District has a marked effect on the river at Cincinnati, and at such times the dissolved oxygen in the water has sometimes been completely exhausted.

Other Factors

The selection of possible sites for sewage treatment works will be no small problem. The Ohio River channel within the Metropolitan Area, is of comparatively recent geological origin. It is narrow, its banks are steep, and there are few low areas of sufficient acreage to accommodate works of the character required. When the comprehensive sewerage plan was investigated in 1912 and 1913, just five locations were considered suitable for treatment plant works. Since then the new Cincinnati Union Terminal has taken possession of one of the areas, and the Municipal Airport has acquired a second. Moreover, one of the remaining locations is near the Great Miami River, about sixteen miles from the central section of the city. To take care of the pollution on the Ohio side of the river, at Cincinnati, would require works costing approximately $15 000 000.

Another factor requiring thoughtful consideration as a part of the pollution problem is that of floods in the Ohio River. The U. S. Weather Bureau Office at Cincinnati has records of river stages from June 1, 1858, to date (January 1, 1938). In this 80-yr period, the maximum flood height was slightly less than 80 ft (79.99 ft at 4 A.M. January 26, 1937); the next highest flood reached approximately 71 ft (February 15, 1884, 71.06 ft); and the third highest was just under the 70-ft stage (69.8 ft, April 1, 1913). The records indicate twelve floods of 60 ft or more during the eighty years, or an average of one such flood every seventh year. To the city, the physical effects of flood waters begin to be felt at an elevation of about 50 ft. Such a flood may be expected to occur once every sixteen months.

CONCLUSION

In conclusion, it may be stated that the Metropolitan District, of which Cincinnati is the largest urban area, realizes that the Ohio River during six months of the average year is grossly polluted. To correct, in part or wholly, this unfavorable situation will require joint action by many incorporated municipalities within the Metropolitan District, as well as by the States of Ohio and Kentucky. It is realized, also, that the possible solution of the problem will be critically affected by two extreme factors—low dry-weather flows and high flood flows. The works required will cost many millions of dollars to construct, and no mean sum annually for satisfactory operation.

WHAT CAN BE DONE ABOUT STREAM POLLUTION?

By Abel Wolman,[25] M. Am. Soc. C. E.

Synopsis

The elements of the stream-pollution problem are reviewed. The possible results of activity in the fields of law, science, finance, and administration are briefly evaluated. The key to present difficulties appears to be primarily in raising money for corrective measures. Opportunities in State or Federal subsidies or grants-in-aid are emphasized.

The revival of interest in the control of stream pollution is one of the significant trends in the field of water resources control. In earlier days, navigation caught the public eye and for many years dominated the scene in opening up the country. Emphasis then shifted at a later date to flood control.

One of the national results (and incidentally one of the penalties) of population and industrial growth appears in the present emphasis on the correction of stream pollution. To sanitary engineers, particularly those engaged in health department practice, this emphatic public interest is somewhat of a curiosity, since for several decades one of the primary concerns of health departments has been the prevention and alleviation of stream pollution. The diagnosis of the causes for this recent and dramatic interest of the public in this field would offer in itself an attractive field for research.

As in all matters of public concern, the answer to what can be done about stream pollution will depend upon the experience, training, temperament, and prejudices of individuals. Those who have had least to do with the problem are prone to over-simplify the solutions, while those who have struggled with the issue for a great many years are accused of delaying the wheels of progress by being too lax in the enforcement of laws and not active enough in public education. Solutions are offered in law, in science, in finance, and in administration. What are the probabilities of success in each of these fields?

Before attempting to analyze these probabilities, a few general facts should be considered.

The intensity of stream pollution is not nation-wide. It is concentrated largely in five or six of the heavy industrial States. Population distribution alone controls the localization of the problem. The degree of difficulty varies from place to place. Blanket elimination and blanket similarity of approach do not seem to fit the case. However, in each area where the situation has become acute, in regard to either domestic or industrial waste, the time has arrived for a definite position on the part of the people as to the balancing of convenience in the use of the stream.

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[25] Chf. Engr., Maryland State Dept. of Health, Baltimore, Md.

Many frank critics of the rush toward stream-pollution abatement are emphasizing the fact that correction, particularly in reference to industry, may militate against the advantage of the region forced to clean up. The weight to be given such considerations should be determined in each case upon the merits of the facts; that the facts should be adequately balanced should certainly be borne in mind.

Law

Law enforcement as a device for the control of any difficulty is frequently the first recourse of certain groups of individuals. The popular phrase "there ought to be a law" exemplifies this faith in correction by legislative fiat. Experience discloses, however, that no greater progress in stream pollution abatement has occurred in those States where rigid legislation has been on the books for many years than where such laws are absent.

This situation has led some people to suggest that the difficulty lies in the inconsistency of individual State Acts. They argue that corrective measures are impossible in one State because similar corrective measures are not executed in another State. This leads to the further suggestion that the only corrective in law is a comprehensive and overlying Federal anti-pollution act. Aside from inherent fallacies in logic, many students of the problem believe that such Federal legislation is unconstitutional. More important is the belief that it would be unwise, not only because the mere passage of an act need not result in correcting difficulties, but because the long and time-consuming negotiations for stream correction are logical fields for local participation and autonomy.

State by State progress in stream-pollution control has been uneven, and uncorrelated with legislative pronouncements. If anything, it has been related primarily to the enthusiasm, ingenuity, and co-operative endeavor of individual State departments and of municipalities and industry.

It is the writer's opinion that the secret of successful stream-pollution control does not reside in the extension of legislative restriction.

Scientific Knowledge

A review of the record discloses that solutions to many of the problems in trade waste treatment and recovery are not yet apparent. In this particular field the solution obviously lies in a more intelligent, consistent, and continued participation of industry in the search for an answer. Inertia and ignorance on the part of industry have perhaps been responsible for more delay and less accomplishment in the control of stream pollution than any other factor. The industrial sewer has been out of sight and hence out of mind. Industry's wastes have not been one of industry's major responsibilities.

However, industry cannot continue to content itself with negative resistance to extension of legislative control; it must begin to traverse more positive paths toward the development of scientific and economical methods of industrial waste treatment. On industry rests the responsibility of evaluating its contribution to the problem, of determining its method of attack, and of demonstrating its willingness to assist in the solution of one of the important problems of national economy. American industry particularly cannot afford to be accused of being unable to solve the technical problems of waste treatment, when its accomplishments in the allied fields of technological production have been as varied, as successful, and as inspiring as they have been.

As for the treatment of domestic wastes, the state of the art is sufficiently advanced to require no similar pressure. Obviously, certain technical facts are still lacking and scientific diagnoses and procedure still need to be developed, but economical and satisfactory processes of treatment are at hand to meet the issue.

MONEY

The key to the primary question of what can be done about pollution, to the writer's mind, lies in money. How can stream-pollution correction best be paid for? "Depression thinking" has given even greater emphasis to the ever-present question of how to raise money for sewage disposal and treatment. Such an enterprise is seldom popular, largely because, in most instances, the sewage nuisance is remote from the average taxpayer's eyes and nose; and in the years of depression, money for such purposes has become increasingly difficult to raise. A revival of a criterion of decency in the American public conscience is a first essential, but no less important is the development of financial techniques for raising funds.

Experience since 1933 has indicated how important an impetus may be given to sewage treatment plant construction by the artificial device of Federal grants-in-aid. More than 25% of existing sewage treatment plants in the United States have been constructed since 1933. It is difficult to believe that this is the result of chance or of a simple natural process of growth, rather than the obvious result of the artificial stimulation of activities in this field by Federal funds. The principle of grants-in-aid to stimulate special types of local endeavor is of course not new; it has been applied for more than fifty years in a variety of highway, navigation, reclamation, public health, and agricultural projects.

This experience leads the writer to suggest that perhaps the most valuable stimulus for correction of stream pollution is in the grant-in-aid principle. It need not follow, of course, that the grant-in-aid from a Federal Government is a pre-requisite, although it makes for uniformity of action throughout the country. The possibility of extending the State grant-in-aid procedure to sanitary projects is an opportunity for real progress. Such a procedure, within the State, again has precedent in the fields of education, agriculture, and highways. It has the merit of continuing and preserving the important local autonomy and responsibility which concerns so many at this time.

In other words, if money is the key to stream-pollution control, a central stimulation of activity—based, of course, upon a reasonable balancing of convenience—offers one of the soundest bases for future success. Correction is cheap, as such expenditures go, but the incentive has been missing.

SUMMARY

This brief discussion leads to the following answers, positive and negative, to the question of "What can be done about stream pollution?"

(1) Standardization of State laws on the subject is desirable. Too much faith in the efficacy of the law, however, is not warranted by past experience,

or the temperament of the American people. If the people are not ready, the law fails.

(2) Industry must show a greater interest in the problem of industrial waste. It must demonstrate that it is attempting to find economical solutions to these problems.

(3) State or Federal financial stimulation of sewage treatment plant construction offers a useful device for increasing the rate of installation of stream-pollution corrective equipment. It is perhaps the most helpful direction in which progress is to be made. Its political implications and its dangers must be carefully scrutinized. With proper control, however, it has demonstrated its possibilities of success.

(4) The permanent control of the situation lies in the co-operative effort of the public official, the private investor, and the man in the street. Intelligent appraisal of the problem and of the benefits to be derived from its correction should be the guide. Hysteria and undue charges in this field, as in any other, are not likely to produce permanent answers to the problem.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

PRELIMINARY DESIGN OF SUSPENSION BRIDGES

By Shortridge Hardesty,[1] M. Am. Soc. C. E., and
Harold E. Wessman,[2] Assoc. M. Am. Soc. C. E.

Synopsis

A rapid and accurate method of analysis for the preliminary design of the stiffening trusses of suspension bridges is presented in this paper. The method demonstrates clearly the related functions of the cable and the stiffening truss, emphasizing the fact that the former is the major structural element and that the latter is added to reduce undesirable grade changes. It is useful in determining quickly the effect of variations in the proportions, chord sections, and materials of the stiffening trusses, of differences in loadings, or of changes in major dimensions of the bridge. The proposed method gives results at the center and quarter-points which agree closely with those obtained by the deflection theory. Typical computations for the Triborough Suspension Bridge, at New York, N. Y., accompanied by detailed explanations, illustrate the procedure.

Introduction

Description of Method.—Certain types of statically indeterminate structures are hybrid[3] in their action. Every time a change is made in dimensions or design sections, a change occurs in the moments at various sections of such structures. The stiffening trusses of a suspension bridge act in this manner. Regardless of arbitrary limits imposed upon deflections or grade changes, the designer has a choice of many different stiffening trusses; and in some cases, with large dead load and small live load, he may omit them entirely, as was done in the George Washington Bridge over the Hudson River at New York, N. Y. There is a definite need for a rapid method of analysis for preliminary design studies of stiffening trusses, so that the engineer may form some intelli-

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[1] Cons. Engr. (Waddell & Hardesty), New York, N. Y.

[2] Prof., Structural Eng., Coll. of Eng., New York Univ., New York, N. Y.

[3] "The Relation of Analysis to Structural Design," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1363.

gent conclusions as to what happens when he varies the depth or changes the chord sections, selects a new sag ratio, changes the weights of flooring, or moves main towers and anchorages to different relative positions. Heretofore, three methods of analysis have been available, but none of them is satisfactory for making rapid studies of a variety of designs.

The so-called "elastic theory" used in earlier days is obsolete. Its equations are misleading and erroneous. True enough, when the structure is very stiff, the elastic theory gives results which are not much in error; but, as the stiffening truss becomes more and more flexible, the error becomes very large. One need only to point to the George Washington Bridge and ask the question, "How much error would have resulted if this structure had been designed by the elastic theory?"

The "deflection theory" is usually termed the exact method because it accounts for the change in shape of the cable under live load and temperature variations and the resulting changes in moments and shears in the stiffening truss. The term, "exact," is relative, however, because, as ordinarily applied, the method involves assumptions which result in some error. On the other hand, it is much more scientific and accurate than the elastic theory. Unfortunately, the exponential formulas of the deflection theory require long and involved computations. It is necessary to use a computing machine or a table of logarithms in order to avoid possibilities of serious error.

The "trigonometric series" method is the third method of analysis. The equations of this method, which are expressed in terms of trigonometric series that converge very rapidly, also account for changes in the shape of the cable. Although it is not commonly used, the trigonometric series method has distinct advantages over the deflection theory. Non-uniform suspender pull is accounted for in the determination of the horizontal component of the cable stress. What is more important, a slide-rule may be used for practically all computations without sacrificing accuracy. This simplifies procedure and saves time, but the computations are still lengthy and involved. Consequently, like the deflection theory, it is not satisfactory for studies of a variety of preliminary designs, although it is an excellent tool for the analysis of the final design. The trigonometric series method has been extended by the writers to provide a basis for precise computations made to support the derivation of certain formulas developed in this paper; but this treatment is not included.

In the proposed preliminary design method, the maximum moments at the quarter-point and at the center of the main span are computed in two major steps: First, the deflections of an unstiffened cable under partial live load, for various ratios of live load to dead load, are computed on the assumption that the cable length is unchanged and the tower tops do not move. An average value of load length is used, which value was determined from a study of the load lengths giving maximum moments at the quarter-point and at the center of the stiffening trusses of several actual bridges.

The effect of adding the stiffening truss is then considered. Evidently the truss will attempt to conform to the curve of deflection of the cable. It will reduce the deflection, however, and assume some compromise position depending upon its stiffness. The bending moment taken by the truss in its attempt to conform to the deflected cable will be a function of the maximum deflection of the cable at certain points and the truss stiffness. A trial moment of inertia is used, and corrected later if found necessary. A single numerical coefficient accounts for non-uniform suspender loading. The moment is evaluated primarily from the important consideration that, if the cable is kept from assuming its natural position as an unstiffened cable, the stiffening truss must provide the bending moment required to hold the cable in its restrained position.

Finally, the changes in the length of the cable due to live load or to rise and fall of temperature, and the sag changes caused by movement of the tower tops or by the interaction between the side spans and the main span, are combined into one change in the center sag. This lowers or raises the truss, causing a moment which is added algebraically to the moment obtained in the first step.

The changes in center sag of the side span are necessarily found in order to evaluate loading conditions that govern the main span. From these sag changes, it is a simple matter to compute the maximum positive and negative moments at the center of the side span trusses.

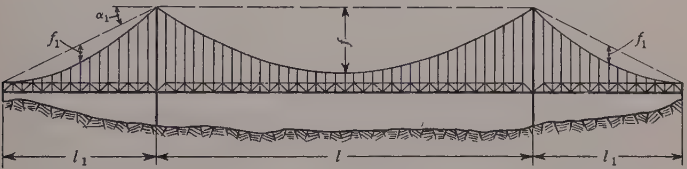

FIG. 1.—SUSPENSION BRIDGE WITH TWO-HINGED STIFFENING TRUSS AND SUSPENDED SIDE SPANS

Fig. 1 is a line diagram of a two-hinged suspension bridge of the type discussed in this paper. The method applies equally well to suspension bridges with straight back-stays.

Notation.—The meanings of the symbols used in this paper are given where they first appear and are assembled, for convenience of reference, in the Appendix.

DEFLECTIONS OF AN UNSTIFFENED CABLE

Cable Ordinates for Dead Load and Mean Temperature.—The cable of a suspension bridge is a flexible member having practically no resistance to bending. Its shape depends upon the initial sag and the loading; in fact, the cable curve is an equilibrium polygon for the loads suspended from it. Consequently, for any condition of loading, the unstiffened cable assumes a curve that represents the moment diagram for the same loading on a simple span. The horizontal component of the cable stress, multiplied by the ordinate at any section, must equal the bending moment at the same section due to the loads on the cable span, considered as a simple beam. With only dead load on the span,

$$H_w\, y = \frac{w}{2}(l\,x - x^2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

in which w = total dead load per unit horizontal length of cable; H_w = horizontal component of cable stress due to dead load and mean temperature;

l = length of main span; and x and y = co-ordinates of any point of cable curve, measured from an origin at the top of the left tower, under dead load at mean temperature. If the center sag is f,

$$H_w = \frac{w\,l^2}{8\,f} \qquad \text{(1b)}$$

Substituting Equation (1*b*) in Equation (1*a*):

$$y = \frac{4\,f}{l^2}\,(l\,x - x^2) \qquad \text{(2)}$$

which is the familiar parabolic formula for the cable ordinates dead load at mean temperature.

The deflections of an unstiffened cable for partial live loads and varying ratios of live load to dead load are first to be found, with cable stretch omitted. Consequently, these deflections are based on an unchanged cable length L, and no relative movement of the tower tops. The equations for the cable ordinates under various live load conditions must first be derived. The differences between the resulting ordinates and those computed from Equation (2) will then give the desired deflections.

Live Load on One End of Span.—When live load moves upon the structure and extends a distance $k\,l$ from the left end, the cable changes shape. The new curve, which represents the moment diagram for the combined live and dead load on a simple span, consists of two parabolas that have a common tangent at the head of the live load. The new ordinates are given by the following equations, which are simply expanded statements of the law of statics, $\Sigma\,M = 0$. It should be noted that H_w has been increased by a value H_a due to live load. When $x < k\,l$ (that is, for the part of the span under live load), there results the following equation, in which w_l = live load on roadway, per unit length of truss,

$$(H_w + H_a)\,y = \frac{w}{2}\,(l\,x - x^2) + w_l\,k\,l\left(1 - \frac{k}{2}\right)x - \frac{w_l\,x^2}{2} \qquad \text{(3a)}$$

or,

$$y = \frac{(w + w_l)\,(l\,x - x^2) - w_l\,l\,(1 - k)^2\,x}{2\,(H_w + H_a)} \qquad \text{(3b)}$$

When $x > k\,l$ (that is, for that part without live load), there is found:

$$(H_w + H_a)\,y = \frac{w}{2}\,(l\,x - x^2) + w_l\,k\,l\left(1 - \frac{k}{2}\right)x - w_l\,k\,l\left(x - \frac{k\,l}{2}\right) \qquad \text{(4a)}$$

or,

$$y = \frac{(w\,x + w_l\,k^2\,l)\,(l - x)}{2\,(H_w + H_a)} \qquad \text{(4b)}$$

With only dead load on the span, the horizontal component, H_w, of the cable stress is easily determined from Equation (1*b*), which involves the initial center sag, f. When live load moves on to the span, the center sag changes. As a consequence, the determination of the new horizontal component, $H_w + H_a$, is not quite so simple. The quantity $H_w + H_a$ will be determined in this step on the assumption that the cable length l does not change. The effect of changes in cable length will be discussed subsequently. If there is no change in length, the external work performed in deflecting the cable must equal zero; hence,

$$\int_0^{kl}\left(w+\frac{w_l}{2}\right)\eta\,dx+\int_{kl}^{l} w\,\eta\,dx=0 \quad \ldots\ldots\ldots\ldots\ldots (5)$$

in which η = the deflection of truss and cable, at any section, from their initial position under dead load at mean temperature. For values of $x < k\,l$, the deflection, η, equals Equation (3b) minus Equation (2); and for values of $x > k\,l$, it equals Equation (4b) minus Equation (2). Making these substitutions in Equation (5), performing the integrations, and combining terms, there results:

$$H_w + H_a = \frac{w\,l^2}{8f}\left[\frac{2+3\dfrac{w_l}{w}k^2(3-2k)+\dfrac{w_l^2}{w^2}k^3(4-3k)}{2+\dfrac{w_l}{w}k^2(3-2k)}\right]=\frac{w\,l^2}{8f}C_1 \quad \ldots (6)$$

Substituting Equation (6) in Equations (3b) and (4b), the following important equations are obtained:

For $x < k\,l$,

$$y=\frac{4f}{l^2}\,\frac{\left(1+\dfrac{w_l}{w}\right)(l\,x-x^2)-\dfrac{w_l}{w}(1-k)^2\,l\,x}{C_1} \quad \ldots\ldots\ldots\ldots (7a)$$

and, for $x > k\,l$,

$$y=\frac{4f}{l^2}\,\frac{\left(x+\dfrac{w_l}{w}k^2\,l\right)(l-x)}{C_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7b)$$

in which C_1 is the quantity within the brackets in Equation (6).

Live Load in Middle of Span.—When the live load occupies the middle of the span, symmetrically placed about the center line with a total length of $k\,l$, the cable curve consists of three parabolas with common tangents at the ends of the live load. The two end parabolas are identical. The derivation of the equations is similar to the preceding case and will not be repeated. The final equations are as follows:

$$H_w + H_a = \frac{w\,l^2}{8f}\left[\frac{2+\dfrac{3\,w_l}{2\,w}(3\,k-k^3)+\dfrac{w_l^2}{w^2}(3\,k^2-2\,k^3)}{2+\dfrac{1}{2}\dfrac{w_l}{w}(3\,k-k^3)}\right]=\frac{w\,l^2}{8f}C_2 \quad \ldots (8)$$

For the end sections under dead load only,

$$y=\frac{4f}{l^2}\,\frac{(l\,x-x^2)+\dfrac{w_l}{w}k\,l\,x}{C_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9a)$$

(in which C_2 is the quantity within the brackets in Equation (8)); and for the middle part subject to both live and dead load,

$$y = \frac{4f}{l^2}\,\frac{\left(1+\frac{w_l}{w}\right)(l\,x - x^2) - \frac{w_l}{w}\frac{l^2}{4}(1-k)^2}{C_2} \quad\ldots\ldots\ldots\ldots(9b)$$

Live Load on Both Ends of Span.—When live load is placed on both ends, covering a length $k\,l$ at each end, the new cable curve consists of three parabolas, the two end ones again being identical. The final equations are:

$$H_w + H_a = \frac{w\,l^2}{8f}\left[\frac{2 + 3\frac{w_l}{w}(6\,k^2 - 4\,k^3) + 8\frac{w_l}{w^2}k^3}{2 + \frac{w_l}{w}(6\,k^2 - 4\,k^3)}\right] = \frac{w\,l^2}{8f}C_3 \ldots.(10)$$

For the end parts, subject to both live and dead load,

$$y = \frac{4f}{l^2}\,\frac{(l\,x - x^2) + \frac{w_l}{w}(2\,k\,l\,x - x^2)}{C_3} \quad\ldots\ldots\ldots\ldots(11a)$$

in which C_3 is the quantity within the brackets in Equation (10). For the middle part under dead load only,

$$y = \frac{4f}{l^2}\,\frac{(l\,x - x^2) + \frac{w_l}{w}k^2\,l^2}{C_3} \quad\ldots\ldots\ldots\ldots(11b)$$

Application of Equations (1) to (11).—Equations (1) to (11) are employed only in computing the basic deflections, and need not be used in applying the approximate method. They are stated in order to demonstrate how the proposed method has been developed.

It will be noted, by examining the governing equations for cable ordinates, that the span of the cable has no effect whatever upon the deflections. When x in the numerator is considered as a proportional part of l, it is evident that the term l^2 in the denominator is cancelled. The deflections of the cable depend only upon the initial sag f, the ratio of live load to dead load $\frac{w_l}{w}$, and the load length factor k. Now k can be made a constant in securing maximum effects at definite points in a span, such as the quarter-point or the center point, regardless of the span length. Hence, curves of deflection for the cable may be plotted for different ratios of live load to dead load, but with a constant proportional load length, $k\,l$. The deflections may be expressed as percentages of the initial sag, f. Then, if the deflection of a cable is wanted for a definite $\frac{w_l}{w}$ ratio, one needs only to select the percentage of deflection corresponding to the proper ratio and multiply the value by the cable sag.

An investigation of various load lengths, $k\,l$, indicated that changes in the $\frac{w_l}{w}$ ratio had practically no effect on the loaded length that would produce maximum deflections at the quarter-point and at the center of the span.

Moreover, it was found that the load lengths which produced maximum downward and upward deflections were about the same as those which caused maximum positive and negative moments in the stiffening trusses. The following values of $k\,l$ were found to be accurate enough for preliminary design purposes:

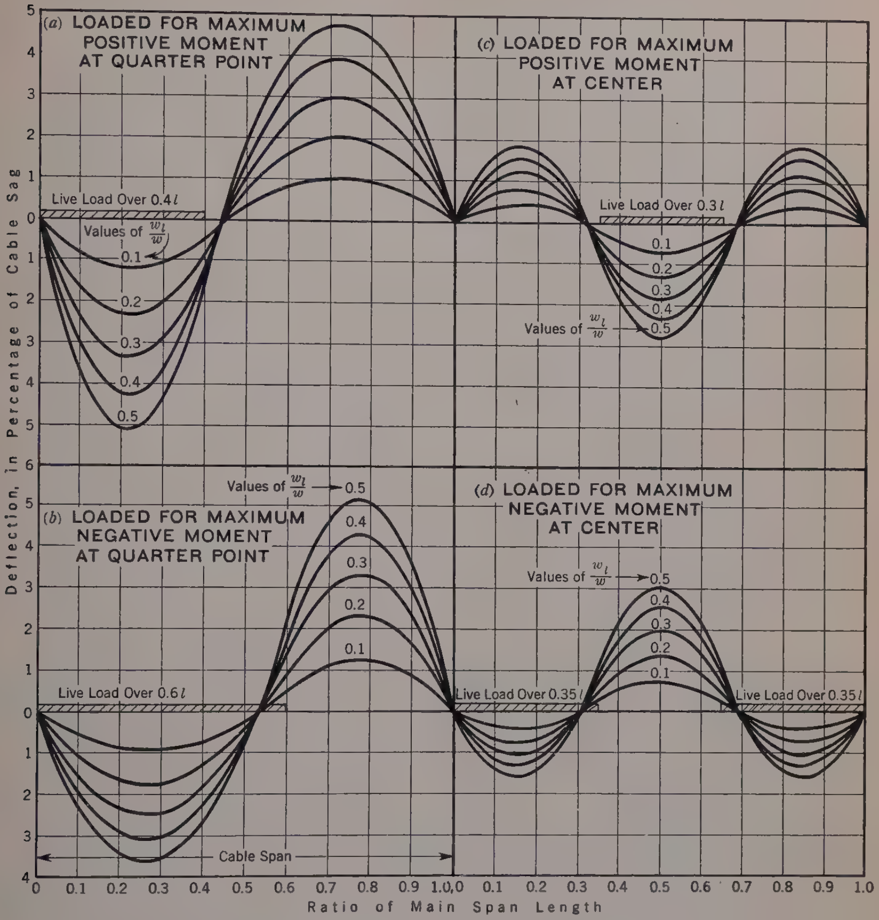

FIG. 2.—DEFLECTIONS OF UNSTIFFENED CABLE DUE TO PARTIAL LIVE LOADS PRODUCING MAXIMUM MOMENTS AT CENTER. DEAD LOAD OVER ENTIRE SPAN

Condition	Load length
Maximum positive moment at the quarter-point,	0.4 l at the same end of span
Maximum negative moment at the quarter-point,	0.6 l at far end of span
Maximum positive moment at the center.......	0.3 l at center of span
Maximum negative moment at the center.......	0.35 l at both ends of span

These load lengths were used to compute the deflections of the unstiffened cable for ratios of live load to dead load ranging from 0.1 to 1.0. Some of the

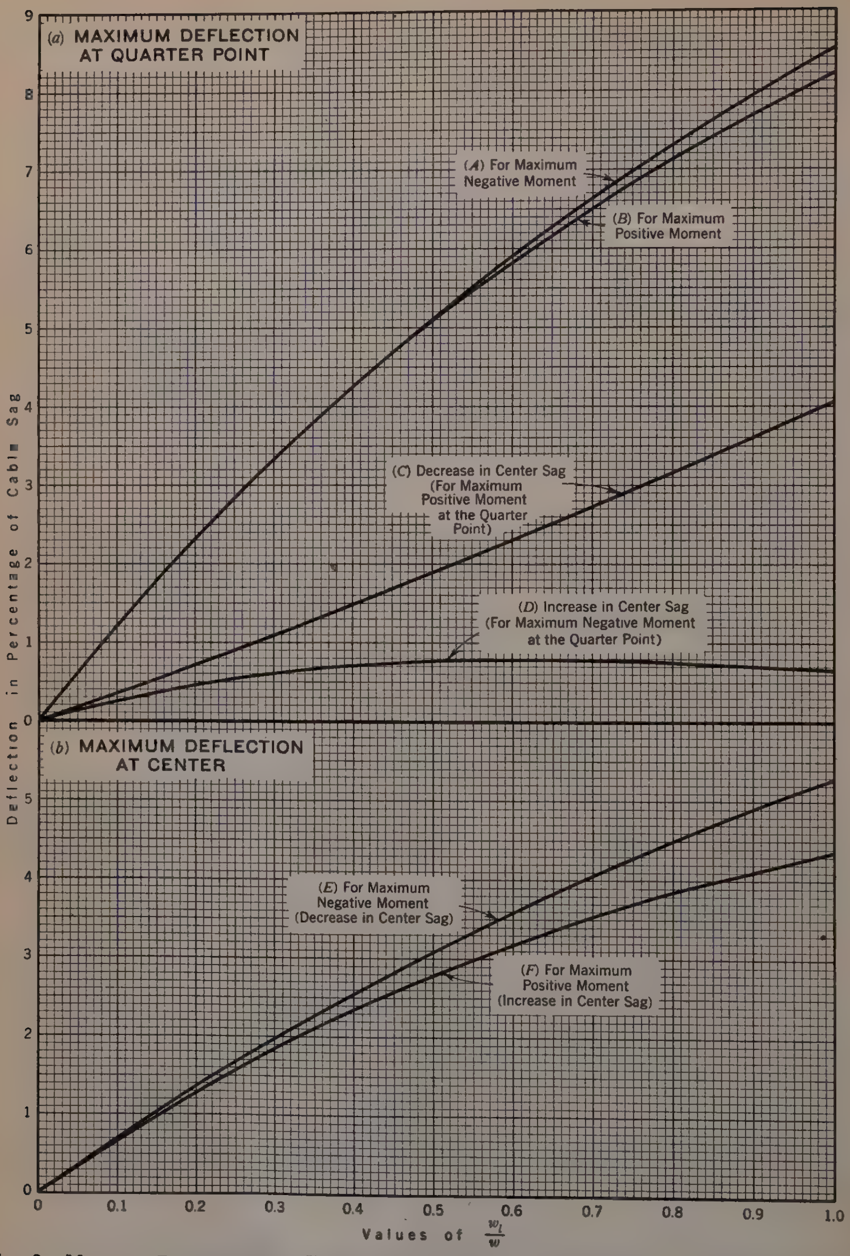

Fig. 3.—Maximum Deflections of Unstiffened Cable at Quarter-Point and Center of Main Span Due to Partial Live Loads

curves showing the procedure at the quarter-point and the center are drawn in Fig. 2. The maximum downward deflections in Fig. 2(*a*) were then plotted against the $\frac{w_l}{w}$ ratio, as Curve *B* in Fig. 3(*a*). This curve obviates interpolation for intermediate $\frac{w_l}{w}$ ratios. The decreases in center sag corresponding to the loading giving maximum positive moment at the quarter-point are also plotted in Fig. 3(*a*) (see Curve *C*). The other curves were obtained in a similar manner.[4]

Figs. 3(*a*) and 3(*b*) are the only curves used in applying the proposed method.

EFFECT OF ADDING STIFFENING TRUSS TO UNSTIFFENED CABLE

Bending Moment, M, in Stiffening Truss.—When a stiffening truss is attached to the cable and the live load is moved on to the span, the cable tends to force the stiffening truss to conform to the deflection curve of the unstiffened cable. If the truss were very flexible, it would practically conform to that curve; whereas, if it were infinitely stiff, it would not deflect at all, and the cable would be forced to remain in its original position. Since the cable offers resistance to anything tending to force it back to its initial position, and since the truss is neither very flexible nor infinitely stiff, the final position of truss and cable will evidently be a compromise between the original dead load position of the cable and the deflection curve of the unstiffened cable, as indicated in Fig. 4(*a*).

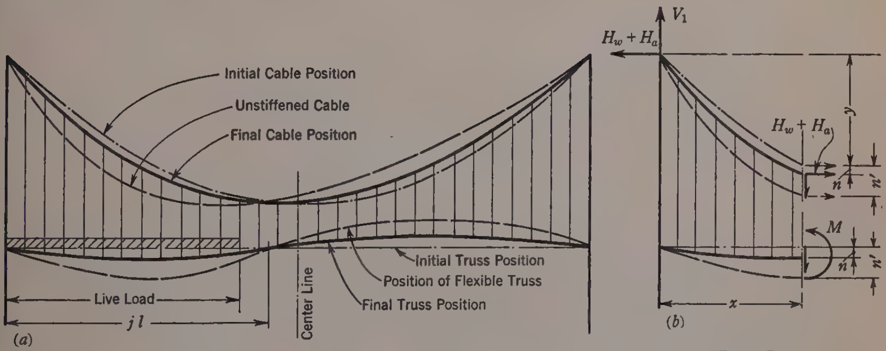

FIG. 4.—DEFLECTION OF CABLE AND STIFFENING TRUSS UNDER PARTIAL LIVE LOAD

In reaching this compromise position, the laws of statics will necessarily be satisfied (note Section *x* in Fig. 4(*b*)). The external bending moment due to dead load at mean temperature is balanced by $H_w\, y$. When live load comes upon the roadway, there is an additional external moment acting at Section *x*. If there were no stiffening truss, or if the truss were so flexible that it could resist no moment, the cable would deflect through a distance η'; and the total external moment would be balanced by $(H_w + H_a)\, y + (H_w + H_a)\, \eta'$. If, on the other hand, the truss had infinite stiffness, the cable deflection would be reduced from η' to 0; in other words, the truss would force the cable back through the entire distance η' to its original dead load position. The change η' in the lever arm of the force, $H_w + H_a$, would upset the balance of moments at Section x, in so far as the cable is concerned; and this balance would be restored by the stiffening truss taking a moment equal to $(H_w + H_a)\,\eta'$, in order to satisfy the equation of statics, $\Sigma\, M = 0$.

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[4] The extensive computations for these curves are not included in the paper, but have been filed for reference in Engineering Societies Library, 33 West 39th Street, New York, N. Y.

Since the truss is neither infinitely flexible nor infinitely stiff, the cable will be forced back only a part of the distance η'. Let the final deflection, shown in Fig. 4(b), be η. The change in lever arm is then $\eta' - \eta$; and the moment resisted by the stiffening truss is,

$$M = (H_w + H_a)\,(\eta' - \eta)\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

From one point of view, this moment resisted or developed by the stiffening truss may be considered as moment taken away from the cable when its position is changed from that of an unstiffened cable. This statement is not intended to convey the idea that the cable itself resists bending moment. The cable is flexible, but the force in the cable, multiplied by a lever arm, most certainly constitutes a moment at a section of the structure.

The deflection η of Equation (12) is found in the following manner. Let M_t be the bending moment at Section x that would be induced in the stiffening truss if it were bent to the deflection η' of the unstiffened cable. Its value will be determined later in terms of the deflection η', the physical properties of the truss, and a loading which accounts for variation in suspender pull. Assuming the final stiffening truss moment, M, to bear the same ratio to M_t as η does to η', there is found:

$$M = M_t\,\frac{\eta}{\eta'}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Equating the right-hand sides of Equations (12) and (13):

$$\eta = \eta'\,\frac{(H_w + H_a)\,\eta'}{M_t + (H_w + H_a)\,\eta'}\ldots\ldots\ldots\ldots\ldots(14a)$$

Hence,

$$M = M_t\,\frac{(H_w + H_a)\,\eta'}{M_t + (H_w + H_a)\,\eta'}\ldots\ldots\ldots\ldots\ldots(14b)$$

Equation 14(b) plays an important part in the proposed method of analysis. From an inspection of this formula, it will be noted that the final moment M in the stiffening truss is less than the moment $(H_w + H_a)\,\eta'$ that would be induced by forcing the cable back to its original position. One may consider either that the ratio $\frac{(H_w + H_a)\,\eta'}{M_t + (H_w + H_a)\,\eta'}$ determines the proportion of the moment M_t taken by the truss, or that the ratio $\frac{M_t}{M_t + (H_w + H_a)\,\eta'}$ represents the proportion of the moment $(H_w + H_a)\,\eta'$ taken by it. The latter ratio shows also the proportion of the deflection η' that is eliminated by the stiffening truss in this step of the process.

Horizontal Component of Cable Stress.—By an inspection of the terms of Equation 14(b), it will be noted that the deflection η' is determined from

Fig. 3(*a*) and Fig. 3(*b*), which leaves two other quantities, M_t and $(H_w + H_a)$, to be evaluated. The latter quantity is the horizontal component of the cable stress due only to dead load and live load on the unstiffened cable. This horizontal component is found by computing the moment M_c' of the dead load and live load at the center of the span, just as if the span were a simple beam, and then dividing by the center sag. In equation form:

$$H_w + H_a = \frac{M_c'}{(f + \eta_c')} \qquad \text{(15a)}$$

in which $(f + \eta_c')$ = the center sag, and η_c' = increase or decrease in the center sag taken from the curves of Fig. 3(*a*) and Fig. 3(*b*). With only the dead load, w, acting:

$$H_w = \frac{w\,l^2}{8f} = 0.125\,\frac{w\,l^2}{f} \qquad \text{(15b)}$$

Evidently, in this case, f is the center sag of the cable in its original position. When live load comes on, however, the center ordinate f changes by an amount η_c'. It should be kept in mind that the effect of cable stretch and temperature is to be considered subsequently. For the time being, deflections which are caused by partial live load are used, with the length of the cable l kept constant.

For the cases of loading summarized in Fig. 2(*c*) and Fig. 2(*d*), the values of $H_w + H_a$ are as follows:

Maximum positive moment at the quarter-point, with live load at the same end, over a length of 0.4 l,

$$H_w + H_a = \frac{1}{f + \eta_c'}\,(0.125\,w\,l^2 + 0.040\,w_l\,l^2) \qquad \text{(16a)}$$

Maximum negative moment at the quarter-point, with live load at the opposite end over a length of 0.6 l,

$$H_w + H_a = \frac{1}{f + \eta_c'}\,(0.125\,w\,l^2 + 0.085\,w_l\,l^2) \qquad \text{(16b)}$$

Maximum positive moment at the center, with live load at the center over a length of 0.3 l,

$$H_w + H_a = \frac{1}{f + \eta_c'}\,(0.125\,w\,l^2 + 0.0638\,w_l\,l^2) \qquad \text{(16c)}$$

Maximum negative moment at the center, with live load over a length of 0.35 l at each end,

$$H_w + H_a = \frac{1}{f + \eta_c'}\,(0.125\,w\,l^2 + 0.0613\,w_l\,l^2) \qquad \text{(16d)}$$

In each of the preceding equations, the quantity within the parenthesis is the bending moment at the center of a simple span due to dead load over the entire span and live load over a part of the span. The deflection η_c' is positive when downward and negative when upward.

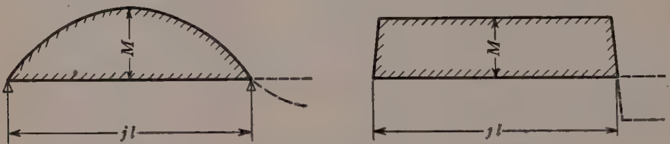
The horizontal component of cable stress may be found in a similar manner for other conditions of loading. Equations (16) are sufficient, however, for preliminary design purposes.

Bending Moment, M_t, in Stiffening Truss.—The bending moment M_t in Equation (14*b*), denoting the moment that would be induced in the stiffening truss if it were bent to the deflection curve of the unstiffened cable, will now be evaluated. For convenience, consider the case of maximum positive moment at the quarter-point. Deflection curves for this case are plotted in Fig. 2(*a*) and Fig. 2(*b*). The stiffening truss will be bent into the two loops shown, one downward and one upward. This is also indicated in Fig. 4. If η' is the maximum deflection of the downward loop and $j\,l$ is the length of the base of the loop, the bending moment may be stated thus, in general terms:

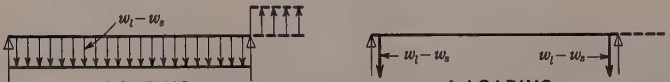
$$M_t = C\,\frac{E\,I\,\eta'}{(j\,l)^2} \qquad (17)$$

in which C is a numerical coefficient depending upon the shape of the moment diagram; or, in other words, the type of loading on the truss. The loading involves both the live load on the roadway and the live load suspender pulls.

It will be well to consider approximate moment diagrams first, in order to show the relationship between M_t and η'.



(*a*) MOMENT (*c*) MOMENT



(*b*) LOADING (*d*) LOADING

FIG. 5.—APPROXIMATE MOMENT AND LOAD DIAGRAMS

If the moment diagram on one loop of the truss is a parabola, as shown in Fig. 5(*a*), the deflection at the center of the loop is,

$$\eta' = \frac{5}{48}\,\frac{M_t\,(j\,l)^2}{E\,I} \qquad (18a)$$

and, the maximum moment is,

$$M_t = 9.6\,\frac{E\,I\,\eta'}{(j\,l)^2} \qquad (18b)$$

The loading creating such a parabolic moment diagram is uniform over the length, $j\,l$, and is designated as $w_l - w_s$ in Fig. 5(*b*). The deflection curve would evidently be a fourth degree curve in this case.

Next suppose that the moment diagram is almost rectangular, as in Fig. 5(*c*), rather than parabolic. Then the center deflection would be,

$$\eta' = \frac{1}{8}\,\frac{M_t\,(j\,l)^2}{E\,I} \qquad (19a)$$

and the maximum moment would be,

$$M_t = 8.0 \frac{E\,I\,\eta'}{(j\,l)^2} \qquad \text{(19}b\text{)}$$

The loading creating such a moment diagram would consist of two equal concentrated loads near the ends of the span, as in Fig. 5(*d*), or two equal end couples. The deflection curve in this case would be a second degree curve or a parabola.

The loadings in the two cases shown are radically different, but the maximum moments, M_t, for the same deflection differ by about 17%; and the final M-values, determined from Equation (14*b*) for these two values of M_t, would differ from one another by less than 8% in most cases.

The actual loading diagram for each loop of the stiffening truss falls somewhere between the two cases shown in Fig. 5. In an approximate method, one might assume arbitrarily that the suspender loading is uniform; but the final moments computed by the deflection theory are not determined from a uniform suspender pull, even if that assumption is used in finding the value of H_a. Hence, to make the proposed preliminary method as accurate as possible, actual moment and load patterns for partial live loads should be studied. Some mathematical relations must first be established to facilitate such a study.

Relationship Between Suspender Pull and Truss Moment.—The initial curve of the cable under dead load and mean temperature is,

$$y = \frac{4\,f\,x}{l^2}\,(l - x) = \frac{w\,x}{2\,H_w}\,(l - x) \qquad \text{(20}a\text{)}$$

The familiar differential equation of this curve is,

$$H_w \frac{d^2y}{dx^2} = -\,w \qquad \text{(20}b\text{)}$$

When an additional load, w_s, due to live load or temperature change, acts through the suspenders upon the cable, the cable ordinates become $y + \eta$, and the differential equation becomes,

$$(H_a + H_w)\,\frac{d^2(y + \eta)}{dx^2} = -\,(w + w_s) \qquad \text{(21}a\text{)}$$

that is,

$$-\frac{H_a}{H_w}\,w - w + (H_a + H_w)\,\frac{d^2\eta}{dx^2} = -\,(w + w_s) \qquad \text{(21}b\text{)}$$

or,

$$w_s = \beta\,w - (H_a + H_w)\,\frac{d^2\eta}{dx^2} \qquad \text{(21}c\text{)}$$

in which β is the ratio $\frac{H_a}{H_w}$. Equation (21*c*) gives the value of the suspender pull w_s.

The differential equation of curvature for the stiffening truss is,

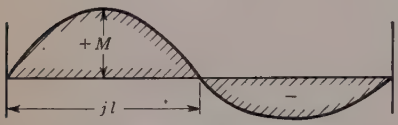
$$\frac{d^2\eta}{dx^2} = -\,\frac{M}{E\,I} \qquad \text{(22)}$$

The curvature of the stiffening truss is the same as that of the curve of deflection of the cable. The value in Equation (22) when substituted in Equation (21*c*) gives the following important relationship:

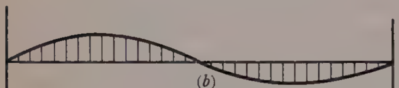
$$w_s - \beta\, w = \frac{H_a + H_w}{E\, I}\, M \qquad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Since $\frac{H_a + H_w}{E\, I}$ is a constant for one loading condition, the final bending-moment diagram on the truss must have the same shape as the diagram of live load pull, w_s, on the suspenders, minus a constant proportion of the dead load pull, w. The diagram may be determined quantitatively from Equation (23), but the major significance of this is qualitative. It is evident that a parabolic moment diagram such as that in Fig. 5(*a*) and a load diagram such as that in Fig. 5(*b*) are not possible.

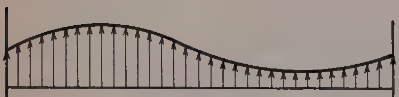
For a live load extending over a length of 0.4 l, the moment diagram on the truss, with no stretch effects included, is a curve of two loops, approximately like that in Fig. 6(*a*). The ($w_s - \beta\, w$) diagram must be similar, as in Fig. 6(*b*). Adding $\beta\, w$ to this curve, the hanger-pull diagram in Fig. 6(*c*), for live load only, is the result. Combining Fig. 6(*c*) with the live load w_l on the roadway (Fig. 6(*d*)) gives the final load diagram, Fig. 6(*e*), for the truss. This is the loading that is associated with the shears and bending moments in the stiffening truss. It should be kept in mind that the dead load, w, has no effect on the stiffening truss at mean temperature, and is not considered, therefore, as a load on the truss. Even if $k\, l$ should be somewhat less than $j\, l$, it is evident that the loading for each loop falls somewhere between the two cases shown in Fig. 5. Consequently, the coefficient C in Equation (17) must lie between 9.6 and 8.0. To verify this, several deflection, moment, and load diagrams, computed precisely by the trigonometric series method, have been studied, omitting the effects of cable stretch due to live load and temperature change, and the effect of side-span interaction. These studies have been presented in detail elsewhere.[5]



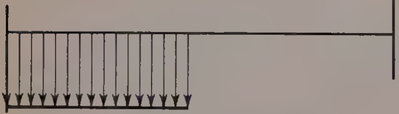
(*a*) MOMENT DIAGRAM



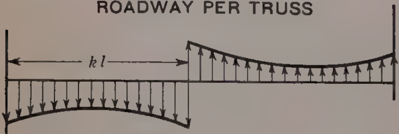
($w_s - \beta w$) DIAGRAM



(*c*) SUSPENDER LOADING, w_s



(*d*) LIVE LOAD, w_l, ON ROADWAY PER TRUSS



(*e*) FINAL TRUSS LOADING, $w_t = w_l + w_s$

FIG. 6.—MOMENT AND LOAD DIAGRAMS FOR STIFFENING TRUSS

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[5] "Related Functions of Cable and Stiffening Truss in Suspension Bridges," by Harold E. Wessman, Assoc. M. Am. Soc. C. E.; Thesis presented to the Univ. of Illinois in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

As a result of these studies and comparisons of results obtained by the preliminary design method with final values for a number of bridges computed by the deflection theory, the following values of the coefficient C, to be used in Equation (17), have been adopted for the preliminary design method:

Maximum positive or negative moment at:	Value of C
Quarter-point	9.1
Center	9.0

Specific Values of M_t.—Definite values of $j\,l$ for Equation (17) have been determined by using the average lengths of the bases of the deflection loops of the unstiffened cable. For positive moment at the quarter-point, $j\,l = 0.44\,l$. For negative moment at the same point, $j\,l = 0.46\,l$. For positive moment at the center, $j\,l = 0.37\,l$, and for negative moment at the center, $j\,l = 0.39\,l$. When these values and the preceding values of C are inserted in Equation (17), the following specific values of M_t are obtained:

Maximum positive moment at quarter-point:

$$M_t = 9.1\,\frac{E\,I\,\eta'}{(0.44\,l)^2} = 47.0\,\frac{E\,I\,\eta'}{l^2} \qquad (24a)$$

Maximum negative moment at quarter-point:

$$M_t = 9.1\,\frac{E\,I\,\eta'}{(0.46\,l)^2} = 43.0\,\frac{E\,I\,\eta'}{l^2} \qquad (24b)$$

Maximum positive moment at center:

$$M_t = 9.0\,\frac{E\,I\,\eta'}{(0.37\,l)^2} = 65.8\,\frac{E\,I\,\eta'}{l^2} \qquad (24c)$$

Maximum negative moment at center:

$$M_t = 9.0\,\frac{E\,I\,\eta'}{(0.39\,l)^2} = 59.2\,\frac{E\,I\,\eta'}{l^2} \qquad (24d)$$

Equations (24) are based upon a constant length, $j\,l$, regardless of the stiffness of the truss attached to the cable. In other words, they imply that the points of zero deflection for partial live loads (with cable stretch and side-span effects omitted) remain practically fixed in position. An inspection of the deflection loops in Fig. 2(a) and Fig. 2(b) indicates that the points of zero deflection do not move much for different ratios of live load to dead load. Consideration was also given to the possible effect of stiffness of truss upon the point of zero deflection, as it was thought that a very stiff truss might possibly cause the point to travel appreciably toward the unloaded end of the truss. To test this hypothesis, the main truss of the Manhattan Suspension Bridge, in New York, N. Y., was replaced by a truss ten times as stiff, and the deflections were then computed for a live load from 0 to $0.4\,l$, with cable stretch and side-span effects omitted. It was found that the point of zero deflection remained practically fixed in position. This check, combined with a number of comparisons of values obtained in actual designs, gave assurance relative to the use of the preliminary design method for a wide range in truss stiffness.

All the values for use in Equation (14*b*) are now established; η' is determined from Fig. 3(*a*) and Fig. 3(*b*); values of $H_w + H_a$ are found from Equations (16); and values of M_t are computed by using Equations (24). This completes the first and major step of the approximate method. It is now necessary to evaluate the effects of cable stretch, temperature change, and side-span interaction.

EFFECT OF CABLE STRETCH, TEMPERATURE CHANGE, AND SIDE-SPAN INTERACTION

Introduction.—When live load moves upon the bridge, or when a temperature change occurs, stress is induced in the cable. This stress causes a change in the normal length of the cable; that is, the length corresponding to full dead load and mean temperature. When a temperature change takes place, the cable lengthens or shortens a distance which is in addition to any length change produced by the stress from live load or temperature change. These changes affect the shape of the cable, increasing or decreasing the center sag and producing positive or negative bending moments in the stiffening truss.

Any change in cable stress or length of the cable also causes a change in the side-span sag. This, in turn, causes a movement of the tower tops which, finally, affects the center-span sag. A change in center sag is accompanied by a change in the cable stress, which will again modify the side-span sag. Evidently, there is an interaction between the side span and the main span, which occurs simultaneously with any change in loading or temperature. When the bending moments in the stiffening truss due to changes in cable length and side-span interaction have been found, the results are to be added algebraically to the moments from live load found by Equation (14*b*).

Change in Length of Cable Due to Stress.—The strain in any section of the cable of differential length, due to stress from live load or temperature, is:

$$\delta = \frac{H_a\, dL}{A\, E_c\, dx}\, dL \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The total strain or change in length of the main-span cable may then be expressed as follows:

$$\Delta L_s = \int_0^l \frac{H_a}{A\, E_c} \frac{dL^2}{dx} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

When evaluated, this gives the following approximate equation to be used for the main span in the preliminary design method:

$$\Delta L_s = \frac{H_a\, L}{A\, E_c\, l}\, L = \frac{H_a\, L^2}{A\, E_c\, l} \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

This simplified equation was used by Allston Dana, M. Am. Soc. C. E., and G. M. Rapp, Assoc. M. Am. Soc. C. E., in the cable calculations for the Delaware River Bridge, from Philadelphia, Pa., to Camden, N. J.[6] It should be noted that l is the horizontal length of the main span and L is the actual length of the cable; L is given quite accurately by Equation (28), as follows:

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[6] "The Delaware River Bridge Connecting Philadelphia, Pa., and Camden, N. J.," Final Rept. of the Board of Engrs., p. 107.

$$L = l\left(1 + \frac{8}{3} n^2\right) \qquad (28)$$

in which n = ratio $\frac{f}{l}$. (D. B. Steinman, M. Am. Soc. C. E., has published[7] a comparison of exact and approximate formulas for cable length.) Likewise, the change in length of the cable over each side span may be expressed as,

$$\Delta L_{1s} = \frac{H_a \, L_1^2}{A \, E_c \, l_1} \qquad (29a)$$

in which l_1 is the horizontal length of the side span, and L_1 is the actual length of the side-span cable, or (see Fig. 1):

$$L_1 = l_1 \left(\sec \alpha_1 + \frac{8}{3} \frac{n_1^2}{\sec^3 \alpha_1} \right) \qquad (29b)$$

There are other short sections of the cable over the tower saddles and at the anchorages, which also change length and contribute to the effects upon the stiffening truss. For one-half the bridge, these sections will be designated by the length term, L'. Then,

$$\Delta L_s' = \frac{H_a \, (L')^2}{A \, E_c \, l'} \qquad (30)$$

in which l' is the horizontal length of the section of cable corresponding to L'. At the tower saddle, l' is virtually equal to L'.

Change in Length of Cable Due to Temperature.—The change in length of the cable over the main span due to temperature variation is,

$$\Delta L_t = \omega \, \Delta t \, L \qquad (31a)$$

The corresponding change over the side span is,

$$\Delta L_{1t} = \omega \, \Delta t \, L_1 \qquad (31b)$$

The change in length of other sections of length, L', for one-half the bridge is,

$$\Delta L_t' = \omega \, \Delta t \, L' \qquad (31c)$$

It should be noted that the changes of length indicated by Equations (27), (29), (30), and (31) are separated into units corresponding to the main span, the side span, and incidental sections of the cable. This separation is convenient in order to evaluate the changes in cable sag due to cable strains.

Relations Between Cable Strain, Cable Sag, and Span Length.—Any change in the length of the cable is accompanied either by a change in the sag or by a change in the horizontal span length. The relation between a small change in sag and a small change in the length of cable, with the span length kept constant, is expressed approximately as follows:[8]

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[7] "Suspension Bridges," by D. B. Steinman, p. 6.

$$df = \frac{15}{16\,n\,(5 - 24\,n^2)}\,dL \quad \ldots\ldots (32a)$$

The relation between a small change in sag and a small change in span length with the cable length kept constant is:

$$df = -\frac{(15 - 40\,n^2 + 288\,n^4)}{16\,n\,(5 - 24\,n^2)}\,dl \quad \ldots\ldots (32b)$$

The relation between a small change in span length and a small change in cable length, with the sag kept constant, is:

$$dl = \frac{15}{(15 - 40\,n^2 + 288\,n^4)}\,dL \quad \ldots\ldots (32c)$$

Equations (32) apply to the main span, and, corresponding equations may be developed for the side span. Usually, the side-span sag ratio is so small that the terms involving n_1^2 and n_1^4 may be omitted without sacrificing reasonable accuracy in a preliminary design method. Then Equations (32) take the following form for the side span:

$$df_1 = \frac{3}{16\,n_1}\sec^3\alpha_1\,dL_1 \quad \ldots\ldots (33a)$$

$$df_1 = -\frac{3}{16\,n_1}\sec^4\alpha_1\,dl_1 \quad \ldots\ldots (33b)$$

and,

$$dl_1 = \frac{dL_1}{\sec\alpha_1} \quad \ldots\ldots (33c)$$

To find the change in the cable sag at the center of the main span due to cable stretch in the main span from stress, or due to change in length of the cable caused by temperature, the values of ΔL_s or ΔL_t from Equations (27) or (31*a*) are substituted for dL in Equation (32*a*); but changes of length, ΔL_1, of the cable in the side spans cause the tower tops to move, thus changing the horizontal length, l, of the main span by a length, dl. The horizontal movement of each tower top is approximately equal to $\Delta L_1 \sec \alpha_1$. If the side spans are symmetrical, the total change is as follows:

$$dl = -2\,\Delta L_1 \sec\alpha_1 \quad \ldots\ldots (34)$$

Any change in length of the part of the cable from the end of the side span to the anchorage must be included in ΔL_1 in Equation (34). Moreover, the change in length of the short section of the cable over the tower saddles causes the tower tops to move horizontally a distance equal to the change in length. This is doubled to account for the other side span and then added to the increment, dl, in Equation (34). When all the horizontal movements of the tower top have been combined, the differential change in sag, df, is found from Equation (32*b*).

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[8] "Modern Framed Structures," by Johnson, Bryan, and Turneaure, Pt. II, Ninth Edition, p. 199.

Effect of a Change in Sag Upon the Main-Span Stiffening Truss.—Whenever the center sag of the main-span cable changes by an amount, df, the stiffening truss is raised or lowered by the same amount at the center. Forces are transmitted to the truss through the suspenders or hangers. If the hanger forces are uniformly distributed, the moment in the truss at the center will be:

$$M = 9.6 \frac{E\,I}{l^2}\,df \qquad (35)$$

Moments at other points would vary as the ordinates of a parabola. The hanger forces are not uniformly distributed, however. An inspection of the $(w_s - \beta\,w)$ diagram, which must look like the moment diagram, will verify this. An investigation of exact load diagrams indicated that the stiffening truss is subject to a non-uniform hanger load of greatest intensity at the ends and decreasing toward the middle. The moment diagrams are slightly elliptic in shape rather than parabolic. Consequently, the following equations are proposed in the preliminary design method for the computation of the center moment in the stiffening trusses caused by changes in sag:

For the main span at the center,

$$M = 9.4 \frac{E\,I}{l^2}\,df \qquad (36a)$$

and, for the side span at the center,

$$M = 9.4 \frac{E\,I_1}{l_1^2}\,df_1 \qquad (36b)$$

For exact results, the value of M given by Equation (36*a*) should be adjusted, such adjustment being carried out by means of equations analogous to Equations (14*a*) and (14*b*); but the correction would be small, and not worth while.

The moment diagrams, as already indicated, are elliptic in shape. For a parabola, the moment ordinate at the quarter-point would be 0.75 of the center ordinate. For a true ellipse, the ordinate at the quarter-point is 0.866 of the center ordinate. A study of calculated moment diagrams for cable-stretch effects indicates that the quarter-point moment should be approximately 0.81 of the center moment for preliminary design purposes. Hence, the following equations are proposed for the computation of the moments at the quarter-points caused by changes in center sag:

For main span at the quarter-point,

$$M = 0.81 \times 9.4 \frac{E\,I}{l^2}\,df = 7.6 \frac{E\,I}{l^2}\,df \qquad (37a)$$

and, for the side span at the quarter-point,

$$M = 7.6 \frac{E\,I_1}{l_1^2}\,df_1 \qquad (37b)$$

Side-Span Interaction.—When the side spans of a suspension bridge are suspended from the cable, the side-span sag, f_1, is made such a value that the horizontal component of the cable stress under dead load and mean temperature has the same value for both the side span and the main span; then:

$$H_w = \frac{w\,l^2}{8f} = \frac{w_1\,l_1^2}{8\,f_1} \quad \text{(38a)}$$

or,

$$f_1 = \frac{w_1\,l_1^2}{w\,l^2} f \quad \text{(38b)}$$

What happens to the side-span sag when live load is placed on the main span, or when there is a change in temperature? First, consider the case with no stiffening truss in the side span. The horizontal component of cable stress has increased to the value $H_w + H_a$. If the tower saddle is free to move, or if the tower has very little resistance to horizontal forces at the top, the sag, f_1, in the side span must decrease by an amount df_1, in order to preserve equilibrium; then:

$$H_w + H_a = \frac{w_1\,l_1^2}{8\,(f_1 + df_1)} \quad \text{(39a)}$$

or,

$$(H_w + H_a)\,(f_1 + df_1) = \frac{w_1\,l_1^2}{8} = H_w\,f_1$$

and,

$$df_1 = -\frac{H_a}{H_a + H_w} f_1 \quad \text{(39b)}$$

In Equation (39*b*) the minus sign indicates a decrease in sag for positive values of H_a. When a stiffening truss is added to the side span, a decrease, df_1, in side-span sag creates negative bending moments in the stiffening truss, as indicated in Equations (36*b*) and (37*b*). At the center of the span, the equation of statics, $\Sigma\,M = 0$, is satisfied when,

$$(H_w + H_a)\,(f_1 + df_1) + 9.4\,\frac{E\,I_1}{l^2}\,df_1 = \frac{w_1\,l_1^2}{8} = H_w\,f_1 \quad \text{(40a)}$$

and,

$$df_1 = -\frac{H_a}{(H_w + H_a) + 9.4\,\frac{E\,I_1}{l_1^2}} f_1 \quad \text{(40b)}$$

Due to the stiffening truss, the change in sag in Equation (40*b*) has a smaller value than the change in sag given by Equation (39*b*). If there is a uniform live load, w_{1l}, over the side span, in addition to the dead load, the equation of equilibrium becomes,

$$(H_w + H_a)\,(f_1 + df_1) + 9.4\,\frac{E\,I_1}{l_1^2}\,df_1 = \frac{w_1\,l_1^2}{8} + \frac{w_{1l}\,l_1^2}{8} \quad \text{(41)}$$

but, since $\frac{w_1 l_1^2}{8} = H_w f_1$, and $\frac{w_{1l} l_1^2}{8} = \frac{w_{1l}}{w_1} H_w f_1$, Equation (41) becomes:

$$df_1 = \frac{- H_a + \frac{w_{1l}}{w_1} H_w}{(H_w + H_a) + 9.4 \frac{E I_1}{l_1^2}} f_1 \ldots\ldots\ldots\ldots (42)$$

It is often convenient to find cable-stretch effects and sag changes in terms of $H_a = 1\,000\,000$ lb, or $H_a = 100\,000$ lb, and then compute the effects for the final value of H_a by direct proportion. This is accurate enough when H_a is small relative to H_w. A trial value of H_a based on Equations (16) is first used. Sag changes are then computed for temperature effect and cable stretch from stress, using this trial value of H_a. From the new sag, a new value of H_a is determined, and the process is repeated until the trial H_a and the final H_a agree, or differ by a very small amount.

The total change in the sag f at the center of the main span due to temperature, cable stretch, and side-span interaction is then consistent with the final value of H_a. From the total sag change, df, the moments in the stiffening truss at the quarter-point and at the center are found from Equations (36*a*) and (37*a*). These moments are added algebraically to the live load moments found from Equation (14*b*), in order to find the final values at the quarter-point and the center to be used in the preliminary design of the stiffening truss.

Since the sag change, df_1, in the side span must be found in order to compute maximum positive or negative moments in the main span, and since the side span is either unloaded or fully loaded with live load to find these main-span moments, it requires little additional computation to find the maximum positive and negative moments at the center of the side span for preliminary design purposes. The value of df_1, when found, is inserted in Equation (36*b*), and the moment is then evaluated.

Computations Illustrating the Preliminary Design Method

Triborough Suspension Bridge.—A definite structure, the Triborough Suspension Bridge, has been used to demonstrate the typical preliminary design computations rather than some fictitious span, in order that the results obtained by the preliminary analysis may be compared with final values for an actual design.

The procedure outlined in Table 2 will be exactly the same as that for the investigation of an entirely new layout, except for the choice of the moments of inertia of stiffening trusses. Preliminary design data and constants are given in Table 1(*a*), Column (2). In the following computations (Tables 1 and 2) final design values for the moments of inertia of the stiffening trusses are used. In a new layout, it is advisable to use an arbitrarily selected moment of inertia, based on a tentative chord section and truss depth, for the initial determination of maximum moments and deflections. Then the procedure should be repeated with other values of moments of inertia, say, one-half as much and two times as much as those used in the first analysis. By doing this, a fairly accurate idea is obtained of the manner in which maximum bending moments vary with truss stiffness. Moreover, a good measure of the reduction in the deflections of an unstiffened cable, caused by each trial truss, is afforded by this process.

Maximum positive and negative moments at the quarter-point and center of the main span, and at the center of the side span, are computed. Considerable space is devoted to the explanations accompanying each step of the method; as a result, the computations may seem to be more extensive than they actually are. Ordinarily, the numerical computations, which are slide-rule results, require very little space and relatively little time when compared

TABLE 1.—COMPARISON OF DESIGN CHARACTERISTICS OF SUSPENSION BRIDGES

Description (1)	Triborough Bridge, at New York, N. Y. (2)	San Francisco-Oakland Bay Bridge, in California (3)	Manhattan Bridge, at New York, N. Y. (4)	Maumee River Bridge (5)	Mount Hope Bridge, in Rhode Island (6)	Moline-Bettendorf Bridge, at Moline, Ill. (7)
(*a*) DESIGN CONSTANTS						
Live load, w_l, in pounds per foot of truss	2 000	3 000	4 000	2 000	750	1 120
Total dead load, w, in pounds per foot of truss	9 580	9 200	5 820	6 500	2 650	2 185
Ratio, $w_l \div w$, of live load to dead load	0.209	0.326	0.687	0.308	0.783	0.512
Dead load, w_1, in pounds per foot of side span truss	9 670	9 700	6 130	6 500	2 650	2 230
Lengths, in Feet:						
Main span, l	1 358	2 294	1 447	777.6	1 188	735
Side span, l_1	660.7	1 152	713.5	230.3	498.3	366.7
Main span cable, L	1 394	2 354	1 485	811.1	1 220	753.9
West side-span cable, L_1	685	1 192	740.0	259.5	521.4	377.6
East side-span cable, L_1		1 184				
Short cable section, L', at tower	22	16.3	31.8	9.7	12.5	5.2
Short cable section, L', at anchorage	108	913.4*	119.1	266.6	341.3	193.4
Sag of cable, f, at center of main span	133.7	228.7	145.3	96.0	118.8	73.0
Sag of cable, f_1, at center of side span	31.6	60.8	37.2	8.41	20.9	18.42
Ratio, $n = f \div l$	0.0985	0.0997	0.1004	0.1234	0.10	0.0994
Ratio, $n_1 = f_1 \div l_1$	0.0478	0.0528	0.0522	0.0365	0.0419	0.0502
Moment of Inertia (Unit, Inches^2 — Feet^2):						
Stiffening truss of main span	15 000	69 500	43 900	2 600	4 259	1 835
Stiffening truss of side span	17 500	99 100	50 860	2 200	4 152	2 115
Horizontal component, H_w, of cable pull, in kips	16 450	25 810	10 480	5 120	3 940	2 030
Modulus of Elasticity, in Kips per Square Inch:						
Stiffening truss and towers, E	29 000	29 000	29 000	29 000	29 000	29 000
Cable, E_c	28 000	28 000	29 000	27 000	29 000	24 000
Area of cable, A, in square inches	276.8	5 000	275	106.6	73.9	44.9
Temperature change, Δt, in degrees Fahrenheit	±55	±30	±55	±55	±60	±60
Coefficient of thermal expansion, in ten-millionths	65	65	66	65	65	65
Secant of Slope Angle, α:						
West span	1.03	1.03	1.03	1.12	1.04	1.03
East span		1.02				
(*b*) COMPARISON OF PRELIMINARY DESIGN METHOD WITH DEFLECTION THEORY (UNITS, IN FOOT-KIPS)						
Main Span; Maximum Positive Moment at:						
Quarter-point, by preliminary design	28 500	111 800	113 300	11 300	10 300	8 400
Quarter-point, by deflection theory	28 000	111 000	119 100	11 580	10 390	9 095
Center, by preliminary design	23 600	90 400	91 300	8 450	8 350	7 060
Center, by deflection theory	22 100	88 830	89 710	7 900	7 954	6 665
Main span; deflection at quarter-point, in feet	4.58	10.1	7.4	3.0	4.5	3.7
Side Span; Maximum Positive Moment at:						
Center, by preliminary design	43 400	209 000	181 000	9 400	12 940	12 750
Center, by deflection theory	42 300	208 230	181 000	9 550	12 970	13 000

* West anchorage, 913.4 ft; and east anchorage, 56.6 ft.

TABLE 2.—PRELIMINARY DESIGN OF TRIBOROUGH SUSPENSION BRIDGE

Step No.	Explanation	Computation	Result	Units
	(a) MAXIMUM POSITIVE MOMENT AT THE QUARTER-POINT OF THE MAIN SPAN			
1	From Fig. 3(a), for $w_l \div w = 0.209$ and $f = 133.7$:			
	η' at the quarter-point	0.0242×133.7	3.24	Feet
	η_c' at the center	-0.0075×133.7	−1.00	Feet
	New center sag, $f + \eta_c'$	$133.7 - 1.0$	132.7	Feet
2	H_w, by Equation (15b)	$2\,200\,000 \div 133.7$	16 450	Kips‡
	$H_w + H_a$, by Equation (16a)	$0.04 \times 2\,000 \times 1\,358^2 = 148\,000$ $2\,348\,000 \div 132.7$	17 680	Kips‡
3	Trial value of $H_a = (H_w + H_a) - H_w$	....	1 230	Kips‡
4	M_t from Equation (24a)	$(47.0 \times 29 \times 10^6 \times 15\,000 \times 3.24) \div 1\,358^2$	35 900	Foot-kips‡
	Moment induced by forcing cable back through the distance, $\eta' = (H_w + H_a)\eta'$	$17\,680 \times 3.24$	57 300	Foot-kips
	Adjusted moment in truss, from Equation (14b)	$(35\,900 \times 57\,300) \div 93\,200$	22 100	Foot-kips
5	Adjusted deflection, η	$3.24 \times 57\,300 \div 93\,200$	1.99	Feet
6	Change in Cable Length, ΔL Due to a Temperature Change, $\Delta t = +55°$ F (see Equation (31)):			
	(a) In the main-span cable	$0.0000065 \times 55 \times 1\,394$	0.498	Feet
	(b) In the side-span cables	$0.0000065 \times 55 \times 685$	0.245	Feet
	(c) In the cable at the tower saddle	$0.0000065 \times 55 \times 22$	0.008	Feet
	(d) In the cable at the anchorages	$0.0000065 \times 55 \times 108$	0.039	Feet
7	Change in Cable, ΔL, Due to $H_a = 1\,000$ Kips:			
	(a) In main-span cable, by Equation (27)	$\dfrac{1\,000\,000 \times 1\,394^2}{277 \times 28 \times 10^6 \times 1\,358} = 0.000129\,\dfrac{1\,394^2}{1\,358}$	0.185	Feet
	(b) In side-span cable, by Equation (29a)	$0.000129 \times 685^2 \div 661$	0.092	Feet
	(c) At the tower saddle, by Equation (30)	0.000129×22	0.003	Feet
	(d) At the anchorage, by Equation (30)	$0.000129 \times 108^2 \div 105$	0.014	Feet
8	Change in Center Sag, df, Due to a Temperature Change, $\Delta t = +55°$ F:			
	(a) From ΔL_t in main-span cable (Equation 32a)	$\dfrac{15 \times 0.498^*}{16 \times 0.0985(5 - 24 \times 0.0097)} = 2.0 \times 0.498^*$	1.00	Feet
	(b) From Equation (34), for ΔL_{lt} in side-span, $dl =$	$-2 \times 0.245^\dagger \times 1.03 = -0.50$	....	....
	and, from Equation (32b), $df =$	$-\dfrac{15 - 0.39 + 0.027}{7.51}(-0.50) = 1.95 \times 0.50$	0.97	Feet
	(c) From Equation (32b) for $\Delta L_t'$ at tower saddle; $df =$	$1.95 \times 2 \times 0.008^a$	0.03	Feet
	(d) From Equation (34), for $\Delta L_t'$ at anchorage; $dl =$	$-2 \times 0.039^b \times 1.03 = -0.081$	....	....
	and, from Equation (32b), $df =$	1.95×0.081	0.16	Feet
	(e) Total, Steps 8(a), 8(b), 8(c), and 8(d)	....	+2.16	Feet

TABLE 2.—(*Continued*)

Step No.	Explanation	Computation	Result	Units
	(*a*) MAXIMUM POSITIVE MOMENT AT THE QUARTER-POINT OF THE MAIN SPAN—(*Continued*)			
9	*df* Due to H_a = 1 000 Kips:			
	(*a*) From Equation (32*a*), for ΔL_s in main-span cable.........	2.0×0.185^c	0.37	Feet
	(*b*) From Equation (34) for ΔL_s in side-span cable, *dl* = ...	$-2 \times 0.092^d \times 1.03 = -0.19$		
	and from Equation (32*b*), *df* =	1.95×0.19	0.37	Feet
	(*c*) From Equation (32*b*), for ΔL_s at tower saddle............	$1.95 \times 2 \times 0.003^e$	0.01	Feet
	(*d*) From Equations (32*b*) and (34), for $\Delta L_c'$ at the anchorages: *dl* = ...	$-2 \times 0.014^f \times 1.03 = -0.029$		
	df = ...	1.95×0.029	0.06	Feet
	(*e*) Total, Steps 9(*a*), 9(*b*), 9(*c*), and 9(*d*).................	....	+0.81	Feet
10	*df* Due to Side-Span Interaction, for H_a = 1 000 Kips:			
	(*a*) From Equation (40*b*), df_1 = ..	$-\frac{1\,000 \times 31.6}{17\,450 \times 10\,900}$	−1.11	Feet
	(*b*) From Equation (33*b*), dl_1 = ..	$\frac{16 \times 0.0478}{3 \times 1.03^4} \times 1.11$	0.252	Feet
	(*c*) $dl = -2\,dl_1$ = ...	-2×0.252	−0.504	Feet
	(*d*) From (32*b*), *df* =..........	1.95×0.504	+0.98	Feet
	Total change, *df* (Step 9(*e*) + Step 10(*d*))	....	+1.79	Feet
11	(*a*) Trial value of H_a (see Step 3)......	....	1 230	Kips‡
	(*b*) *df* due to Δt = + 55° F, and H_a = 1 230 kips	$2.16 + 1.23 \times 1.79$	4.36	Feet
	(*c*) Corrected center sag............	$132.7^g + 4.36$	137.1	Feet
	(*d*) $H_w + H_a$......................	$2\,348\,000^h \div 137.1$	17 130	Kips‡
	(*e*) $H_a = (H_w + H_a) - H_w$.........	$17\,130 - 16\,450$	680	Kips‡
	(*f*) New trial, H_a..................	Between 680 and 1 230; assume, H_a =	780	Kips‡
	(*g*) *df* due to Δt = + 55° F, and H_a = 780 kips	$2.16 + 0.78 \times 1.79$	3.55	Feet
	(*h*) Revised center sag, *f* = ...	$132.7 + 3.55$	136.3	Feet
	(*i*) $H_w + H_a$ =..................	....	17 230	Kips‡
	(*j*) H_a (this value checks Step 11(*f*)) ..	....	780	Kips‡
	(*k*) Therefore, the final change, *df*, and the deflection of the truss, is.....	$2.16 + 0.78 \times 1.79$	+3.55	Feet
12	Moment at quarter-point of truss (Equation 37(*a*))................	$7.6 \times 29 \times 10^6 \times 15\,000 \times 3.55 \div 1\,358^2$	+6 370	Foot-kips‡
13	Final moment, from Steps 4 and 12..	$22\,100 + 6\,370$	+28 500	Foot-kips‡
14	Final moment, by the deflection theory..........................	....	+28 000	Foot-kips‡
15	Total deflection, η.................	$1.99^i + 0.73^j \times 3.55^k$	+4.58	Feet
	(*b*) MAXIMUM POSITIVE MOMENT AT THE CENTER OF THE MAIN SPAN			
1	From Fig. 3(*b*), for $w_l \div w$ = 0.209 and *f* = 133.7: η' at the center =.............	0.0133×133.7	1.78	Feet
	New center sag, $f + \eta'$ =..........	133.7×1.78	135.5	Feet
2	H_w, by Equation (15*b*) =..........	$2\,200\,000 \div 133.7$	16 450	Kips‡
	$H_w + H_a$, by Equation (16*c*) =.....	$0.0638 \times 2\,000 \times 1\,358^2 = 235\,000$		Kips‡
		$2\,435\,000 \div 135.5$	17 950	Kips‡
3	Trial value of $H_a = (H_w + H_a) - H_w$	....	1 500	Kips‡
4	M_t, from Equation (24*c*) =..........	$(65.8 \times 29 \times 10^6 \times 15\,000 \times 1.78) \div 1\,358^2$	27 600	Foot-kips‡
	Moment induced by forcing cable back through the distance, η' $= (H_w + H_a)\eta'$................	$17\,950 \times 1.78$	32 100	Foot-kips‡
	Adjusted moment in truss, by Equation (14*b*)......................	$27\,600 \times 32\,100 \div 59\,700$	+14 800	Foot-kips‡

TABLE 2.—(*Continued*)

Step No.	Explanation	Computation	Result	Units
	(*b*) MAXIMUM POSITIVE MOMENT AT THE CENTER OF THE MAIN SPAN—(*Continued*)			
5	Adjusted deflection, η	$1.78 \times 32\,100 \div 59\,700$	0.95	Feet
6–10	Same as Table 2(*a*)	....	....	....
11	(*a*) Trial value of H_a (see Step 3) (*b*) df due to $\Delta t = +55°$ F, and $H_a = 1\,500$ kips (*c*) Corrected center sag (*d*) $H_w + H_a$ (*e*) $H_a = (H_w + H_a) - H_w$ (*f*) New trial, H_a = (*g*) df due to $\Delta t = +55°$ F and $H_a = 1\,000$ kips (*h*) Revised center sag, f = (*i*) $H_w + H_a$ (*j*) H_a (this value checks Step 11(*f*)) (*k*) Therefore, the final change, df, and the deflection of the truss, is	.... $2.16 + 1.5 \times 1.79$ $135.5 + 4.85$ $2\,435\,000^h \div 140.4$ .... Between 880 and 1 500; assume H_a = $2.16 + 1 \times 1.79$ $135.5 + 3.95$	1 500 4.85 140.4 17 330 880 1 000 3.95 139.5 17 450 1 000 3.95	Kips‡ Feet Feet Kips‡ Kips‡ Kips‡ Feet Feet Kips‡ Kips‡ Feet
12	Moment at center of truss (Equation (36*a*))	$9.4 \times 29 \times 10^6 \times 15\,000 \times 3.95 \div 1\,358^2$	+8 760	Foot-kips‡
13	Final moment, from Steps 4 and 12	....	+23 600	Foot-kips‡
14	Final moment, by the deflection theory	....	+22 100	Foot-kips‡
15	Total deflection, η	$0.95 + 3.95$	4.90	Feet
	(*c*) MAXIMUM NEGATIVE MOMENT AT THE QUARTER-POINT			
1	From Fig. 3(*a*), for $w_l \div w = 0.209$ and $f = 133.7$: η' at the quarter-point = η_c' at the center = New center sag, $f + \eta_c'$ =	 -0.0242×133.7 0.0048×133.7 $133.7 + 0.64$	 −3.24 0.64 134.3	 Feet Feet Feet
2	H_w by Equation (15*b*) = $H_w + H_a$, by Equation (16*b*) =	$2\,200\,000 \div 133.7$ $0.085 \times 2\,000 \times 1\,358^3 = 314\,000$ $2\,514\,000 \div 134.3$	16 450 18 700	Kips‡ Kips‡
3	Trial value of $H_a = (H_w + H_a) - H_w$	....	2 250	Kips‡
4	M_t, from Equation (24*b*) Moment induced by forcing cable back through the distance, $\eta' = (H_w + H_a)\eta'$ Adjusted moment in truss, by Equation (14*b*)	$(-43.0 \times 29 \times 10^8 \times 15\,000 \times 3.24) \div 1\,358^2$ $18\,700 \times 3.24$ $-32\,800 \times 60\,600 \div 93\,400$	−32 800 −60 600 −21 300	Foot-kips‡ Foot-kips‡ Foot-kips‡
5	Adjusted deflection, η	$-3.24 \times 60\,600 \div 93\,400$	−2.10	Feet
6	ΔL when $\Delta t = -55°$ F	(Same as Step 6, Table 2(*a*), with all negative values)	....	.....
7	ΔL due to $H_a = 1\,000$ kips	(Same as Step 7, Table 2(*a*))	....	....
8	df when $\Delta t = -55°$ F	(Same as Step 8, Table 2(*a*), with opposite signs) $\Sigma\, df$ =	−2.16	Feet
9	df due to $H_a = 1\,000$ kips	(Same as Step 9, Table 2(*a*)); $\Sigma\, df$	+0.81	Feet
10	df Due to Side-Span Interaction, for $H_a = 1\,000$ kips: (*a*) From Equation (42), df_1 = (*b*) dl_1 = from Equation (33*b*) (*c*) $dl = -2\,dl_1$ = (*d*) From Equation (32*b*), df =	 $\frac{-2\,250^m + (2\,000 \div 9\,670)16\,450}{18\,700 + 10\,900} \times 31.6$ $-\frac{16 \times 0.0478 \times 1.23}{3 \times 1.03^4}$ 2×0.28 -1.95×0.56	 1.23 −0.28 0.56 −1.09	 Feet Feet Feet Feet

TABLE 2.—(*Continued*)

Step No.	Explanation	Computation	Result	Units
	(*c*) MAXIMUM NEGATIVE MOMENT AT THE QUARTER-POINT—(*Continued*)			
11	(*a*) Trial value of H_a (see Step 3) .	....	2 250	Kips‡
	(*b*) df due to $\Delta t = -55°$ F, and $H_a = 2\,250$ kips..........	$-2.16 + 2.25 \times 0.81 - 1.09$	−1.42	Feet
	(*c*) Corrected center sag.........	$134.3^g - 1.42$	132.9	Feet
	(*d*) $(H_w + H_a)$................	$2\,514\,000^h \div 132.9$	18 900	Kips‡
	(*e*) $H_a = (H_w + H_a) - H_w$.....	....	2 450	Kips‡
	$(f)^n$ Repeat Steps 10 and 11 to find H_a.................	....	2 400	Kips‡
	(*k*) $\Sigma\, df$ for $\Delta t = -55°$ F and $H_a = 2\,400$ kips...........	$-2.16 + 2.4 \times 0.81 - 0.94$	−1.14	Feet
12	Moment at quarter-point of truss (Equation (37*a*))...............	$7.6 \times 29 \times 10^6 \times 15\,000 \times (-1.14) \div 1\,358^2$	−2 000	Foot-kips‡
13	Final moments from Steps 4 and 12..	....	−23 300	Foot-kips‡
14	Final moments, by deflection theory .	....	−22 700	Foot-kips‡
15	Total deflection, η, from Steps 5 and 11.	$-2.10 + 73^i(-1.14)$	−2.93	Feet
	(*d*) MAXIMUM NEGATIVE MOMENT AT THE CENTER OF THE MAIN SPAN			
13	Final moment from Steps 1 to 12....	$-15\,000 - 3\,300$	−18 300	Foot-kips‡
14	Final moment by the deflection theory	....	−16 500	Foot-kips‡
15	Total deflection, η.................	$-1.08 - 1.52$	−2.60	Feet
	(*e*) MAXIMUM POSITIVE MOMENT AT THE CENTER OF THE SIDE SPAN^o			
1	H_w, by Equation (15*b*) =..........	$2\,200\,000 - 133.7$	16 450	Kips‡
2	df due to $\Delta t = +55°$ F.............	....	2.16	Feet
3	df due to $H_a = 100$ kips...........	....	0.08	Feet
4	df Due to Interaction of Loaded Side Span, for $H_a = +100\,000$ lb:			
	(*a*) From Equation (42), df_1 =...	$\frac{-100 + 3\,400}{16\,550 + 10\,900}\, 31.6 =$	+3.80	Feet
	(*b*) From Equation (33*b*), dl_1 = ..	$-0.226 \times 3.80 =$	−0.858	Feet
	(*c*) From one side span, $dl = -dl_1$..............	....	+0.858	Feet
	(*d*) From Equation (32*b*), df =...	$-1.95 \times 0.858 =$	−1.67	Feet
	df Due to Interaction of Unloaded Side Span:			
	(*e*) From Equation (40*b*), df_1 = ..	$\frac{-100 \times 31.6}{16\,550 + 10\,900}$	−0.115	Feet
	(*f*) By proportion from Step 4(*d*), df =.....................	....	0.05	Feet
	Total change, $\Sigma\, df$, from both side spans (Step 4(*d*) + Step 4(*f*))......	....	−1.62	Feet
5	(*b*) df due to $\Delta t = +55°$ F, and $H_a = +100$ kips..............	$2.16 + 0.08 - 1.62$	0.62	Feet
	(*c*) Revised center sag, f	$133.7 + 0.62$	134.3	Feet
	(*d*) $H_w + H_a$ =................	$2\,200\,000 \div 134.3$	16 360	Kips‡
	(*e*) H_a..........................	....	−90	Kips‡
	(*f*) New trial H_a = ...	Between −90 and +100; assume, H_a =	−50	Kips‡
	(*g*) Repeat Steps 4 and 5; H_a = (practically)......................	....	−40	Kips‡
6	df_1, due to $H_a = -40$ kips = (Equation (42))....................	$\frac{+40 + 3\,400}{16\,410 + 10\,900}\, 31.6$	+3.98	Feet

TABLE 2.—(*Continued*)

Step No.	Explanation	Computation	Result	Units
	(*e*) MAXIMUM POSITIVE MOMENT AT THE CENTER OF THE SIDE SPAN[o]—(*Continued*)			
7	Moment at center of side span (Equation (36*b*))	$(9.4 \times 29 \times 10^6 \times 17\,500 \times 3.98) \div 661^2$	+43 400	Foot-kips‡
8	Moment from deflection theory	....	+42 250	Foot-kips‡
	(*f*) MAXIMUM NEGATIVE MOMENT AT THE CENTER OF THE SIDE SPAN[p]			
6	Final change, *df*, in sag of unloaded side span	....	−3.30	Feet
7	Moment at center	....	−36 000	Foot-kips‡
8	Moment from deflection theory =	....	−34 200	Foot-kips‡

* See Step 6(*a*). † See Step 6(*b*). ‡ The result is given in kips, or thousands of pounds to conserve space. [a] See Step 6(*c*). [b] See Step 6(*d*). [c] See Step 7(*a*). [d] See Step 7(*b*). [e] See Step 7(*c*). [f] See Step 7(*d*). [g] See Step 1. [h] See Step 2. [i] See Step 5. [j] At the quarter-point, η is about 0.73 times its value at the center. This was determined from a study of precise deflections due to cable stretching. [k] See Step 11(*g*). [m] In Equation (42) use a trial value of $H_a = 2\,250\,000$ lb from Step 3, Table 1(*c*), because this formula is not a linear function of H_a. [n] Steps 10(*f*), 10(*g*), 10(*h*), 10(*i*), and 10(*j*). [o] The side span under consideration is fully loaded with live load; the main spans and the other side span are not loaded; and the value, Δt, is + 55° F. [p] The span under consideration is not loaded; the remainder of the bridge is fully loaded; and the value of Δt is − 55° F.

to the procedure necessary in using the deflection theory or the trigonometric series method. A study of the computations emphasizes the importance of Fig. 3(*a*) and Fig. 3(*d*).

Preliminary Design Values for Other Suspension Bridges.—Maximum moments and corresponding deflections have been computed by the preliminary design method for five additional suspension bridges, ranging from a relatively short and light span to a long and heavy span. Detail computations are omitted from the paper; but the constants for each bridge are given in full in Table 1(*a*). The final results are tabulated in Table 1(*b*), together with the corresponding moments computed by the deflection theory.

Effect of Variation in Truss Stiffness.—In preliminary design studies of suspension bridges, it is important to know approximately what happens to moments and deflections when the truss stiffness is varied. The results in Table 1(*b*) for the San Francisco-Oakland Bay Suspension Bridge are based upon the equivalent uniform moments of inertia of the main span and side-span trusses as actually designed. If an unstiffened cable had been used, the downward deflection at the quarter-point would have been approximately 14.2 ft. Consequently, the truss as designed has reduced the deflection of the cable at the quarter-point from about 14.2 ft to about 10.1 ft.

If the moments of inertia of the main-span and side-span trusses are doubled, the moments become larger and the deflections decrease. If the moments of inertia are decreased, the moments decrease, but the deflections become greater. Fig. 7 shows theoretical values plotted for moments of inertia, I_t, four times, twice, one-half, and one-quarter as much as the moment of inertia I_a used in the actual design. The curves are not exact; nevertheless they give the designer a fairly accurate picture of what happens.

Fig. 7(*a*) indicates that the most economical stiffening truss is no stiffening truss whatever; but Fig. 7(*b*) indicates that the deflections of an unstiffened cable may be too great, and that, therefore, a stiffening truss may be needed. The maximum grade change is probably of more importance than the maximum deflection; even so, Fig. 7(*b*) gives a clue as to what may be expected with trusses of different stiffness, since the deflections and grade changes will vary in about the same ratio. What constitutes the limiting grade change, however, is still a subject for research. Present tendencies are to make stiffening trusses more flexible. Studies like those embodied in Fig. 7 may then be of great help in selecting a proper design. As stated[9] by Hardy Cross, M. Am. Soc. C. E.: "Where the action is hybrid there are many possible structures; the designer must make his choice." The problem becomes one of intelligent selection.

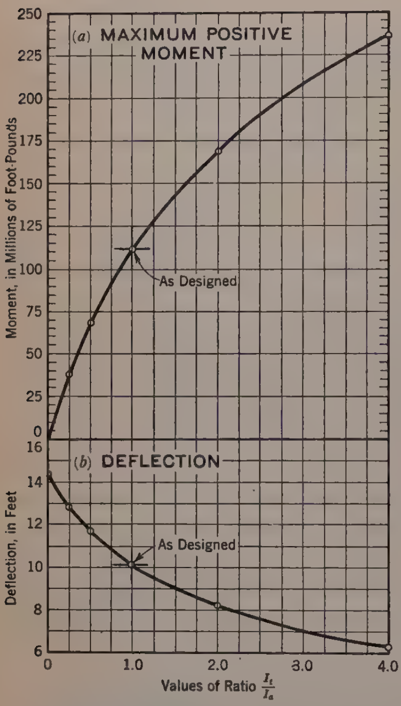

FIG. 7.—EFFECTS AT QUARTER-POINT OF MAIN SPAN OF SAN FRANCISCO-OAKLAND BAY BRIDGE DUE TO CHANGES IN TRUSS STIFFNESS

CONCLUSION

A rapid and accurate method of analysis for the preliminary design of suspension bridges is presented in this paper. The method begins logically with an unstiffened cable and then shows what happens when a stiffening truss is added. The effects of cable stretch from stress and from temperature, and the effect of side-span interaction, are obtained separately, thus giving the designer a good idea of the contributions of the various factors to the final moment and deflection in the stiffening truss. The method demonstrates clearly the related functions of cable and stiffening trusses in suspension bridges.

The method emphasizes the fact that there is no single design for a stiffening truss. There are any number, depending upon what limits are established for deflection and change of grade. The method will also prove a valuable aid in making additional studies which are needed to "throw light" upon suspension bridge design. These studies are indicated in the following questions: When may the stiffening truss be omitted? What is the maximum permissible grade change? Is it more economical to obtain stiffness by increasing the chord section or by increasing the depth of the truss? May one count on the floor system to help the stiffening truss in reducing cable deflections? If so, how must the floor system be connected to insure participation? Will participation "hurt" the floor system more than it will "help" the truss? When should one use alloy steels with higher working stresses than those for ordinary structural steel? Is there a place for alloys with different moduli of elasticity? What are the economics of using light-weight floors?

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[9] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1369.

Acknowledgments

The writers greatly appreciate the valuable suggestions made during the course of this study by Professor Cross. They also acknowledge the generous co-operation of the following engineers, who supplied final design data for various suspension bridges: Allston Dana, Leon S. Moisseiff, D. B. Steinman, Montgomery B. Case, and C. E. Paine, Members, Am. Soc. C. E., and Ralph A. Tudor, Assoc. M. Am. Soc. C. E.

APPENDIX

Notation

The following symbols, introduced where they first appear in the paper, conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Materials" compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:[10]

A = area; cross-sectional area of cable;

C = constants;

E = modulus of elasticity of the stiffening truss and towers; E_c = modulus of elasticity of the cable;

f = sag of cable at the center of the main span; f_1 = sag of cable at the center of the side span;

H = horizontal component of force; H_w = component of cable stress due to dead load and mean temperature; H_a = additional component due to any change from the condition of dead load at mean temperature;

I = moment of inertia of the stiffening truss of the main span; I_1 = moment of inertia of the stiffening truss of the side span;

k = a ratio defining loaded length of stiffening truss;

L = length of cable in main span; L_1 = length of cable in side span;

l = horizontal length of main span; l_1 = horizontal length of side span;

M = moment; actual bending moment in the stiffening truss; M_t = bending moment induced in the stiffening truss if it were bent to the deflection η' of the unstiffened cable; M' = bending moment in the stiffening truss, assuming it to be a simply supported beam;

t = temperature; Δt = temperature change, or variation;

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[10] A. S. A.—*Z*10*a*—1932.

V = actual shear in stiffening truss at any section; V' = shear at any section in the stiffening truss, assuming it be a simply supported beam;

w = total dead load per unit horizontal length of cable; w_l = live load on the roadway per unit length of truss; w_s = suspender pull expressed as a load per unit of horizontal length of cable due to live load or temperature, or both; w_t = final load acting on truss per unit length.

x = co-ordinate horizontal distance of any point of the cable curve, measured from an origin at the top of the left tower, under dead load at mean temperature;

y = co-ordinate vertical distance corresponding to x;

α = slope of the straight line joining the tower tops; α_1 = slope of the chord of the side-span cable;

β = a ratio, $\frac{H_a}{H_w}$;

δ = unit elongation; strain;

η = deflection of truss and main cable, at any section, from its initial position, under dead load at mean temperature; η' = corresponding deflection of unstiffened main cable; and

ω = coefficient of thermal expansion.

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PAPERS

STRUCTURAL BEHAVIOR OF BATTLE-DECK FLOOR SYSTEMS

By Inge Lyse,[1] M. Am. Soc. C. E., and Ingvald E. Madsen,[2] Jun. Am. Soc. C. E.

Synopsis

Results of two years of investigating the behavior of battle-deck flooring are reported in this paper. Four one-third sized models and one full-sized floor panel, designed on the basis of the results obtained on the preliminary models, were tested under the action of a concentrated wheel load, such as the specifications recommended for H-20 loading. The battle-deck flooring acted as an integral unit distributing the wheel load over various stringers, with the amount of load taken by the several stringers depending on their spacing. The plate acted with the stringer so as to form a T-beam which, if taken into account in the design, might result in an economy of 10 to 15 per cent. The width of plate contributing to the T-beam action was also found to depend on the stringer spacing. When the stringers were coped in on the floor-beams, partial fixation resulted with further economy in design. The models were loaded with dead weights, and the full-sized floor panel was tested by means of a jack and spring device. The test results gave a basis on which to formulate a rational design method for battle-deck floor systems.

Introduction

Experiments were made on four one-third sized models, based on a prototype bridge floor consisting of two panels 20 ft long and 10 ft wide. The first model represented a floor with a $\frac{3}{8}$-in. plate on stringers, spaced at 24 in. The tests showed that such a floor was not capable of supporting an H-20 loading. Consequently, the second model had stringers welded between those of the first model, making the prototype a floor with a $\frac{3}{8}$-in. plate on stringers, spaced

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Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **May 15, 1938.**

[1] Research Prof. of Eng. Materials, in Chg., Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

[2] Asst. on Eng. Corps, P. R. R., Supervisor's Office, Dennison, Ohio; formerly Am. Inst. of Steel Constr. Research Fellow, Lehigh Univ., Bethlehem, Pa., in Immediate Chg. of Battle-Deck Investigation.

12 in., center to center. This model proved adequate in supporting the load, and the next step was to determine the more economical design. Comparative cost data indicated that although a widening of the stringer spacing might increase the weight of the floor slightly, the decrease in welding cost would more than offset the gain in weight. Thus, for the third floor model, a prototype was selected which consisted of a $\frac{3}{4}$-in. plate laid on stringers spaced at 30 in., and for the fourth model, a $\frac{9}{16}$-in. plate on stringers spaced at 24 in.

The results of the model tests showed that the floors behaved according to certain relationships. Design methods and procedure, as established from these results, served for the construction of the full-sized floor panel. The agreement between the design stresses and the measured stresses in this full-sized floor indicated a check on the design assumptions. The full-sized floor consisted of 12-in. standard I-beams, spaced on 26-in. centers, with a $\frac{11}{16}$-in. plate. The floor was 16 ft 9 in. in span and 9 ft 5 in. wide. This span length was adopted because the full-sized panel was tested as a simple beam with the stringers resting on the floor-beams, representing the distance between the points of contra-flexure of a 20-ft panel length with the stringers in the bridge floor coped into the beams.

THE PROBLEM

The problem of determining the behavior of battle-deck flooring may be divided broadly into two parts: (1) The stresses in the plate; and (2) the action of the stringer; that is, the amount of load carried by each stringer and the interaction of plate and stringers in resisting flexural stresses. The investigation was limited to the study of these conditions in relation to battle-deck flooring for highway bridges subjected to concentrated loads. For an adequate solution of the problem it was necessary to determine:

(*a*) The strength and deflection of battle-deck flooring under concentrated loads;

(*b*) The properties of the floor-plate in distributing the load over various stringers;

(*c*) The width of plate acting with the stringers as the compression flange of a T-beam;

(*d*) The length of plate under concentrated load affected by the load;

(*e*) The effect of changing the distance between the stringers;

(*f*) The degree of fixation at the ends of the stringers; and,

(*g*) The degree of fixation at the ends of the plate.

Little work has been done in this field, although some mathematical studies have been made on the width of plate acting in a T-beam under various loading conditions.[3] Even less is known of the distribution of load among the various stringers of the floor. For uniform loads over a floor, simple relationships obtain, but for concentrated wheel loads such as were used in this investigation, the problem becomes very difficult.

What actually happens to a stringer floor under load is readily visualized, but a strictly mathematical analysis is practically impossible because the floor is statically indeterminate to a high degree. As the load is applied the stringer beneath it deflects and the plate deflects with it, acting as a beam to transmit shear to the next stringer. This second stringer will also deflect and carry on the distribution. Theoretically, the distribution will go to all the stringers, although, practically, the effect may become so small after being distributed over three or four of them that any further distribution may be neglected. This action is quite different from that usually assumed in design, namely, that the load is spread equally over a certain definite number of stringers. Since the deflection of a beam varies as the cube of the span (that is, the spacing between the stringers) and inversely as the cube of its thickness, a floor with equal spacings between the stringers has a constant proportion of the shear transmitted between each stringer. In other words, the load on a floor is distributed throughout the floor in a geometrical ratio.

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[3] "Die Mittragende Breite," by Theodor von Kármán, M. Am. Soc. C. E., in Beiträge zur Technischen Mechanik und Technischen Physik, August Föppl Festschrift, Berlin, 1924.

The action of the plate under a concentrated load cannot be determined readily. The plate may be regarded as a rectangular plate supported along the edges, but the behavior is complicated by the fact that the deflection of the stringers varies along the plate, giving it an elastic support; and, in addition, the rotation of the stringers makes the degree of fixation in the plate an uncertain quantity.

In this investigation strain readings were taken before and after loading to determine the stresses. Slope readings were also taken along the stringers before and after loading. These readings were plotted, and the resulting curves were differentiated once to obtain the moments, and then a second time to obtain the shears. They were also integrated to obtain the deflections, and the deflections were measured directly for a check. From mechanics, the slope curve, differentiated twice, and multiplied by $E\,I$, gives the shear. Since the sums of all the shears of the stringers must be equal to the applied load, the value of the moment of inertia, I, was selected so that this became true for the experimental results. Knowing the value of I, it was easy to determine how much plate was acting with the stringer as a T-beam. The values of the shears on each stringer indicated how much load was carried by the various stringers. Furthermore, when the T-beam action of the stringer was known, the section modulus could be computed, and the stress could be determined from the moment curves which had been computed. These stresses were compared with the measured stresses, and their agreement served as a check on the work. The stresses in the plate were determined by means of Huggenberger tensometers.

PROGRAM

The four models were tested with the dead loads in various positions. In what is called a typical run, strain readings were taken along, and transversely between, the stringers, and level-bar readings were taken along the stringers both before and after the load was applied. The difference between the initial and final readings was the effect due to the applied load. The strain, multiplied by the modulus of elasticity, gave the stress at any point in the stringer; and the difference in the level-bar readings gave the slope due to the applied load. Such tests were made on the models with the load in all parts of the floor. However, the load in the middle of the stringer caused the larger stresses and, therefore, governed the design. In the full-sized panel, tests were made only with the load in the middle of the floor.

For these tests, the sum of all the slope readings along a stringer at any point would give the deflection of the stringer at that point. However, this was supplemented by tests in which Ames dials were placed along the stringers to measure the actual deflections. These results agreed quite closely with the integration of the slope readings and served as a check on them.

The plate stresses were obtained by measuring the strains in the plate with the tensometers. The load was placed in various parts of the plate, and tensometers were located at all points where any effect was noticed, thus recording the distribution of the plate stresses. In making some of the models, strain measurements were also taken on the stringers before and after welding to furnish an idea as to the severity and the effect of the welding stresses.

TESTING

The models were made of standard rolled-steel sections and plate. They were constructed on the basis that the model should be one-third size. However, since the prototype could not be reduced in all proportions without expensive machining, the stringers were machined so as to keep the clear spans of the plate in proportion, and they were designed so that the stresses in the model would be approximately the same as in the prototype. The ideal could not quite be attained since the smallest I-beams rolled gave about a 25% excess over the computed section modulus. The panels in the model were 80 in. by 40 in. The first two models consisted of two panels welded together. The last two models consisted of only one panel each, but they were welded in the same frame and their floor-beams consisted of channels. After the

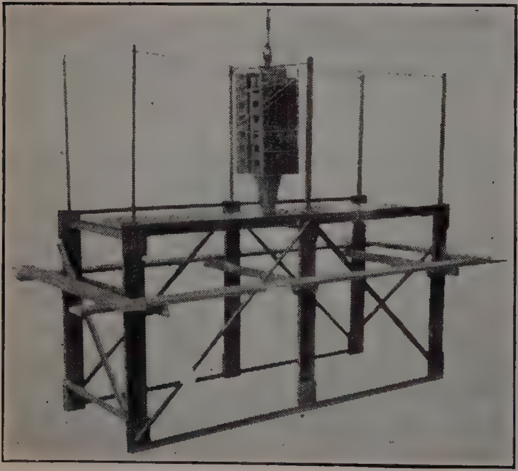

FIG. 1.—FLOOR MODEL AND LOAD

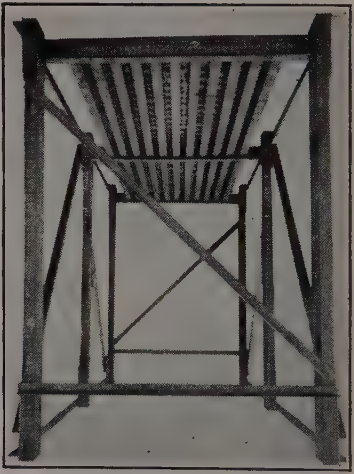

FIG. 2.—BOTTOM VIEW OF SECOND FLOOR MODEL

testing of the last two models was completed, a filler plate was inserted between the two adjacent channels, and weld metal was deposited so as to make an I-beam and cause the panels to be continuous. A view of the first two models is shown in Figs. 1 and 2. All the models had the same type of stringers, but the stringer spacing and the plate thickness varied. The plate in the first two models was steel metal sheeting of a low strength and yield point. This type was selected because the uniformity of thickness which sheeting possesses was an important factor in the $\frac{1}{8}$-in. plate thickness used in these models. The $\frac{1}{4}$-in. and $\frac{3}{16}$-in. plates in the third and fourth models were of the regular structural grade of steel and passed the specifications.

Great care was necessary in welding the models in order to avoid warping, particularly in the first two where the plate was so thin. They were fabricated by first tacking the stringers on to the plate, after which the weld metal was deposited, alternating from one spot to another on the floor so as to minimize the heat and thus decrease the tendency to warp.

The models were loaded by a dead-weight loading rig which is shown in Fig. 1. The Bureau of Public Roads, United States Department of Agriculture, has approved [4] a loading area of 20 in. by 10 in. for the rear wheel of an H-20 loading. This assumes a tire 20 in. wide, with a pressure of 112 lb per sq in., giving 10 in. of longitudinal bearing.

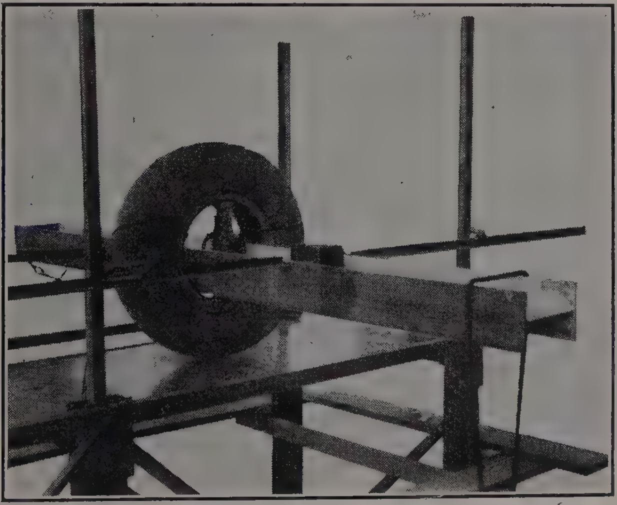

FIG. 3.—TIRE LOADING RIG

The loading rig consisted of a cast-iron block to which a frame was fastened to carry the additional dead weight. The initial weight of the rig was 400 lb. The load was applied, in 50-lb increments, through a steel bearing plate which was one-third size, or $6\frac{2}{3}$ in. by $3\frac{1}{3}$ in. A piece of soft rubber, $1\frac{1}{2}$ in. thick, was placed under the bearing plate to keep the load constantly uniform as the plate deflected, and a piece of cellotex was placed under the rubber to keep the area in contact with the plate constant.

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[4] Specifications for Highway Bridges, Am. Assoc. of State Highway Officials, 1935, p. 173.

A load of 2 489 lb should cause the same stresses in the model as would the rear wheel of an H-20 truck in the full-sized panel. A load of 2 500 lb was used in the model tests.

To show that the loading rig gave a uniform load distribution on the plate, and to compare its action with that of a tire, comparative load tests were made.

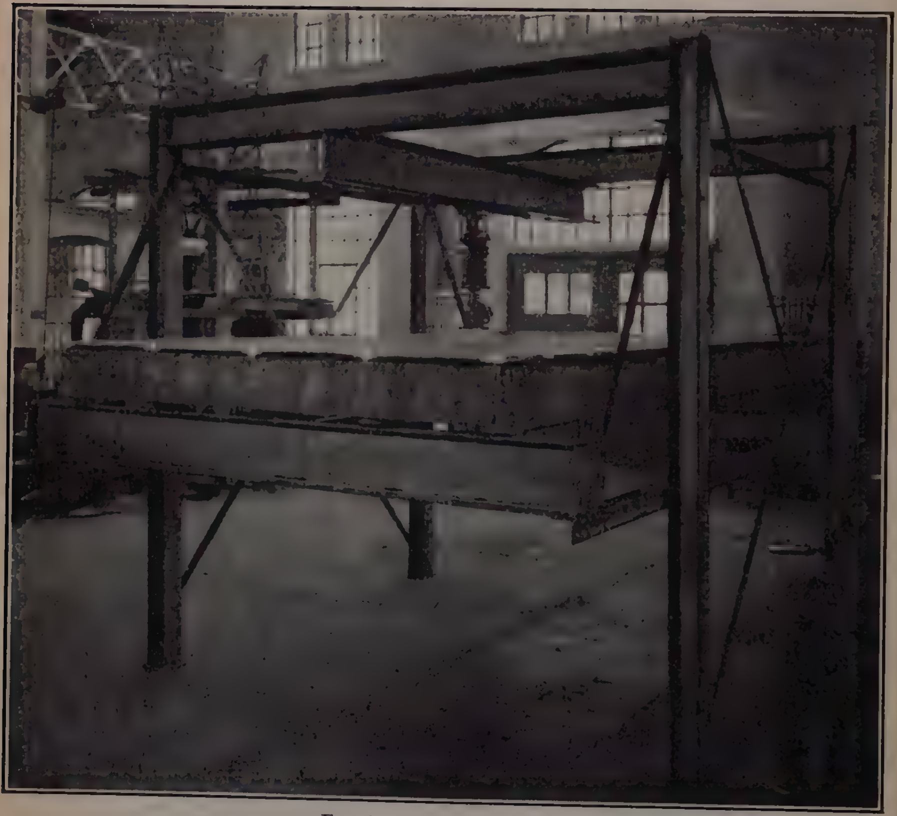

Fig. 4.—Full-Sized Floor

The floor was tested by means of the loading rig, and strain measurements were taken around the load at a number of critical points. Then the floor was tested by means of a tire, as shown in Fig. 3. The results of these tests were compared and found to be equal within the limits of experimental error, showing that the loading rig gave essentially the same results as an actual wheel load.

No trouble was experienced with warping in the full-sized floor panel, due to welding, so that it was unnecessary to adopt any special welding procedure. The plate was so thick that it dissipated the heat rather quickly. The structural grade of steel was used in the floor.

The floor was set in a frame as shown in Fig. 4. It was tested by jacks such as that shown in Fig. 5, the load being measured by the deflection of calibrated springs. It was possible to ascertain the load on the floor by this method to within 1 per cent. In order that the pressure on the floor be kept uniform as the plate deflected, the deflection of the plank on which the spring set-up was placed, was computed, and the plank was made of such a thickness that its deflection would be approximately equal to that of the plate. The deflection of the plank was computed on the basis of a beam on a yielding foundation under uniform pressure.

The models were held in a frame consisting of vertical posts made of 8-in. channels braced with angles (see Figs. 1 and 2). The floor-beams of the models were welded to the vertical channels to simulate the connections of beams to hangers in an actual bridge construction. The full-sized panel was held in a truss made of beams and angles, the details of which are shown in Fig. 4.

The strains in the stringers were measured by a fulcrum-type Whittemore strain-gage, equipped with a 0.0001-in. Ames dial. It is accurate to about 600 lb per sq in., as a tolerance of 0.0002 was allowed in repeating a reading. Since temperature changes cause strains in a structure that would be measured by the gage, the observations were made when these changes were at a minimum. Usually, therefore, temperature could be neglected, but when variations occurred, corrections were applied to the strains from observations on mild steel standards.

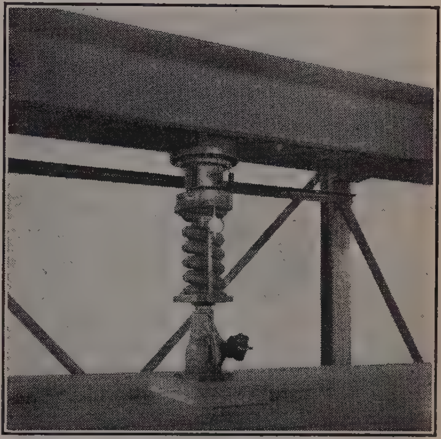

Fig. 5.—Jack Used on Full-Sized Floor

The tensometers which were used in the plate tests had gage lengths of 1 in. and 0.5 in. They were accurate to within about 500 lb per sq in., depending on the gage length.

Two level-bars were used. The one in the model tests had gage lengths varying from 1 in. to 6 in. However, the 5-in. gage was used almost entirely. It was fitted with a very sensitive bubble so that readings could be repeated to 0.0001 in., if the point hit the same spot on the floor. Hitting the exact spot was practically impossible, and, therefore, the accuracy was limited by the irregularities in the plate surface. For the full-sized model, the bubble was mounted so that the level-bar had a 15-in. gage length. This would make the instrument three times more accurate than it was on the 5-in. gage length, but in this case, again, the accuracy was limited by the irregularities in the plate surface. These irregularities were worse in the full-sized plate than in the models. In all cases the spots where the micrometer point of the level-bar rested were ground smooth and polished with an emery cloth.

Test Data and Relationships

Many tests were made on all the models and on the full-sized panel. It is impossible to describe all these tests, however, and only the results of the significant and important runs will be given.

Tests on First Model of 24-Inch Stringer Spacing.—A series of nine runs was made on the first model. Some of the load positions included the quarter-points of the stringers, the center of the span, and the points adjacent to the floor-beams. As far as stringer stresses are concerned, the load in the center of the span caused the critical stresses.

In Series 16 (see Fig. 6), the load was placed between Stringers E and G, in the middle of the first panel. The results of the tests are shown in Tables 1 and 2. The two adjacent stringers, E and G, received 80% of the load, whereas Stringers C and I received about 10 per cent. The stringers that were away from the direct influence of the load carried over about 25% of their load to the adjacent stringer. The beams acted as if they were partly fixed, the average fixation factor on the left being about 15% and that on the right about 39 per cent. Due to this fact, the shear on the left was less than that on the right. These fixation factors were the ratios of the moments at the supports to those of a fixed-end beam.

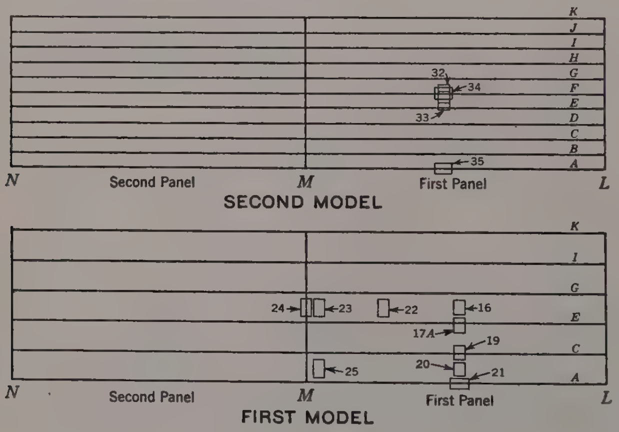

Fig. 6.—Loading Positions for First and Second Models

In order that the shears should equal the applied load, the moment of inertia of a stringer had to be 3.54 in.^{4}. This required 5.50 in. of plate acting in the compression flange. With these values the computed center moment was found from the slope curves, and the fiber stresses were computed. The measured stresses did not agree with those computed as well as expected, and the reason for this will be discussed subsequently.

The center deflection was computed from the formula:

$$y = \frac{W L^3}{192 E I} (-4 + 2 F_l + F_r) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which: W = wheel load; L = span length; E = modulus of elasticity; I = moment of inertia; F_l and F_r are the fixation factors at the left and right ends of the span, respectively. Equation (1) was derived from moment-area theorems and, although it is approximate, it is reasonably accurate, if the fixation factors do not differ by more than 50 per cent.

TABLE 1.—TEST RESULTS

Stringer*	SHEAR, IN POUNDS		Load, in pounds	Ratio to next stringer	Percentage of total load	PERCENTAGE FIXATION		Computed center moment, in thousands of inch-pounds‡	Ratio to next stringer	CENTER STRESS, IN THOUSANDS OF POUNDS PER SQUARE INCH		CENTER DEFLECTION, IN INCHES			Ratio to next stringer*
	Left	Right				Left, F_l	Right, F_r			Computed	Measured	Computed	From slope readings	Measured	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(a) SERIES 16, FIRST MODEL															
C..	101	119	220	0.21	8.8	13	42	3.8	0.21	+2.3 −1.3	0 −1.5	0.0185	0.0178	...	0.23
E..	507	555	1 062	1.00	42.6	18	34	18.5	1.00	+10.3 −6.0	+9.6 −2.3	0.0880	0.0769	...	1.00
G..	470	488	958	1.00	38.3	15	26	17.2	1.00	+9.6 −5.6	+10.1 −2.2	0.0828	0.0854	...	1.00
I..	122	138	260	0.27	10.4	11	55	4.3	0.25	+2.4 −1.4	+0.8 −1.8	0.0211	0.0205	...	0.24
T†.	1 200	1 300	2 500	...	100.1	...	...	...	...	...	...	...	...	...	...
(b) SERIES 17A, FIRST MODEL															
A..	61	75	136	0.23	5.4	27	29	2.3	0.25	+1.1 −1.4	0 −2.5	0.0135	0.0129	...	0.30
C..	265	315	580	0.47	23.2	38	42	9.3	0.43	+5.3 −3.4	+5.2 −2.6	0.0434	0.0426	...	0.41
E..	538	699	1 237	1.00	49.5	11	35	21.6	1.00	+12.2 −7.9	+15.4 −4.0	0.1120	0.1075	...	1.00
G..	201	241	442	0.38	17.7	15	27	7.9	0.37	+4.5 −2.9	+4.5 −3.1	0.0402	0.0386	...	0.36
I..	47	58	105	0.24	4.2	23	43	1.7	0.22	+1.0 −0.6	+0.8 −0.9	0.0087	0.0083	...	0.22
T†.	1 112	1 388	2 500	...	100.0	...	...	...	...	...	...	...	...	...	...
(c) SERIES 21, FIRST MODEL															
A..	717	918	1 635	1.00	65.4	32	55	25.1	1.00	+15.5 −15.5	+20.0 −9.2	0.161	0.149	...	1.00
C..	336	350	686	0.42	27.4	33	36	11.1	0.44	+6.5 −5.4	+5.7 −5.4	0.062	0.061	...	0.41
E..	89	86	175	0.25	7.0	51	57	2.5	0.23	+1.5 −1.2	0 −2.3	0.013	0.011	...	0.18
T†.	1 142	1 354	2 496	...	99.8	...	...	...	...	...	...	...	...	...	...

TABLE 1.—(*Continued*)

Stringer*	SHEAR, IN POUNDS		Load, in pounds	Ratio to next stringer	Percentage of total load	PERCENTAGE FIXATION		Computed center moment, in thousands of inch-pounds‡	Ratio to next stringer	CENTER STRESS, IN THOUSANDS OF POUNDS PER SQUARE INCH		CENTER DEFLECTION, IN INCHES			Ratio to next stringer*
	Left	Right				Left, F_l	Right, F_r			Computed	Measured	Computed	From slope readings	Measured	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(*d*) SERIES 32, SECOND MODEL															
C..	44	52	96	0.37	3.8	6	31	1.7	0.39	+1.0 −0.7	+0.5 −1.2	0.0092	0.0089	...	0.41
D..	129	134	263	0.50	10.5	23	35	4.5	0.45	+2.6 −1.7	+1.3 −2.2	0.0226	0.0218	...	0.44
E..	263	258	521	0.68	20.8	7	23	10.1	0.77	+5.7 −3.8	+6.3 −2.4	0.0508	0.0500	...	0.76
F..	352	416	768	1.00	30.7	19	37	13.1	1.00	+7.5 −5.0	+9.8 −3.0	0.0672	0.0659	...	1.00
G..	233	238	471	0.61	18.8	10	25	8.6	0.65	+4.9 −3.2	+6.4 −2.5	0.0440	0.0440	...	0.67
H..	109	124	233	0.50	9.4	18	43	3.9	0.46	+2.2 −1.5	+2.5 −1.5	0.0201	0.0190	...	0.43
I..	49	49	98	0.42	3.9	0	40	1.8	0.45	+1.0 −0.7	+0.3 −1.0	0.0095	0.0090	...	0.47
T†.	1 179	1 271	2 450	...	97.9	...	...	...	...	...	...	...	...	...	...
(*e*) SERIES 6, THIRD MODEL															
A..	17	11	28	0.06	1.1	−16	−5	0.6	0.06	+0.3 −0.2	+0.5 −0.9	0.0028	0.0037	0.0045	0.08
B..	220	234	454	0.31	18.5	−38	−27	10.4	0.39	+5.4 −2.3	+5.8 −2.0	0.0458	0.0450	0.0435	0.48
C..	735	722	1 457	1.00	59.4	+27	+24	25.4	1.00	+13.1 −5.5	+14.2 −2.8	0.0944	0.0938	0.0935	1.00
D..	234	240	474	0.32	19.3	−47	−36	11.4	0.45	+5.9 −2.5	+6.6 −2.6	0.0495	0.0497	0.0490	0.53
E..	28	15	43	0.09	1.7	+5	−23	1.2	0.10	+0.6 −0.3	+1.0 −1.2	0.0037	0.0038	0.0045	0.07
T†.	1 234	1 222	2 456	...	100.0	...	...	...	...	...	...	...	...	...	...

The deflection was obtained by adding the areas under the slope curve. The agreement between the observed and measured deflections served as a check on the accuracy of the differentiation and the computations.

In Table 1(*b*) are shown the results of Series 17A on the first model. In this run, the load was placed directly on Stringer *E* (see Fig. 6), at its center. However, the loading area was so wide that the bearing block overlapped the plate and some of the load was transferred directly through the plate to Stringers *C* and *G*. The ratio between the shear carried from one stringer to another

TABLE 1.—(*Continued*)

Stringer* (1)	Shear, in Pounds		Load, in pounds (4)	Ratio to next stringer (5)	Percentage of total load (6)	Percentage Fixation		Computed center moment, in thousands of inch-pounds‡ (9)	Ratio to next stringer (10)	Center Stress, in Thousands of Pounds per Square Inch		Center Deflection, in Inches			Ratio to next stringer* (16)
	Left (2)	Right (3)				Left, F_l (7)	Right, F_r (8)			Computed (11)	Measured (12)	Computed (13)	From slope readings (14)	Measured (15)	
(f) Series 19, Fourth Model															
A..	92	82	174	0.30	6.7	+16	+6	3.3	0.26	+1.8 −1.2	0 −2.0	0.016	0.017	0.012	0.30
B..	288	301	589	0.47	22.8	−16	−10	12.6	0.57	+6.7 −3.3	+6.0 −2.8	0.058	0.057	0.054	0.63
C..	625	625	1 250	1.00	48.4	+23	+24	22.1	1.00	+11.9 −5.7	+12.8 −2.8	0.091	0.091	0.091	1.00
D..	235	214	449	0.36	17.4	−4	−13	9.3	0.42	+5.0 −2.4	+4.4 −1.8	0.042	0.042	0.043	0.46
E..	63	63	126	0.28	4.9	+7	+7	2.4	0.26	+1.3 −0.6	0 −0.7	0.011	0.011	0.010	0.26
T†.	1 303	1 285	2 588	...	100.2	...	...	...	...	...	...	...	...	...	...
(g) Series 31, Full-Sized Floor															
A..	...	...	405	0.08	1.6	...	...	16.8	0.07	+0.4 −0.2	0 −0.5	0.006	0.006	0.009	0.09
B..	...	...	4 850	0.37	20.8	...	...	250.0	0.41	+5.6 −1.8	+5.4 −1.6	0.066	0.070	0.075	0.42
C..	...	...	13 120	1.00	56.2	...	...	610.0	1.00	+13.6 −4.4	+13.9 −2.1	0.179	0.165	0.170	1.00
D..	...	...	4 820	0.37	20.6	...	...	252.0	0.41	+5.6 −1.8	+5.2 −1.3	0.066	0.070	0.080	0.42
E..	...	...	199	0.04	0.8	...	...	12.3	0.05	+0.3 −0.1	0 −0.3	0.003	0.004	0.003	0.06
T†.	...	...	23 394	...	100.0	...	...	...	...	...	...	...	...	...	...

* See Fig. 6. † *T* = total. ‡ Values actually computed to nearest 10 lb; recorded as shown to conserve space.

in the direction away from the load is 0.24. The average value of F_l was 23% and that of F_r, 35 per cent. The value required for the moment of inertia of the stringers to make the shear equal to the load was 3.35 in.^{4} for the interior stringers, and 2.84 in.^{4} for the exterior stringer, *A* (see Table 2). These values for *I* were developed as follows: First, the shears were computed using a value of *I* corresponding to full T-beam action; that is, using 8 in. of plate for the top part of the T in the interior stringers, and 4 in. for the exterior stringers. This made the value of the shears too large. Next, the values for the moment of inertia were multiplied by a factor so that the shears would equal the load,

TABLE 2.—PROPERTIES OF SECTIONS USED IN COMPUTATIONS FOR TABLE 1

Description	Series 16, first model	Series 17A, First Model		Series 21, First Model		Series 32, second model	Series 6, Third Model			Series 19, Fourth Model		Series 31, Full-Sized Floor	
		Interior stringers	Exterior stringer, *A*	Interior stringers	Exterior stringer, *A*		Stringer *A*	Stringers *B*, *C*, and *D*	Stringer *E*	Stringer *A*	Stringers *B*, *C*, *D*, and *E*	Stringers *A* and *E*	Stringers *B*, *C*, and *D*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
T-beam action*...	5.5	4.25	1⅞	2.0	0.5	4.0	3.25	6.0	4⅝	3.25	5.75	12.0	17.5
I†.............	3.54	3.35	2.84	2.93	2.53	3.30	3.82	4.43	4.15	3.51	4.02	392.0	427.0
S‡.............	{1.79 3.07	1.77 2.72	1.67 2.00	1.70 2.06	1.62 1.62	1.76 2.64	1.86 3.20	1.93 4.62	1.89 3.90	1.80 2.82	1.87 3.86	43.6 106.0	44.7 137.0

* Inches of plate in T-beam action. † Moment of inertia = 1 in.^{4} ‡ Section modulus = *S* in.^{3}

and then the width of plate required for this value of the moment of inertia was computed.

The left reactions were again found to be less than the right, as would be expected, since the left fixation factor, F_l, was the smaller. It is seen that Stringer *E*, directly beneath the load, received fully 50% of the load.

In Series 19, the load was placed on Stringer *C*, which was next to the exterior stringer. Consequently, the load could not be distributed so widely. In this case, the loaded Stringer *C* carried 55% of the load, or 5% more than in the former series.

The worst loading condition for the floor occurred in Series 21, Table 1(*c*), when the load was placed directly over Stringer *A*, which thus received 65% of the load. However, this loading would not be attained in an actual bridge, since, in order to place the wheel directly over an exterior stringer, a considerable part of the wheel would project beyond the floor. When the load was placed as close to the exterior stringer as is practical, it took about 50% of the load, and its design would be about the same as that of an interior stringer.

Referring to the stringer stresses of Series 21, the measured stress gave values that would require much more plate in T-beam action than the total plate width between stringers. This shows that there were other stresses in the stringer besides those due to bending. The observed slopes of Stringer *A* during this test are shown in Fig. 7(*a*), and the corresponding stresses in Fig. 7(*b*).

The results of quarter-point loading on the floor were quite interesting. The length of plate in T-beam action was the same as that in the corresponding center of span loadings, but the shear transferred from one stringer to another was only 0.08 of the shear in the stringer, instead of the 0.25 which occurred in the former cases. This agrees with the fact that the shear transferred from one stringer to another depends on their relative deflection; and since the stringers deflected less at the quarter-point, the difference in deflection between two adjacent stringers was less. Carried to the extreme, when the load is at the end of the stringer, one stringer will take all the load and must be designed for it.

Supplementary tests were also made on this model. In Fig. 8 the deflection of Stringer *E* is plotted against the load, which was placed in a position corresponding to Series 16. It is seen that the load-deflection relation is a straight line (as would be expected), showing that the width of plate acting as a T-beam was constant and did not vary with the load. The center deflection diagram for the plate is also given in Fig. 8. The curve slopes upward to the left showing that the load required for equal increments of deflection increased with the increase in the load on the plate. In other words, catenary action helped to support the plate.

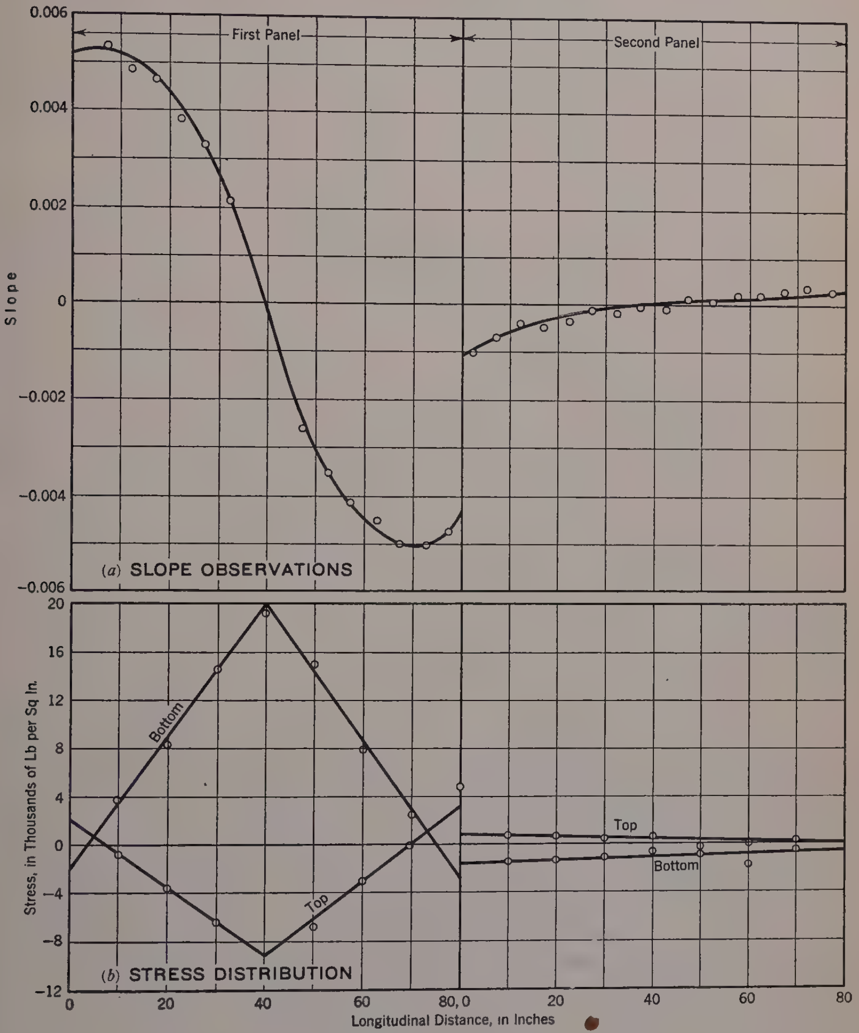

FIG. 7.—TESTS OF STRINGERS *A*

In Fig. 9, the deflections of the plate and Stringer *E* are plotted against the longitudinal axis of the stringer for a 2 500-lb load. The deflection curves for quarter-point loading are also given. In the stringers, there is a slight initial reverse curvature of the deflection curve at the ends of the span. This shows a slight degree of fixation. The curve for the plate, in sharp contrast to that of the stringers, slopes gradually and then changes quite sharply beneath the load.

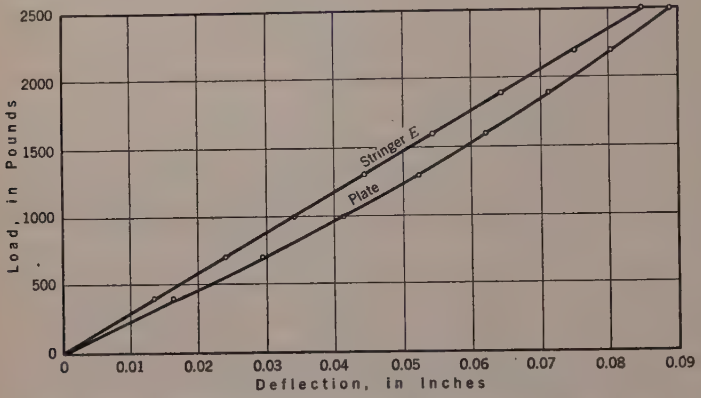

Fig. 8.—Load-Deflection Relations for Center Stringer and Plate

In all these tests the stringers rotated considerably. However, the top of the stringer showed little or no rotation, which indicated that the stringers must have rotated about the plate. This rotation is at least partly due to the deflection of the floor plate. Evidently, the stringers were subjected to torsional forces and thus contributed to the load distribution.

Tests on Second Model of 12-Inch Stringer Spacing.—The second model was the same as the first, except that the stringers were spaced 4 in., center to center, instead of the former 8 in. The results were similar to those of the first test, but due to the decreased stringer spacing, the floor was stiffer so that the load was distributed over more stringers.

The results of Series 32 (see Fig. 6), in which the load was placed directly on top of Stringer *F*, are shown in Table 1(*d*). The width of the loaded area was so great, relatively, to the stringer spacing that when Stringer *F* was loaded, the area overlapped on the two other stringers, *E* and *G*. Full T-beam action was present in this series. Stringer *F* took the largest proportion of the load—about 30 per cent. The ratio of shear carried from one stringer to another, in the direction away from the load, was about 0.43, instead of the 0.25 found in the first model. The average fixation factor was about 12% at the left (F_l) and about 35% at the right (F_r).

The center moment, stress, and deflection were computed for the second model in the same manner as for the first. The slope readings checked the deflection. However, the ratios between the various stringers did not stay quite as constant for the shear, moment, and deflections, as they did in the first model. This shows that the slope curves were not exact second-degree curves, although they were so assumed, and, consequently, a slight error occurred in the differentiation and integration. The load on the other sections of the floor gave results similar in nature to those shown for Series 32.

After the regular runs had been made on the second model an attempt was made to test it to destruction. It was loaded by means of a 20-in. I-beam extended out as a cantilever from an 800 000-lb testing machine. It was first loaded to 15 710 lb, which is more than six times the design load, at which time the deflection under the load was 0.531 in. The load was released to 4 700 lb and the deflection was 0.240 in., of which 0.118 in. was permanent set.

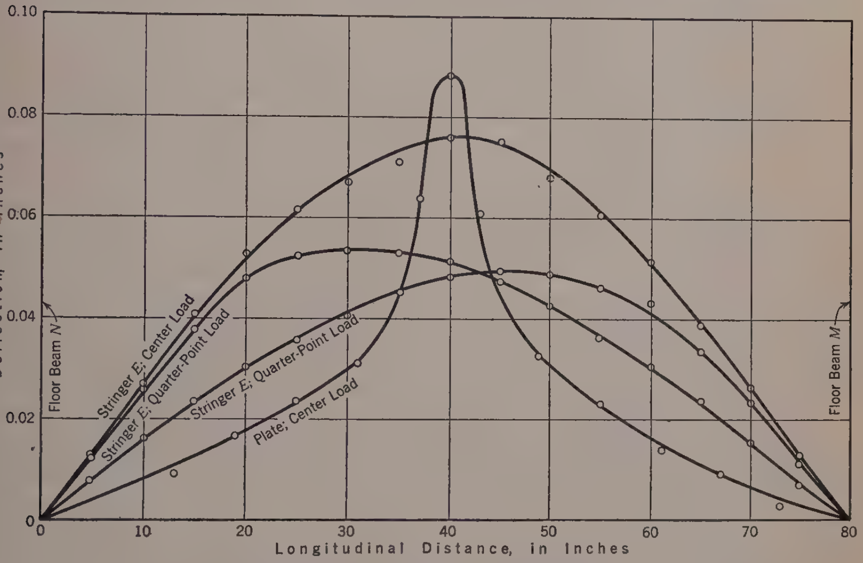

FIG. 9.—LOAD-DEFLECTION RELATIONS FOR STRINGER *E* AND PLATE

The model was loaded again to 28 510 lb, when the testing had to be discontinued due to the incipient yielding of the loading beam and the bowing of the vertical legs of the frame holding the floor. At this load, the total deflection was about 1 in. The stringer under the load had yielded and thrown much of the load on to the two adjacent stringers, which were also beginning to yield. Upon removal of the load, a permanent set of about 0.5 in. was observed.

There was no sign of yielding in the plate, and the only sign of failure was a scaling of the whitewash in one of the welds holding the floor-beam in the supporting frame.

Tests on Third Model of 30-Inch Stringer Spacing.—The third model was based on a prototype with a $\frac{3}{4}$-in. plate welded to stringers spaced on 30-in. centers. This model, as well as the fourth, consisted of only one span, and since the stringers were coped into the floor-beams there was a partial degree of fixation. In these models, the plate was extended to the center of the exterior stringer, *A*, whereas it overlapped 2 in. from the center of the exterior stringer, E. This was done to see whether the overlapping plate was efficient in T-beam action.

The results of the tests on the third model were very similar to those of the first two. The results of the case in which the load was placed on top of Stringer C are given in Table 1(e). Stringer C took 59.4% of the load, which is a greater proportion than in the first two models. This was due to the greater stringer spacing. For a similar reason the width of plate in T-beam action was not as large as in the first two models. The fixation factor for the loaded stringer was small, as would be expected. For the other stringers the value of F is negative, denoting an applied moment at the end of the stringer. The reason it is negative is that, due to the partial fixation of the loaded stringer, the floor-beam rotates and imposes applied moments on the adjacent stringers.

A number of tests were made in a study of the plate stresses in the third and fourth models. Similar tests were made on the full-sized floor and since these were more complete and gave essentially the same results as those on the models, only the results on the full-sized floor plate are given in this paper. Full T-beam action was not present in these models. Accordingly, the compression in the top part of the T-beam would be expected to decrease away from the stringer. This decrease was found to be linear, as shown in Fig. 10, for the variation of the compression in the plate between Stringers C and D when the load was placed between them.

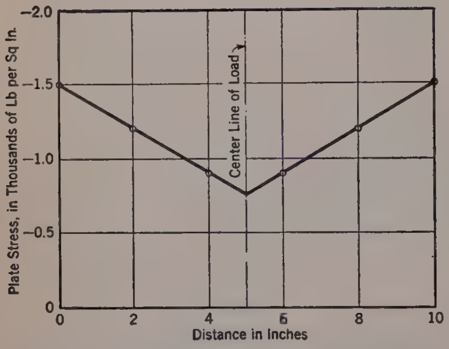

FIG. 10.—STRESS DISTRIBUTION IN FLOOR PLATE WHEN LOAD IS PLACED BETWEEN STRINGERS C AND D

Cross-bracing was welded between the stringers in one of the bays of this model. It consisted of $\frac{1}{4}$-in. by 2-in. plates welded to the stringers and the plate, at 12-in. intervals. Although the secondary bracing was spaced this closely, the plate stresses were reduced very little.

Tests on Fourth Model of 24-Inch Stringer Spacing.—The fourth model was based on a prototype with a $\frac{9}{16}$-in. plate welded to stringers at 24-in. centers. In this model, an attempt was made to evaluate the welding stresses. Strain readings were taken on the stringers and on the part of the plate over the stringers, before and after the floor was welded together. The welding stresses were greatest in the plate because most of the welding was done there. The longitudinal welding stresses in the plate varied from about 4 000 lb per sq in. over the exterior, to 9 000 lb per sq in. compression over the interior, stringers. The stress along the bottom of the stringers was about 2 000 lb per sq in. in tension. The transverse welding stress in the plate between the stringers varied from 15 000 lb per sq in. to 20 000 lb per sq in. in compression. These stresses did not seem to affect the test results.

The results of the various tests of the fourth model were similar to those of the previous models. Table 1(*f*) gives the results for the case in which Stringer *C* was loaded. This stringer is seen to take 48% of the load.

After the regular runs had been made on the third and fourth models, they were welded together to form a two-panel floor. A run was taken similar to that in the third model for which the results are given in Table 1(*e*), and the results for the two-panel model were very similar to those for the third model except that the fixation factor at the intermediate floor-beam was increased to 50 per cent. The stresses and deflections in the floor-beams were found to be smaller in accordance with the increased fixation factor.

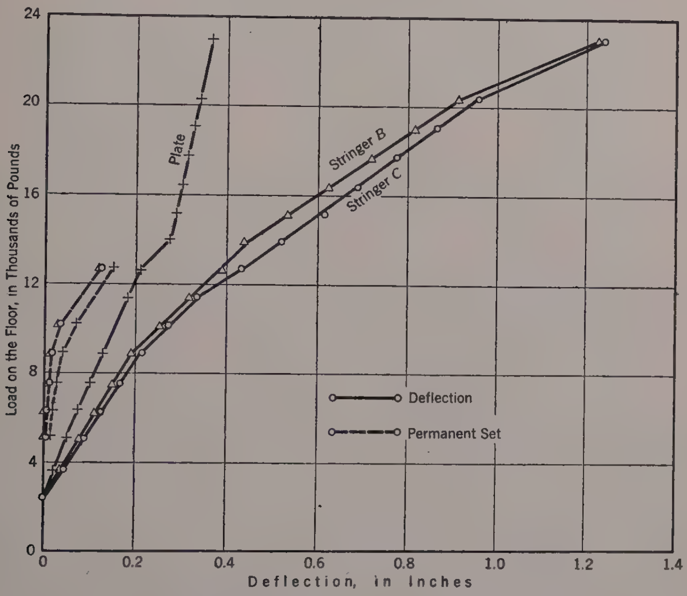

FIG. 11.—DEFLECTION AND PERMANENT SET OF PLATE AND STRINGERS *B* AND *C*. BREAKING TEST

Both the third and fourth models were tested to destruction. They were loaded in a manner similar to that used for the second model; that is, a cantilever load from the big testing machine. The third model was loaded up to 26 744 lb and the fourth, to 27 911 lb. In each case, the testing had to be discontinued due to incipient yielding of the loading beam. At these maximum loads, which were about eleven times the design load, large deflections were present although nothing broke. The large reserve strength is due to the fact that as one stringer yields, the increase in load is taken by the adjacent stringers while the original stringer still holds its yield-point load. This process continues until all stringers have yielded.

In the third model the load was placed on the plate between Stringers *B* and *C* (Fig. 6). Fig. 11 shows a diagram of the load-deflection and permanent-set curves of the stringers and the plate. At the maximum load, the deflection was 2 in. and the permanent set was 1 in.

The fourth model was tested with the load directly over Stringer *C*. The results were about the same as for the third model. A diagram of the deflection of the centers of the stringers under various load increments is shown in Fig. 12.

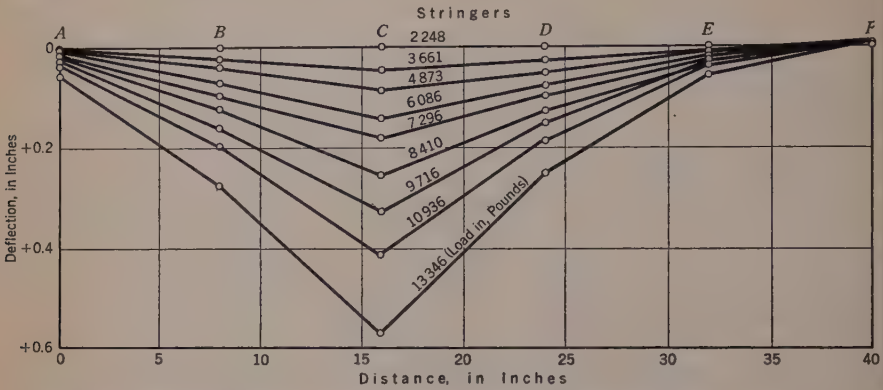

FIG. 12.—DEFLECTION CURVES FOR CENTER OF STRINGERS. BREAKING TEST, MODEL NO. 4

Tests of Full-Sized Floor Panel.—The full-sized floor panel was built according to a design procedure, determined by a study of the results of the model tests. The panel was 16.75 ft long, center to center bearing, and 9.42 ft wide. This floor was built so that it would act as a test on the design procedure, and serve as a check on the model tests. How well it did this is illustrated in the comparison of the design values with the measured values of stress and deflection for the critical sections of the floor, shown in Table 3. The check is very good considering that the properties of structural sections may vary as much as 5 per cent. The tests on this full-sized floor were similar to those of the models. Table 3 gives the results when the load was on top of Stringer *C*.

TABLE 3.—TEST OF FULL-SIZED FLOOR

Description (1)	Maximum stress in plate, in pounds per square inch (2)	Maximum deflection of plate, in inches (3)	Maximum stress in loaded stringer, in pounds per square inch (4)	Maximum deflection of loaded stringer, in inches (5)	Percentage of wheel load taken by loaded stringer (6)	Width of plate in T-beam action, in inches (7)	Weight of floor (stringers and plate), in pounds per square foot (8)
Design value.....	27 900	0.113	13 300	0.169	53	17.5	44.0
Measured value...	26 000	0.111	13 900	0.170	56	17.5	...

Because of the large dimensions of the full-sized floor, it was possible to obtain more data concerning the stresses in the plate than had been possible in the models. A large number of tensometer readings were taken in order to obtain the stress distribution in the plate. Fig. 13(*a*) shows the distribution of the transverse stress in the plate along the length of the floor. This stress is the greatest in the plate, since it lies along the short span between the stringers. It is seen to have a peak of 26 000 lb per sq in. at the center of the load, rapidly decreasing asymptotically along the plate. The compression in the top of the plate is seen to be similar to the tension in the bottom. Complete readings could not be obtained for the compression side since the load was in the way. The distribution of the transverse stress in the bottom of the plate, across the width of the floor, is shown in Fig. 13(*b*). In the loaded span, the stress in the plate was zero at the edges of the stringer flanges. The stress changed to compression along the flange, and rapidly dropped to zero at the next stringer.

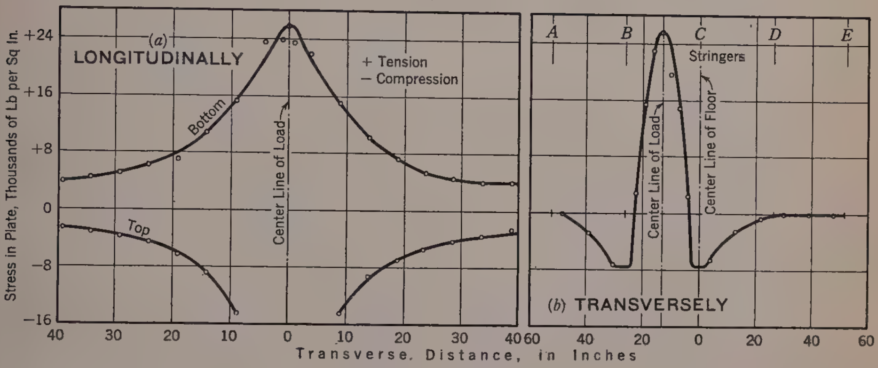

Fig. 13.—Transverse Stress in Plate of Full-Sized Floor

An over-load test was made on the full-sized floor. It was loaded up to 56 000 lb, or two and one-half times the design load. The deflection of the floor increased linearly with the increase in load. At a 50% over-load, there was an over-all permanent set under the plate of 0.005 in. At the maximum load of two and one-half times the design load the permanent set was 0.025 in. No signs of failure or yielding could be determined with the exception of a scaling of the whitewash near one of the welds, between the plate and the stringer. After the load was removed, the floor appeared just as perfect as ever, and one could never have detected by eye that it had been overloaded. The maximum deflection of the plate had been 0.269 in.

Discussion of Results

The results of these tests show that the battle-deck floors acted as an integral unit. The load was distributed from one stringer to another by means of the plate, which acted as a cantilever beam, in proportion to the relative deflection of the stringers. This distribution factor was a constant for a definite stringer spacing and plate thickness. In all cases, as shown by the tables, the carry-over factor was larger for the stringers close to the load than for those away from the load. Considering the case in which the load was directly over one stringer, the adjacent stringers took, not only the load carried over by their relative deflections, but also some of the load itself, as the wheel load was so wide that it overlapped the loaded stringer.

In some cases, the stresses in the stringer did not check the computed stresses, particularly for the smaller stringer spacings, because the computed stresses only included bending stresses. Since the stringers were welded to the floor-beams, direct tension could occur in addition to the bending moments. Four types of stresses are probably added to the simple bending stresses along the tension flange: First, due to the tension in the lower flange, its length is increased, imposing compression on the ends of the beam, and thus tending to reduce the flange tension; second, due to the deflection of the beam, the longitudinal axis shortens and causes tension along that axis; and, third, due to the rotation of the bottom flange, shortening takes place which causes tension along the gage line. Finally, these effects in any one stringer have a reaction on the floor-beam which, in turn, applies a couple and a tensional stress to the other stringers.

The effect of these stresses is greatest on the loaded stringer as it reduces the compression and increases the tension. This stringer has a great effect on the rotation of the floor-beam which, in turn, tends to offset the secondary stresses in the adjacent stringers. In the stringers a distance away from the load, the tension is reduced in some cases to zero, and the compression is increased. Table 1(*b*) illustrates this phenomenon.

The width of plate in T-beam action was found to increase as the stringer spacing decreased, whereas the load taken by a stringer increased with the increase in spacing. Both these results are logical and Fig. 14 presents the relationships obtained. These relationships are useful for the design of battle-deck floors.

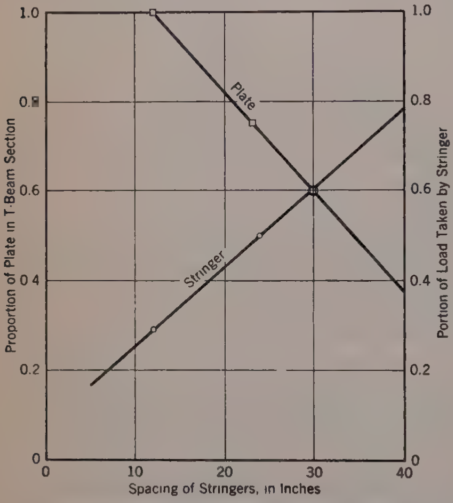

Fig. 14.—Amount of Plate in T-Beam Action and Load Taken by Stringer

A stringer needs to be designed only for the effect of one rear wheel load since the usual axle spacing on trucks is so large that the effect of one wheel is not carried over to the other. There seems to be no need of making the exterior stringer larger than the interior stringers since it is practically impossible for the center of the wheel to come over the center of the exterior stringers. Usually, the exterior stringer will not be stressed higher than the interior stringers if it is the same size. The stringer next to the exterior will take about 5% more of the load than any of the other interior stringers, because a full wheel load can rest on it and the exterior will not help support it as much as the interior stringers. Thus, it will be overstressed about 5%, if it is the same size as the other interior stringers.

When coped into the floor-beams of a single span, the stringers had a fixation factor of about 25 per cent. When the spans were continuous, the factor was as high as 50 per cent. A substantial saving in material can be effected if this partial fixation is taken into account in design. The foregoing statements apply to web-plate connections; the fixation factors for web-angle connections were slightly smaller.

In designing battle-deck floors the smaller the stringer spacing is made, the lighter will be the resultant floor. However, the increased welding cost of the lighter floors will probably make them uneconomical unless the importance of light weight is particularly great, such as in lift spans.

The over-load tests showed that the battle-deck floor had a large reserve strength and was practically impossible to break. The tests on the full-sized floor panel would indicate that, although the measured stress was fairly high, the plate thickness could be reduced to $\frac{5}{8}$ in., or $\frac{9}{16}$ in., and still be amply strong to take an H-20 load.

Most plates in battle-deck flooring have been designed on the assumption that the plate under the load acts as a fixed-end beam. No account has been taken of the longitudinal distribution of the load. Fig. 13 shows the distribution of the plate stress. The longitudinal distribution is seen to extend over about four times the clear span of the plate, or about 84 in.

The point of contra-flexure of the plate fell close to the edge of the stringer flange. One of two assumptions may explain this: The first is that the plate acts as a simple span between the flange of stringers; and, the second is that the plate acts as a fixed beam with the point of contra-flexure at the edge of the stringer flange. In either case the result is the same, and the first assumption is the easier to use in computation. The second assumption is probably closer to what actually takes place since the plate forms a fixed beam of varying cross-section, the depth between the flanges being the depth of the plate itself, and the depth over the stringer flanges being the thickness of the plate plus the flange. The increased depth of beam at the stringers decreased the stress over the stringer far below what would be expected. The tension stress was larger than the compression stress, probably because a catenary stress of about 1 000 lb per sq in. was present.

The curve in Fig. 13(*b*) shows that the longitudinal distribution of stress varied from a peak at the center of the load, decreasing asymptotically. Neglecting the small curve at the peak, this variation may be assumed to be parabolic. Thus, the computation of the stresses in the plate becomes very simple: First, the total moment which the plate must support is computed on the assumption that the plate is a simple beam between the edges of the stringer flanges. Next, the average stress in the plate is computed over the length of four times the clear span of the plate, since the load is distributed over that distance. For the average stress this gives:

$$s_{\text{ave}} = \frac{M}{4\,S\,L} \quad \ldots\ldots\ldots\ldots (2)$$

in which M is the moment; L, the clear span; and S is the section modulus of the plate per inch of plate. The maximum stress in the plate will be three times the average since the distribution of stress is parabolic. This semi-empirical method of determining the plate stress was used in computing the stresses in the full-sized floor and was checked by the observed stresses. The values of the stresses in the one-third sized models checked even more closely than those for the full-sized floor. This method of determining plate stresses may be expected to give quite accurate results for one-way slabs under concentrated wheel loads.

RECOMMENDED METHOD OF DESIGN

The results of the four models and the full-sized floor panel indicated that battle-deck floors may be designed by the following procedure:

(1) When the stringer spacing is determined, obtain from Fig. 14 the load taken by T-beam action of one stringer and its contributing plate width. With this information, the moment on the stringer is easily found and a trial stringer is selected. The properties of the T-beam section can be determined as soon as the plate thickness is found.

(2) When the trial stringer section has been selected for the T-beam, the clear span between the stringers is known. The trial section can be determined quite closely in the first step since great changes in the top of the T-beam change the value of the section modulus only slightly.

(3) The required plate thickness, t, is determined by the formula:

$$s_{\max} = \frac{3\,M}{4\,S\,L} \qquad \text{(3)}$$

and is found directly by changing Equation (3) to,

$$t = 3\sqrt{\frac{M}{2\,s_{\max}\,L}} \qquad \text{(4)}$$

(4) Knowing the plate thickness, the section of the T-beam is determined, and the stresses in the stringer are computed. If the stresses are not satisfactory another section is selected.

(5) Design the stringer connections for the full load in shear.

(6) Partial fixation may be taken into account and resulting economies effected by using a fixation factor of 25% when the span is simple and 50% when it is continuous.

Wide-flanged beams, in general, will be economical since they reduce the clear span of the plate. However, the lightest wide-flanged beam may not meet the specification that the web must be at least $\frac{3}{8}$ in. in thickness. In general, lateral bracing should not be necessary for the floor when the stringers are coped in on the floor-beams.

When the plate and stringers are selected, the remainder of the floor is designed according to the usual methods. The welds between the plate and the stringers are designed for longitudinal shear. The resulting welds will also be strong enough to take care of the horizontal and catenary stresses in the plate.

SUMMARY

The results of the tests indicated that:

1.—Battle-deck flooring acts as an integral unit.

2.—Inherent welding stresses may be found in the plate but they do little harm under static load. Care in welding will minimize these stresses.

3.—A tire imposes an essentially uniform load over the area upon which it rests.

4.—The plate acts with the stringers to form a T-beam, reducing the stringer stress about 15 per cent.

5.—The load taken by a stringer and the width of plate acting with the stringer vary with the stringer spacing.

6.—The stringers distribute the load in proportion to their relative deflections; thus, the distribution is greatest when the load is in the center of the panel and decreases as the load approaches the floor-beams. The distribution factor varies with the thickness of the plate and the distance between the stringers.

7.—The plate acts as a simple beam between the edges of the stringers. The load is distributed longitudinally over a distance equal to four times the clear span. The distribution is parabolic, with the maximum stress three times the average stress.

ACKNOWLEDGMENTS

The American Institute of Steel Construction, through its Technical Research Committee, sponsored this investigation of the action of concentrated loads on battle-deck flooring, in order to obtain information for design purposes. A research fellowship supported by the Institute was established at the Fritz Engineering Laboratory of Lehigh University, at Bethlehem, Pa., and the investigation was conducted under the guidance of the Committee. Acknowledgment is due all members of this Committee for their active interest in the work and their advice and guidance. Acknowledgment is also due to the members of the Fabricating Division of the Bethlehem Steel Company for constructive criticism and valuable suggestions during the investigation, and to H. J. Bowles, Welding Supervisor, Bethlehem Steel Company, for advice and assistance in all the construction work, and to C. C. Keyser, Instructor at the Laboratory, for his assistance in the construction and testing. The full-sized floor panel was fabricated in the shops of the Bethlehem Steel Company.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

RELATIVE FLEXURE FACTORS FOR ANALYZING CONTINUOUS STRUCTURES

By Ralph W. Stewart,[1] M. Am. Soc. C. E.

Synopsis

An effective method of analyzing the moments in continuous structures, by using the geometrical properties of elastic curves, is offered in this paper.

The area of any $\frac{M}{E\,I}$-diagram for a beam or part of a beam represents the curvature of the beam or the part to which the $\frac{M}{E\,I}$-area is related. This curvature is measured by the angle between the tangents to the elastic curve at the ends of the beam or of the part considered. It may be treated as a rotation of the tangent at one end of the curve with reference to the tangent at the other end, the center of rotation being the point where the center of gravity of the $\frac{M}{E\,I}$-area is projected upon one of these tangents to the elastic curve. In using the method, a graph consisting of a series of lines connecting these centers of rotation is substituted for a graph consisting of a series of moment diagrams. Each angle between the lines represents a moment area, both as to its magnitude and also as to the location of its center of gravity. It is not necessary that the graph be drawn to scale as the work is analytical. The method also involves use of the familiar mechanical requirements that the moments about a joint must balance, for equilibrium, and that the shear in a member must equal its combined end moments divided by its length. The series of lines referred to is designated as a "traverse of the elastic curves," because the lines which pass through points of support or joints in the structure will be, in fact, tangents to the curves formed by the flexed structure.

Although the method of analysis used in the paper is new, the construction of the type of graph used was described by the writer in a paper published in 1934.[2]

NOTE.—Written comments are invited for immediate publication; in order to ensure publication the last discussion should be submitted by **May 15, 1938.**

[1] Engr. of Bridges and Structural Design, City of Los Angeles, Los Angeles, Calif.

[2] "Analysis of Continuous Structures by Traversing the Elastic Curves," by Ralph W. Stewart, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 105.

TRUE TRAVERSE AND EQUIVALENT TRAVERSE OF A SERIES OF ELASTIC CURVE UNITS

Fig. 1(*a*) represents a column with one end deflected laterally; Fig. 1(*b*) and Fig. 1(*c*) represent the induced bending moments and the true traverse of the elastic curves; and, Fig. 1(*d*) and Fig. 1(*e*) represent equivalent moment diagrams and an equivalent traverse. The moment diagrams (Fig. 1(*d*)) are considered as running the full length of the member, thereby fixing their centers of gravity at known fractions of the lengths. If the member is of uniform section, these points will be at the third points. If intermediate loading is applied, the simple moment diagram due to the loading, and its corresponding angle in the traverse, are added, as in Fig. 1(*f*) and Fig. 1(*g*). This assembly of moment diagrams is the same as that used in treatises on slope deflection and other forms of analysis.[3] The traverse diagrams express the same data as the moment area constants, but in a much more workable form.

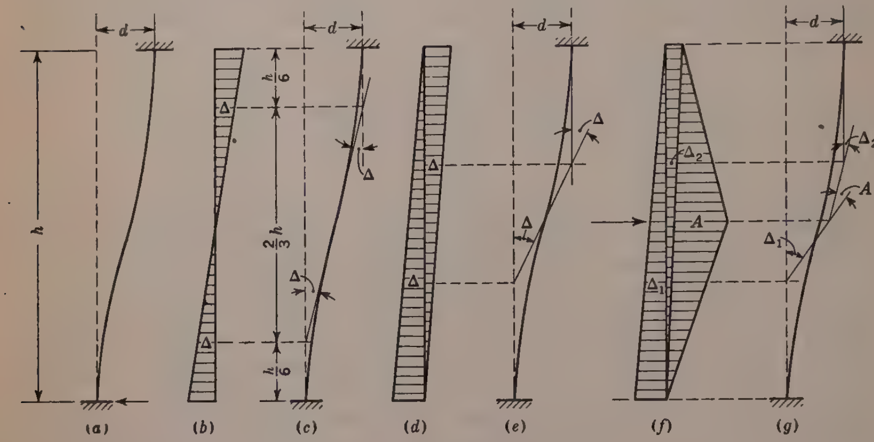

FIG. 1

In the following solutions, the symbol, Δ, represents the relative curvature in the entire length of a member due to an end moment, the different Δ-angles being identified by numerical subscripts. The symbol, θ, represents the relative angular rotation of a joint, the location being identified by a subscript. Symbols for fixed-end moments will follow the practice standardized for slope deflection.[3] Simple moment areas or their corresponding traverse angles will not be used, as solutions will be based on the fixed-end moments produced by any loading.

The stiffness of a member, as used in this paper, is measured by the moment, applied to one end of the member, necessary to produce a unit Δ-angle when the other end is hinged; or, briefly, is the ratio of an end moment to its Δ-angle. Under this definition, stiffnesses of a group of non-tapering members will have the same relative values as the stiffnesses used in the end-moment distribution method.[4] For tapering members, however, the relative stiffnesses will be different from those used in the end-moment distribution method, and will be easier to compute.

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[3] *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois; also "Moments in Restrained and Continuous Beams," by F. E. Richart, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 51.

The values of the separate items of curvature in the members, which are the angles in the traverse of the elastic curves, constitute the flexure factors mentioned in the title.

APPLICATION TO A CONTINUOUS BEAM OF NON-TAPERING SPANS

In Fig. 2 it is desired to determine influence lines for a load moving along Span *B C*. The relative stiffnesses of the spans are indicated by the numerals in circles.

Apply a moment at Joint *B* as shown in Fig. 2(*b*). The next step is to draw the traverse for the entire structure. This is easily done by the following procedure: (1) Draw Line 31-51 representing the rotated position of the tangent to the elastic curve at Joint *B*; (2) from Point 51 (which is under the third point of the span) draw Line 51-9 which, if produced, would pass over Joint *C*, and under the third point from the right end of Span *B C* (directing Line 51-9 to pass over Joint *C* is done because there is restraint at Joint *C*, and passing it under the third point mentioned indicates that the restraint is less than fixture); (3) from Point 9 draw Line 9-18 representing the tangent to the elastic curve passing through Joint *C*, and continue the traverse to Point 6, all angles being opposite the third points of the beam; and (4), since the hinged joint at End *E* offers no restraint, the traverse is finished by drawing a straight line from Point 6 to Point 2.

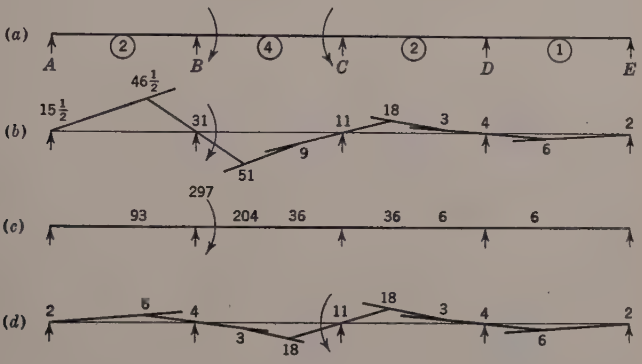

FIG. 2

Now, assign the arbitrary values of 2, 6, and 4, to the angles of the triangle in the traverse of the right span, these values being proportional to the opposite sides of the triangle. Since $M_{DC} = M_{DE}$ and since Span *C D* is twice as stiff as Span *D E*, the angle due to M_{DC} will be one-half the angle due to M_{DE} giving it the value, 3, as shown. (The reason for assigning the even number, 2, to the rotation at End E was to avoid fractional values in Span $C\ D$, this division by 2 being foreseen). The next Δ-angle to the left of 3 is quickly obtained by the following deflection computation: Using one-third the length of the span as the unit of length, $(4 \times 3) + (3 \times 2) - \Delta \times 1 = 0$. Therefore, $\Delta = 18$. The rotation of Joint C is $4 + 3 - 18 = -11$. Again, by stiffness ratios, the angle at the left of Joint C is 9 and, proceeding as described, the rotation at Joint B is found to be 31. The two remaining angles in Span $A\ B$, being proportional to the opposite sides, are, respectively, 1.5 and 0.5 of 31, or 46.5 and 15.5.

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[4] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

Fig. 2(*b*) now shows all relative flexure factors for the beam due to a moment at Joint B. To reduce the relative flexure factors to relative moment factors, multiply them by the stiffness factors of their respective members. All the moments thus obtained resist the clockwise rotation of Joint B. The applied moment that produces these resisting moments is the numerical sum of the resisting moments acting at Joint B, and is equal to 297, as shown in Fig. 2(*c*). Note that if this moment, 297, acts through Member $B\ C$ as a medium, the actual moment, M_{BC}, will be the applied moment, 297, less the resisting moment, 204, leaving an actual moment of 93, which balances $M_{BA} = 93$. For any fixed-end moment, M_{BC}, the moments throughout the beam can now be computed by one setting of the slide-rule, as each will have the same ratio to its corresponding relative moment in Fig. 2(*c*) that the fixed-end moment has to 297.

Fig. 2(*d*) and Fig. 2(*e*) represent the same procedure for a moment at Joint C. In this case, another traverse beginning with an arbitrary angle assumption at End A, is run to Joint C. Owing to the accidental sequence of stiffness factors, each traverse yields the same end slope, 11, at Joint C. Ordinarily, the computed slope at Joint C, due to the traverse starting with an arbitrary angle assumption at the right end would not be the same as the computed slope at C resulting from the traverse starting with an arbitrary angle assumption at the left end; and it would be necessary to eliminate the difference in slopes at C with one slide-rule setting by multiplying all flexure factors on one side of the joint by the ratio necessary to equalize the end slopes at this joint. With the data in Fig. 2(*c*) and Fig. 2(*e*), all joint moments can now be determined quickly for any set of fixed-end moments in Span $B\ C$.

This beam was selected because it has heretofore been used as a demonstration problem and this solution may be compared with slope deflection solutions heretofore published.[5] The superiority of the solution by flexure factors will become greater if more spans are added. The extension of the traverse computation across another span is a small item, whereas, with slope deflection, and also with methods based on virtual work, another simultaneous equation is added to an already cumbersome group.

For a complete solution of this structure, for influence lines involving any fixed-end moments in any span, it is only necessary to run one traverse computation in each direction the entire length of the beam and then obtain all other values by simple ratios requiring only a single setting of the slide-rule for any series of moments. These right and left traverse computations have an automatic check by Maxwell's theorem of reciprocal deflections since the final moment at Joint D for an angle assumption at End A will be the same as the final moment at End A due to the same angle assumption at End E. As far as the writer knows, this elegant check, which avoids duplication of computations, is not available in automatic form with any other method.

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[5] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 347, 356, and 1048.

For influence-line computations, involving side-sway of an unsymmetrical frame, as illustrated by Fig. 3, it is necessary to determine a set of relative flexure factors for each moving joint and the unbalanced shear resulting from them. The remainder of the work is by direct proportion and summation.

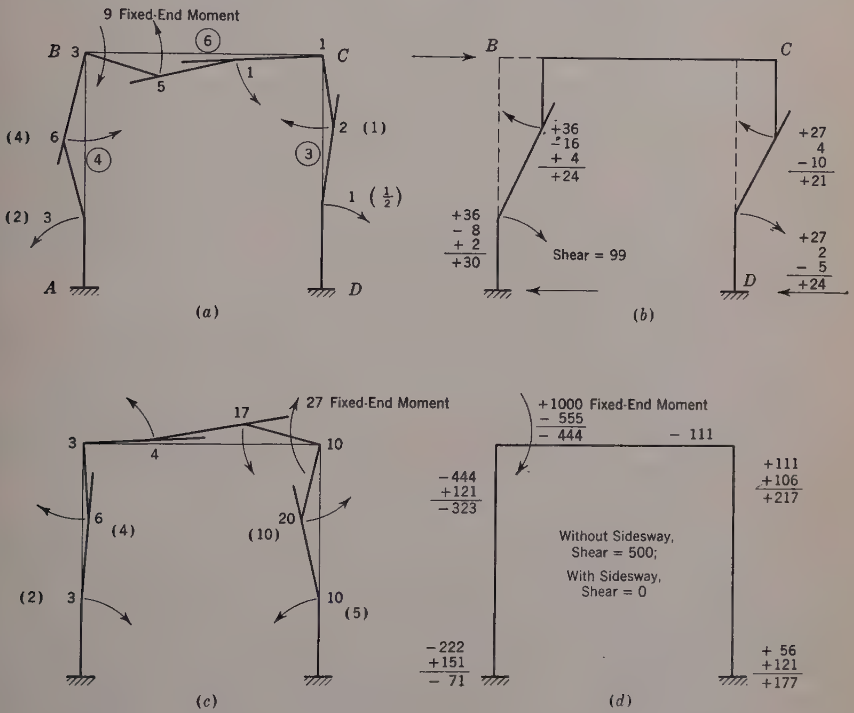

FIG. 3

To illustrate: Fig. 3(*a*) shows a set of flexure factors, computed easily by mental arithmetic, beginning with the relative value of 1 for the lower angle in the right column and progressing around the frame in the direction, $D\ C\ B\ A$. The numerals in parentheses opposite the column flexure factors are the corresponding moment factors computed on the basis of accepting the beam as representing the standard of stiffness, so that its flexure factors and moment factors are equal. The column moment factors are then equal to their flexure factors multiplied by the ratio of column stiffness to beam stiffness. Adding the moment factors for M_{BA} and M_{BC} it is found that the fixed-end moment acting on Joint B to produce these relative moment factors is 9. If this fixed-end moment acts through the beam as a medium, the actual moment in the beam is equal to the fixed-end moment, 9, minus the resisting moment, 5 = 4, which balances the moment, 4, at the top of the column. The upper numbers of each group beside the columns in Fig. 3(*d*) (namely, 222, 444, 111, and 56) are the moments without side-sway for a fixed-end moment of 1 000, being $\frac{1\,000}{9}$ times the moment factors in Fig. 3(*a*).

For side-sway correction the unbalanced shear due to these moments is computed assuming the height of the frame as equal to 1. This unbalanced shear of 500 is illustrated by the arrow at Joint *B*, in Fig. 3(*b*), and the structure must deflect to the right until resisting moments sufficient in magnitude to hold the structure in equilibrium are developed. To obtain these moments, first consider the frame deflected as illustrated by Fig. 3(*b*), without rotation at Joints *B* or *C*, causing moments of as yet unknown, and of unassumed, magnitude but with directions illustrated by the curved arrows. In order to transform this into a true picture of flexure by releasing Joints *B* and *C* so they can rotate, it is necessary to obtain a series of flexure and moment factors for Joint *C*, as illustrated by Fig. 3(*c*). This is done by the same procedure as for Fig. 3(*a*), beginning with the angles in the left column. These assumed column flexure factors were selected so that in crossing Joint *C* from the column to the beam, the multiplication by the stiffness ratio of $\frac{2}{3}$ would give a whole number. It is found by easy mental arithmetic that the flexure and moment factors shown in Fig. 3(*c*), are produced by a fixed-end moment of 27.

The flexure of Fig. 3(*b*), may now be evaluated. The method usually given in existing treatises is to distribute the unbalanced shear between the columns in proportion to their $\frac{I}{l^3}$-values, or to distribute the moments due to the shear in proportion to $\frac{I}{l^2}$-values. The writer considers this method somewhat undesirable because it requires remembering formulas and also because it is not a universal method applicable to tapering members. The following procedure is more easily remembered, equally easy to use, and is applicable to all types of members. Note that the equal deflections at Joints *B* and *C* in Fig. 3(*b*), are governed entirely by the inclined courses in the traverse along the centers of the columns. This will also be true if tapering members are used. For the case in hand, since each column has a constant value of *I*, the Δ-angles will be at the third points, and since the columns are of equal length all Δ-angles will be equal and the distribution of moments will be in proportion to the stiffness factors. For the general case, the distribution of moments is obtained by taking relative Δ-values directly from the geometry of the figure and reducing them to relative moment values by multiplying them by the respective stiffness factors. For the case in hand the most convenient set of relative moment values is obtained by adopting the trial moment of 27 in Fig. 3(*c*), as the relative end moment for Column *C D* in Fig. 3(*b*). For M_{BA} and M_{AB} in Fig. 3(*b*), then, $\frac{4}{3} \times 27 = 36$, which is four times the trial moment in Fig. 3(*a*). Joints *B* and *C* are now held in their deflected position, but are made free to rotate. They will then turn until the moment at the top of each column is in balance with its adjoining beam moment and in accomplishing this adjustment the flexure at the bottoms of the columns will also receive an adjustment. The adjustments of the column moments due to the rotation of Joint A are obtained by multiplying the moments in Fig. 3(a), by $\frac{36}{9} = 4$, and are written on Fig. 3(b). The moments due to the adjustment of Joint C are copied directly from Fig. 3(c).

It may be noted that the adjustment of moments at the bottoms of the columns is one-half that at the tops of the columns, corresponding to a carry-over factor of one-half. However, this factor of one-half is for the special case of non-tapering members and the method of this paper does not use or advocate carry-over factors.

By summation, the final relative moments for side-sway influence are 30, 24, 21, and 24, corresponding to a side-sway force, or the shear of their sum = 99. The side-sway moments due to any shear can now be determined by direct proportion.

For a fixed-end moment, $M_{BC} = 1\,000$, the column moments without side-sway are $\frac{1\,000}{9}$ times the column moments in Fig. 3(a), giving a computed shear of 500. The correction moments for side-sway will be $\frac{500}{99}$ times the moments in Fig. 3(b). The column moments without side-sway, the side-sway correction moments, and the final column moments, are shown on Fig. 3(d).

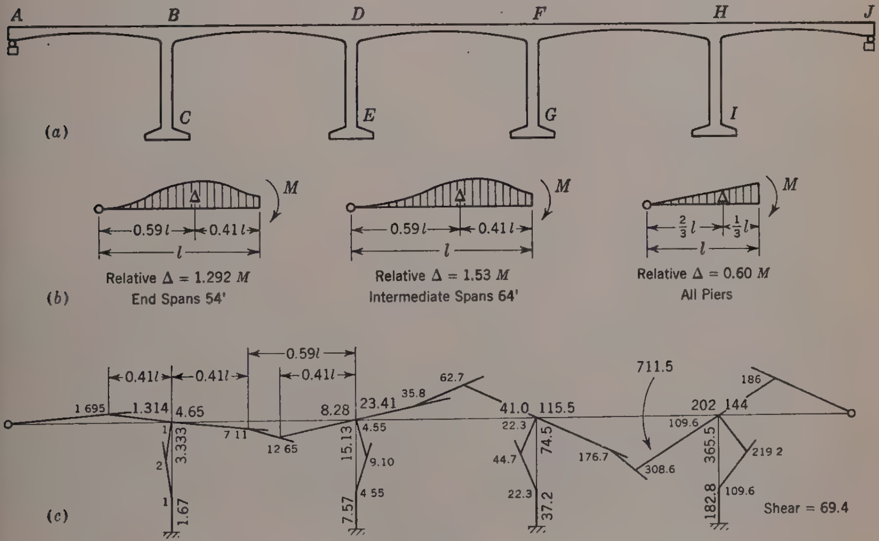

FIG. 4

Multi-Span Rigid Frame with Variable Moment of Inertia.—Fig. 4(a) represents a five-span rigid-frame bridge, the deck spans having parabolic soffits. The moment-area constants for each member may be obtained by the method described by George E. Large, Assoc. M. Am. Soc. C. E.,[6] or by tables based on integrations. In this structure all crowns are 27 in. deep, the haunches over the centers of piers are 54 in. deep, the piers are 40 in. thick, and d^3 is assumed to represent relative I-values. Young's modulus, E, being constant throughout, is omitted from the computations.

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[6] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 104, Fig. 48.

It is desired to obtain influence lines for any loading. Rotate Joint H as shown in Fig. 4(*c*) and draw the traverse diagram. Designate the relative values of the angles in Column $B\ C$ as 1, 2, and 1, as shown. For Span $A\ B$, the angles in the traverse triangle, being proportional to the opposite sides are computed, from the assumed relative joint rotation of 1, to be 0.69 and 1.69. The Δ-angle of 1.69 corresponds to a moment of 1.31 and the Δ-angle of 2 for the column corresponds to a moment of 3.33, these values being entered in their respective places on the traverse diagram.

These two moments, which act contra-clockwise, must be balanced by a clockwise moment, M_{BD}, equal to their sum, or 4.65. Enter this in Fig. 4(*c*) and compute and enter its corresponding Δ-angle of 7.11. The regular form of traverse equation from Joint B to Joint D now gives $1 + 7.11 \times 0.59 - 0.41\ \Delta = 0$, from which Δ for $M_{DB} = 12.65$ and its moment $= 8.28$ as shown. Enter these in Fig. 4(*c*) and by summation of angles obtain $\theta_D = 4.55$. The traverse angles for Column $D\ E$ are now added to the figure, the top moment in the column $= 15.13$ computed, entered on the figure and combined with the 8.28 deck moment to get $M_{DF} = 23.41$. This process is then repeated across the structure, and it is found that M_{HF} must equal 711.5 to produce the relative flexure factors and relative moments shown in Fig. 4(*c*). Since the structure is symmetrical the same relative factors may be reversed to compute the effect of a moment at Joint B. If the structure were not symmetrical a right to left traverse would be computed beginning with an angle of 1 to Joint H and ending, if correctly computed, with an external moment of 711.5 at Joint B. The distribution factors for fixed-end moments at Joints D and F are computed as already explained for Fig. 2. The effect of any fixed-end moment is now computed by simple ratios, and the effect of any combination of fixed-end moments can easily be tabulated.

Application to Closed Box-Frame.—In a closed box-frame each flexure factor is governed by stress flow along two separate routes. The solution is illustrated by Fig. 5, which shows such a structure, subjected to unsymmetrical loading, consisting of a single fixed-end moment at Joint B. Designate the rotation at Joint D by χ and traverse from Joint D to Joint B by Routes $D\ C\ B$ and $D\ A\ B$. The first traverse gives $\theta_B = 5.5\ \chi + 3$, and the second traverse gives $\theta_B = 7\ \chi - 4$. Equating these two values of θ: $\chi = 4.667$; from which value all flexure factors may be evaluated and the moments determined. The solution can then be extended to include side-sway corrections by the procedure applied to Fig. 3.

Application to Partial Fixture.—An easily understood and easily used method for treating partial fixation is available. In Fig. 6, let it be desired to analyze the effect of a partly fixed-column base. Add to the figure an imaginary underground member, as shown in dotted lines. Any desired degree of fixation can now be taken into account in the design by a proper adjustment of the stiffness factor, K_0, of the imaginary member, with reference to the stiffness factor, K, of the column. For 50% fixture, $K_0 = K$. For any ratio of fixation, designated as r, $r = \frac{K_0}{K_0 + K}$; and, $\theta = \frac{1 - r}{2\,r}\,\Delta_1$.

Application to Large Buildings.—Buildings like the Empire State Building, in New York City, and even those of much lesser size, have so many members that it is impracticable to analyze them by extending moment distribution over their entire width and height. This is especially the case when slow convergence of the successive approximations is encountered. For such cases a plausible method is to assume a degree of partial fixation at the various boundary joints around the part that is to be analyzed. For example, in Fig. 7, the moments in Girder $G\,H$ can be determined accurately with only ordinary judgment as to the degrees of fixation at Joints A, B, C, D, E, and F. This is because a given percentage of error in assuming the moment at Joints A, B, C, D, E, or F will produce a smaller percentage of error in M_{GH} or M_{HG}.

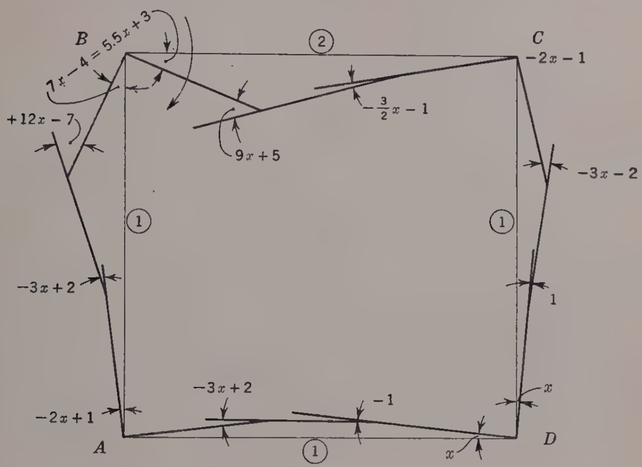

FIG. 5

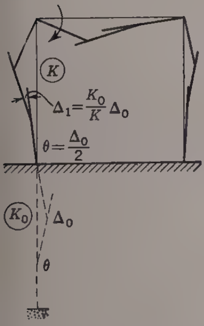

FIG. 6

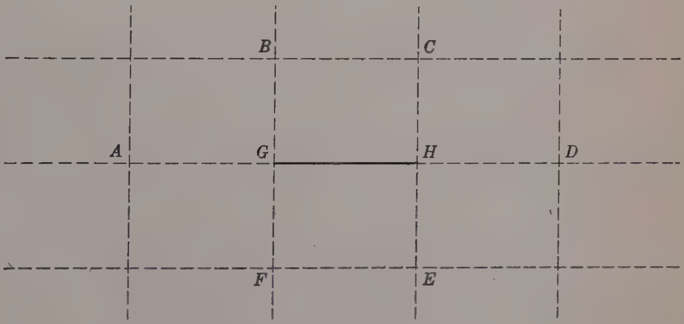

FIG. 7

SUMMARY AND CONCLUSIONS

It has been noted that the essential feature of the method set forth herein is to establish, as centers of rotation, the points of intersection of the tangents to elastic curves. Slope deflection is based on rotations at the joints where, due to continuity, the governing tangents to elastic curves do not establish angles. No simultaneous equations were used in this paper, whereas with slope deflection and also with the Maxwell-Mohr method of virtual work, very annoying groups of simultaneous equations would be needed to solve some of the problems presented.

A distinction between the method of relative flexure factors and end-moment distribution (which also avoids simultaneous equations) is that the latter begins the computation at the member affected by the load and works outward toward unloaded members, whereas the former begins the computation at a point remote from the loaded member and works toward it. The relative flexure-factor method does in a "once-over" computation what end-moment distribution accomplishes by a series of computations.

The writer feels quite confident that, for the class of structures illustrated in this paper, the method of treatment set forth will prove to be superior, not only in reduced arithmetical work but also in other respects, to the other methods mentioned.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

WATER-HAMMER PRESSURES IN COMPOUND AND BRANCHED PIPES

By Robert W. Angus,[1] Esq.

Synopsis

In two previous papers,[2,3] the writer has derived the principal equations for water hammer and has given the theory in some detail, so that only the barest outline of the general theory is given herein. Beginning with the two general equations derived by L. Allievi,[4] a graphical construction has been explained in some detail and illustrated in two simple problems of uniform pipes.

For the application of the method to simple pipes, the reader should consult the earlier papers, as this paper deals with other problems, and particularly those on compound and branched pipes. Illustrations are given of the application to parallel pipes, pipes with dead ends, surge tanks in systems, pipes leading from reservoirs and having two branches, each discharging water, the effect of gate closure on turbines and draft-tubes, and the case of a pumping system in which the pressure falls so low as to cause the column to separate. In all these cases the variation of pressure and velocity with time is found for various points in the system.

All these problems, and many more, may be solved by a careful worker, both accurately and quickly, on the drafting-board, although their solution by analytical means is almost impossible.

Notation.—The letter symbols used in this paper are defined where they are first mentioned and, for convenience of reference, a complete list is given in the Appendix.

Note.—Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **May 15, 1938.**

[1] Head of Dept. of Mech. Eng., Univ. of Toronto, Toronto, Ont., Canada.

[2] "Simple Graphical Solution for Pressure Rise in Pipes and Pump Discharge Lines," by Robert W. Angus, *Journal*, Eng. Inst. of Canada, 1935, Vol. 18, pp. 72, 264.

[3] "Water Hammer in Pipes, Including Those Supplied by Centrifugal Pumps: Graphical Treatment," by R. W. Angus, *Proceedings*, Inst. M. E., 1936.

[4] "Theory of Water Hammer, Notes I–V," by L. Allievi; Tr. by E. E. Halmos, M. Am. Soc. C. E.; Distributed by A. S. M. E., 1925.

OUTLINE OF GENERAL THEORY

When a closed pipe is filled with moving liquid (water will be referred to hereafter), the laws governing the changes of pressure and discharge will depend upon the conditions under which flow occurs. If the motion is steady (by which is meant that the volume per second passing through any section of the pipe remains constant as time goes on), the Bernoulli equation may be applied.

When the motion is unsteady (that is, when the discharge at each section is varying from one instant to the next), the Bernoulli equation no longer serves, and the pressures and velocities are not connected by this equation. Sometimes during unsteady motion there is a mass flow of the entire column of water in the pipe, such as occurs when the water surges back and forth between two reservoirs, or between a reservoir and a tank; such cases are common and there must be an open tank or reservoir, or an air chamber, in the line.

On the other hand, variable motion may occur in a fully closed system provided there is anything to start it, the motion being then due to the elasticity of the water and pipe. Thus, variable motion may occur in a section of a pipe system with a "dead end," due to some change in the system, such as the closing of a valve somewhere, although there may be no delivery of water through the pipe.

The three cases are distinct and important: The first is used, for example, in finding the size of a pipe for given service; the second case is common in water-power plants where surge tanks are necessary to store and restore the water during load changes; and, the third case is of common occurrence in all systems. Quite frequently, the latter two cases occur together; there is a mass surge of the water in the system, but at the same time the elasticity of the water and pipes produces independent action, often building up high pressures at different points and not infrequently producing pressures at the bottom of an open tank many times greater than that corresponding to the depth of water in the tank. Under certain conditions, too, the elasticity of the water and pipe walls induces extremely low or collapsing pressures, and may even cause separation of the water column itself.

This paper refers to the latter case of unsteady motion which produces the phenomenon of water hammer. It will be best to start with the simple case of a horizontal uniform pipe connected at one end to a reservoir that has a fixed level and terminating in a nozzle, gate, or valve (the three terms are used in the paper for the same type of device) which may be used to control the flow. The equation of continuity is used and will be specifically applied to branch pipes, but it is to be noted that at a given instant the elasticity of the water and of the pipe walls enables the discharges at different points on the same pipe to differ at the same time.

In many problems, velocity head is relatively small, as is also friction loss, and the cases under consideration are simplified if the two terms are omitted; the pressures computed when friction loss is omitted are nearly always greater than the actual pressures and, therefore, the results are on the safe side. Where pipes burst, or in other unusual conditions, both friction and velocity may have to be taken into account, and the method of doing this is explained subsequently. For convenience, the reservoir level is assumed constant, although variations in it are easily allowed for; these variations are very slow, however, in comparison with the pressure wave velocity.

The case described is illustrated by Pipe $A\,B\,C$ of length L in Fig. 1, in which Point A is at the discharge end, Point C, at the reservoir end, and Point B, at the center. Under conditions of steady flow with the gate fully open, H_0 will represent the pressure on the gate as well as the elevation of the water surface above the gate, and if the pipe area is A_a, the steady velocity, V_0, in the pipe with the discharge, Q_0, is given by,

$$Q_0 = A_a\,V_0 \qquad (1)$$

in which the units are in feet and seconds. Suppose, now, that a closing movement of the gate is begun which would effect complete closure in T sec; any law of closure may be used, but if it is such that the "effective gate area" (by which is meant the actual gate area multiplied by the proper coefficient of discharge) decreases at a uniform rate with time, the motion is referred to as "straight-line" or "linear," a nomenclature used in the remainder of this paper.

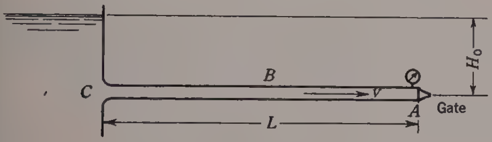

Fig. 1

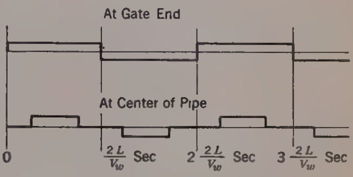

Fig. 2

The actual gate movement may be assumed made in small steps, each one of which causes a small change in the pipe velocity with a corresponding pressure change along the pipe. The mechanics of the phenomena are fairly well understood and need not be repeated, except to state that the first movement causes a direct pressure wave to travel up the pipe with velocity V_w, the round trip of this wave requiring $\frac{2\,L}{V_w}$ sec, and it returns a second time up the pipe from the gate as an indirect pressure wave unchanged in magnitude, etc. These waves continue to follow one another, each small gate movement producing corresponding waves, the ultimate pressure at any point being the resultant of those due to the separate waves. Fig. 2 illustrates what is meant by each wave, and its effect at the end and center of the pipe.

The magnitude of the pressure waves at the gate may be determined from the following considerations: After a small closing movement of the gate, a pressure wave travels up toward the reservoir with velocity, V_w, and in the time, δT, it will have traveled a distance (see Fig. 3),

$$L_w = V_w\,\delta T \qquad (2)$$

the pressure rise for such change being indicated by δh. Denoting the part of the velocity extinguished in this movement by δV, Newton's second law gives,

$$-w\,A_a\,\delta h = \frac{m\,\delta V}{\delta T} = \frac{A_a\,w\,V_w\,\delta T}{g}\,\frac{\delta V}{\delta T} \dots\dots\dots\dots\dots (3)$$

in which m is the mass of the water changed in velocity; and w is the weight of a cubic foot of water. The minus sign appears in Equation (3) because an increase in δh corresponds to a decrease in δV. The formula reduces to,

$$-\,\delta h = \frac{V_w}{g}\,\delta V \dots\dots\dots\dots\dots (4)$$

Integration of Equation (4) gives, for the first interval:

$$h = H - H_0 = \frac{V_w}{g}(V_0 - V) \dots\dots\dots\dots\dots (5)$$

in which H is the new reading of the gage at the gate. The maximum value of h results when $V = 0$; in other words, when the entire velocity is extinguished in the first interval, in which case the pressure rise is,[5]

$$h = H - H_0 = \frac{V_w}{g}\,V_0 \dots\dots\dots\dots\dots (6)$$

and this shows the interesting fact that the maximum pressure rise is independent of the dimensions of the pipe and of the head, and depends only on the velocity extinguished.

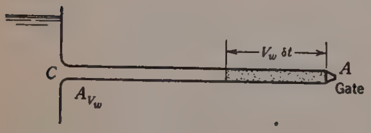

FIG. 3

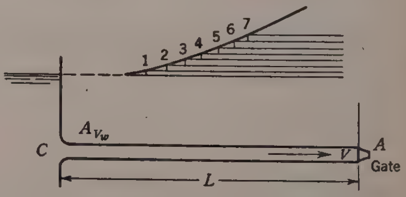

FIG. 4

Since Equation (6) refers to the direct waves and takes no account of the reflected ones, it can only be applied to find the direct change in pressure corresponding to an individual gate movement and not to find the pressure rise in the pipe, directly, unless closure is effected in one interval, in which case no reflection occurs. The equation applies naturally within any interval provided the proper velocities are used. The magnitude of the direct wave in the nth movement of the gate will be indicated by F_n and is given by,

$$F_n = H_n - H_{n-1} = \frac{V_w}{g}(V_n - V_{n-1}) \dots\dots\dots\dots\dots (7)$$

Having traced the course of events following the first gate movement, it is now possible to see what happens during subsequent movements. If the closing time, T, is less than $\frac{L}{V_w}$, the events are as shown in Fig. 4, where the numbers indicate the pressures computed from Equation (5) corresponding to the small movements of the gate; the combined series of movements produces a wave of sloping (curved) front traveling first from the gate, and this wave will reach its maximum height before its "toe" reaches Point C, as represented in Fig. 5(*a*). When the closure requires $\frac{L}{V_w}$ sec, the wave front is shown in Fig. 5(*b*), the wave front covering the entire length of the pipe; but, in Case 5(*c*) in which the closure time exceeds $\frac{L}{V_w}$, the returning opposition wave neutralizes some of the positive pressures and the maximum pressure is exerted on the pipe until Point g is above Point j. In Case 5(*d*) the maximum pressure is reached only, but does not remain, as the opposition wave front cancels it, whereas, in Case 5(*e*), the pressure is never as high as in the former cases. The maximum pressure reached is the same in Cases (*a*) to (*d*) where $T \leqq \frac{2\,L}{V_w}$ (provided the same velocity is extinguished at each step); but the quicker the closure the longer the time will be during which this maximum pressure continues.

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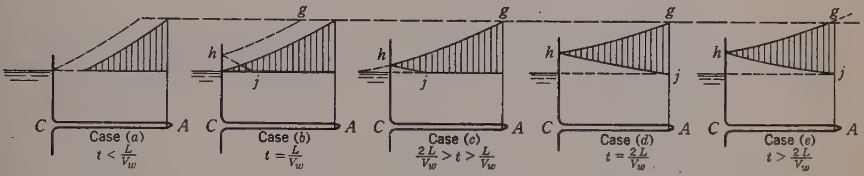
[5] A summary of the work done by N. Joukowsky, of Moscow, in 1898, has been given in a paper entitled "Water Hammer" by Miss O. Simin, *Proceedings*, Am. Water Works Assoc., 1904, p. 341.



Case (*a*) $t < \frac{L}{V_w}$ Case (*b*) $t = \frac{L}{V_w}$ Case (*c*) $\frac{2\,L}{V_w} > t > \frac{L}{V_w}$ Case (*d*) $t = \frac{2\,L}{V_w}$ Case (*e*) $t > \frac{2\,L}{V_w}$

FIG. 5

For the case in Fig. 5(*e*) the same maximum pressure is not reached. Furthermore, it may be shown that for linear gate movement the curvature of the wave front is not great; it is convex to the horizontal axis as drawn, so that in such cases as those in Figs. 5(*b*), 5(*c*), 5(*d*), and 5(*e*), the total pressure rise has roughly a straight-line variation from zero at the reservoir to a maximum value at the gate, a fact also confirmed by many computations.

The general proposition may then be stated that the maximum pressure rise is produced for closing times equal to, or less than, $\frac{2\,L}{V_w}$; this is commonly referred to as sudden closure. For short pipes the closure may easily be too slow to produce the most serious results, but it is quite possible in the longer pipes, and, for this reason, it is frequently stated that the danger of damage from water hammer is greater in long pipes than in short ones; this statement, however, is true only in the sense that the time of closure to produce the worst results is often within the range of operation of the valves on long pipes.

FUNDAMENTAL EQUATIONS FOR VARIABLE FLOW IN CLOSED PIPES

In order to establish the fundamental relations, the variation in flow is considered to be due to closing a valve in a pipe line, thereby causing a decrease in the flow. The equations thus established apply equally well to a gate-opening.[4] The pipe is assumed to be of a uniform diameter, D, and a thickness, t; and the problem frequently occurs in one or the other of the two forms shown in Fig. 6. The former (Fig. 6(*a*), with the enlarged section in Fig. 6(*b*)) corresponds to closing a nozzle on the end of a pipe, whereas Fig. 6(*c*) represents a pumping line with a control valve, decreased flow being caused by closing the valve or by cutting off power to the pump. Elsewhere,³ the writer has demonstrated that, at any point, situated a distance, x, from the gate (Case (*a*), Fig. 5) and at a time, T, after closure begins:

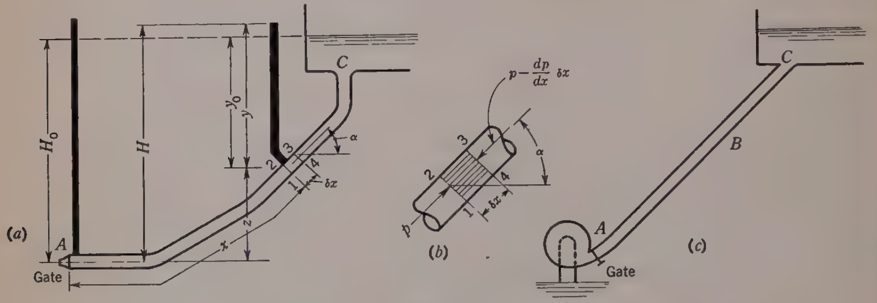


FIG. 6

$$\frac{\delta V}{\delta T} = g\,\frac{\delta H}{\delta x} \qquad (8a)$$

and,

$$\frac{\delta V}{\delta x} = \frac{g\,\delta H}{V_w^2\,\delta T} \qquad (8b)$$

In Case (*c*), Fig. 5, both left-hand members have a negative sign and, therefore, Equations (8) apply, with proper correction to the sign. Furthermore,

$$V_w = \frac{\sqrt{\dfrac{g\,K}{w}}}{\sqrt{1 + \dfrac{K\,D}{E\,t}}} = \frac{4\,665}{\sqrt{1 + \dfrac{K\,D}{E\,t}}} \qquad (9)$$

in which K is the bulk modulus of elasticity for water; and, E is Young's modulus for the material in the pipe. Some approximate values of the ratio, $\frac{K}{E}$, are:

Steel plate	0.0097 (usually assumed as 0.01)
Cast iron	0.02 to 0.022
Concrete	0.10
Wood	0.20

In Equations (8) and (9), $\frac{\delta V}{\delta T}$, $\frac{\delta H}{\delta x}$, $\frac{\delta V}{\delta x}$, and $\frac{\delta H}{\delta T}$, refer to changes in a small cylinder (1 2 3 4, Fig. 6(*b*)) situated a distance, x, from the gate. The general integrals of Equations (8) were first obtained by Allievi as follows:

$$H - H_0 = F\left(T - \frac{x}{V_w}\right) + f\left(T + \frac{x}{V_w}\right) \qquad (10a)$$

and,

$$V_0 - V = \frac{g}{V_w}\left\{F\left(T - \frac{x}{V_w}\right) - f\left(T + \frac{x}{V_w}\right)\right\} \qquad (10b)$$

and the corresponding integrals for Case (*c*), Fig. 6, are similar except for the change in sign of the left-hand member in each case.

Equations (10) have been illustrated in the Appendix hereto, since they are essential to a proper understanding of this paper. In the Appendix the writer has used an example from the paper published in 1919 by Norman R. Gibson, M. Am. Soc. C. E.[6] One point deserves special emphasis: The pressure head, H, is measured above the horizontal plane of the nozzle, so that at Section 1–2, Fig. 6(*a*), the actual pressure in the pipe is only y and is connected with H by the relation,

$$H = y + z \qquad (11)$$

The pressure rise at this section is $H - H_0 = y - y_0$; but the ratio of the pressures after, and before, gate movement is,

$$\frac{y}{y_0} = \frac{H - z}{H_0 - z} \qquad (12)$$

In Equations (10) (which will be referred to as Allievi's equations), H represents the pressure head, in feet, and V the velocity, in feet per second, at a point distant x from the gate, at a time, T, after closure begins; $F\left(T - \frac{x}{V_w}\right)$ denotes the sum of all direct pressure waves (positive, in closing), each persisting for a time, $T = \frac{2\,L}{V_w}$, at the gate; and $f\left(T + \frac{x}{V_w}\right)$ denotes the sum of all reflected pressure waves (negative, in closing), each of which quantities also persists for a time, $T = \frac{2\,L}{V_w}$ sec, at the gate. A little consideration will show that the direct wave in any interval is equal in magnitude, but of opposite sign, to the reflected wave, $\frac{2\,L}{V_w}$ sec later, if the reflected and direct waves each correspond to the same gate movement. This has been shown in the general discussion of the pressure waves in the pipe and also in the illustration given in the Appendix.

Allievi's equations (Equations (10)) may be used to find the conditions at any point on the pipe, but if the pressure at a point, $x = 0.75\,L$, is desired, the steps of the valve motion must be not more than $\frac{1}{4}\left(\frac{2\,L}{V_w}\right) = \frac{L}{2\,V_w}$ sec apart, and a similar statement applies to other points; for the gate the movements may be $\frac{2\,L}{V_w}$ sec apart. Similar equations may be written[2] where friction and velocity head are considered if the following quantity, H_T, is substituted for H:

$$H_T = H + \frac{V^2}{2\,g} + h_r \qquad (13)$$

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[6] "Pressures in Penstocks Caused by the Gradual Closing of Turbine Gates," by Norman R. Gibson, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 707.

in which h_r is the friction loss, in feet, up to the point considered. The equations applying to Fig. 6(*a*) would then be:

$$H_T - H_0 = F\left(T - \frac{x}{V_w}\right) + f\left(T + \frac{x}{V_w}\right) \ldots\ldots\ldots\ldots (14a)$$

and,

$$V_0 - V = \frac{g}{V_w}\left\{F\left(T - \frac{x}{V_w}\right) - f\left(T + \frac{x}{V_w}\right)\right\} \ldots\ldots (14b)$$

and for the case of Fig. 6(*c*) they are easily written. The solution of the problem is virtually the same whether H_T or H is used, and hence the graphs will be constructed using Equations (10).

The analytical method of dealing with water hammer has been used to some extent in practice. For example, Dr. Charles Jaeger[7] has described the analytical solution in considerable detail and has illustrated it by some examples. It must be evident, however, that: (*a*) Much care is to be exercised in its application; (*b*) some of the calculations are long and somewhat involved; and, (*c*) there is a possibility of error in using it unless one is doing it very frequently. It is quite difficult to apply in case of branched and complicated pipes, but has proved of great value in establishing basic relationships.

In order to simplify and hasten the answers to problems in water hammer, a number of graphical methods have been devised of which probably that of Allievi[4] was the first; but his method was only applied to uniform pipes, without friction, and usually with straight-line valve movement. Furthermore, the method is not practicable on account of the large scale to which the drawings should be made. Allievi used it to develop his useful series of charts. In 1926, Dr. Hans Kreitner[8] described a method of merit which was translated and summarized by the writer in 1935.[9] This method has been used by Hruschka.[10]

The construction described and applied in this paper is based on that of earlier writers, but its development is due very largely to Dr. O. Schnyder,[11] of Klus, Switzerland, and to Professor L. Bergeron,[12] of Paris, France, who have shown that the method is general and may be very simply used in solving the most complicated problems quickly.

GRAPHICAL METHOD

The graphical method proposed herein proceeds from Equations (10) and as these formulas are also used in the analytical development the same results will be obtained by the graphical as by the analytical solution. By addition and subtraction, they reduce to:

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[7] "Theorie General du Coup de Belier," by Charles Jaeger, Dunod, Paris, 1933.

[8] "Druckschwankungen in Turbinenrohrleitungen," *Die Wasserwirtschaft*, 1926, No. 10; a translation by Professor Angus has been filed for reference in Engineering Societies Library, 33 West 39th St., New York, N. Y.

[9] *Mechanical Engineering*, December, 1935.

[10] "Druckrohrleitungen der Wasserkraftwerke," by A. Hruschka, Julius Springer, Berlin, 1929.

[11] "Über Druckstösse in Rohrleitungen," by Dr. O. Schnyder, *Wasserkraft und Wasserwirtschaft* (Berlin), 1932, Vol. 5/6; also, "Über Druckstösse in verzweigten Leitungen mit besonderer Berücksichtigung von Wasserschlossanlagen," by O. Schnyder, *Wasserkraft und Wasserwirtschaft* (Berlin), 1935, Vol. 12.

[12] "Variations de régime dans les conduites d'eau," by Prof. L. Bergeron, *Comptes Rendus* des Travaux de la Société Hydro-technique de France, May, 1932, p. 605H; also, "Etude des variations de régime dans les conduites d'eau: Solution graphique generale," by Prof. L. Bergeron, *Revue Generale de l'Hydraulique* (Paris), 1935, p. 12, etc.

$$H - H_0 = -\frac{V_w}{g}(V_0 - V) + 2F\left(T - \frac{x}{V_w}\right) \quad \text{.........(15a)}$$

and,

$$H - H_0 = +\frac{V_w}{g}(V_0 - V) + 2f\left(T + \frac{x}{V_w}\right) \quad \text{.........(15b)}$$

If Equations (15) are applied to the gate end of the pipe, where $x = 0$, they will be written:

$$H - H_0 = -\frac{V_w}{g}(V_0 - V) + 2F(T) \quad \text{.............(16a)}$$

and,

$$H - H_0 = +\frac{V_w}{g}(V_0 - V) + 2f(T) \quad \text{.............(16b)}$$

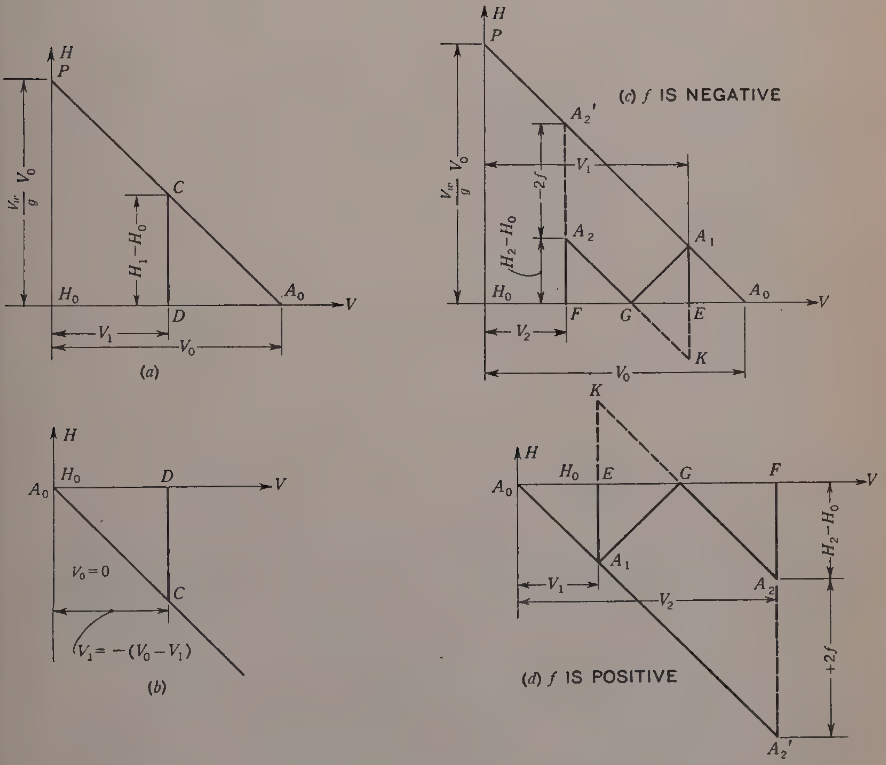

FIG. 7

In Fig. 7 the axes of H and V have been drawn as shown, and Point A_0 represents the initial steady velocity, V_0, and static head, H_0. In the first interval there is no reflected wave, or $2f(T) = 0$, and Equation (16b) represents a straight line through A_0 with a tangent $-\frac{V_w}{g}$ as represented at $A_0\,P$ in Fig. 7(*a*). For a closing motion of the gate, reducing the velocity to V_1 in $T \leqq \frac{2\,L}{V_w}$, the rise in pressure is given by Line $C\,D$ which corresponds to $-\frac{V_w}{g}(V_0 - V_1) = H_1 - H_0$, as shown in Fig. 7(*a*), and Equation (16*a*) shows that the direct wave, $C\,D$, equals $F_1\,(T)$ (see also Equation (7)), since,

$$2\,F_1\,(T) = H_1 - H_0 + \frac{V_w}{g}(V_0 - V_1) \ldots\ldots\ldots\ldots\ldots (17)$$

When the opening occurs from a closed gate, $V_0 = 0$, at $H = H_0$, and the line, $A_0\,C$, is the locus of the values of H as before. Both $V_0 - V_1$ and $H_1 - H_0$ are negative in this case (see Fig. 7(*b*)). If, however, the velocity, $V = V_2$, is not reached until an interval beyond the first—for example, in the time $2\left(\frac{2\,L}{V_w}\right) > T > \frac{2\,L}{V_w}$—then the pressure reached is modified by the reflected wave. When the gate is being closed, the previous discussion shows that the reflected wave produces a negative pressure equal in magnitude, but opposite in sign, to the direct or F-wave produced in the previous interval. In Fig. 7(*c*) let V_1 be the velocity reached at the end of the first interval, then the pressure corresponding to this velocity will be represented at Point A_1 and Line $A_1\,E$ is the pressure rise, or the value of $F_1\,(T)$ in the first phase. It is also the numerical value of the reflected pressure or f-wave in the second interval; that is,

$$\overline{A_1\,E} = -f_2\,(T) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Equation (16*b*) with Fig. 7(*c*) shows that,

$$A_2'\,F = +\frac{V_w}{g}(V_0 - V_2) \ldots\ldots\ldots\ldots\ldots\ldots (19a)$$

and,

$$2\,A_1\,E = A_1\,K = -2\,F_1\,(T) = 2\,f_2\,(T) \ldots\ldots\ldots\ldots (19b)$$

Hence, $H_2 - H_0 = A_2\,F$; and, A_2 represents the conditions corresponding to the change of velocity, $V_0 - V_2$, since $A_2'\,A_2 = A'\,K$. Point A_2 is determined graphically either by making Line $E\,K$ = Line $A_1\,E$ and drawing Line $K\,A_2$ parallel to Line $A_0\,P$; or, by drawing Line $A_1\,G$ at the slope $+\frac{V_w}{g}$ (that is, $\overline{A_0\,E} = \overline{E\,G}$) and drawing Line $G\,A_2$ parallel to Line $A_0\,P$.

When the gate is being opened, the result is quite similar. Suppose the gate is opened from a closed position with $V_0 = 0$ and the velocity, V_1, is reached in the first interval, it is evident that Line $C\,D$, Fig. 7(*b*), represents $H_0 - H_1$. If the velocity, V_2, is reached in the second interval, the construction would be as shown in Fig. 7(*d*), in which both $V_0 - V_2$ and $H_2 - H_0$ are negative, the slope of the line has the same tangent, $-\frac{V_w}{g}$, as before, and

A_1 represents the pressure at the end of the first interval. Therefore, $F_1(T)$ is negative and $f_2(T)$ is positive and equal to $A_1 E$, and the point, A_2, representing H_2 lies at A_2 above A_2'. Equations (16), therefore, are easily represented by this graphical construction.

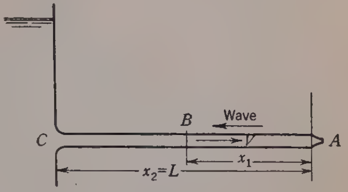

FIG. 8

These two formulas, Equations (15), then, define the laws of the graphical method, and it only remains to re-arrange them into a more useful form. Applying them to the pipe shown in Fig. 8 (in which case T_0, T_1, and T_2, denote the times that the same direct wave passes, respectively, Point A on the pipe at the origin, Point B at a distance, x, and Point C at the reservoir, which is a distance, $x_2 = L$ from Point A), it is to be noted that,

$$T_1 = T_0 + \frac{x_1}{V_w} \quad \ldots\ldots\ldots\ldots (20a)$$

and,

$$T_2 = T_0 + \frac{x_2}{V_w} \quad \ldots\ldots\ldots\ldots (20b)$$

Therefore, while the direct wave is proceeding from Point A on the pipe to Point C, the following equations apply (using H_{A0}, etc., to denote the steady condition):

$$H_{AT0} - H_{A0} = -\frac{V_w}{g}(V_{A0} - V_{AT0}) + 2\,F\,(T_0) \quad \ldots\ldots (21a)$$

$$H_{BT1} - H_{B0} = -\frac{V_w}{g}(V_{B0} - V_{BT1}) + 2\,F\left(T_1 - \frac{x_1}{V_w}\right)$$

$$= -\frac{V_w}{g}(V_{B0} - V_{BT1}) + 2\,F\,(T_0) \quad \ldots\ldots\ldots\ldots (21b)$$

and,

$$H_{CT2} - H_{C0} = -\frac{V_w}{g}(V_{C0} - V_{CT2}) + 2\,F\left(T_2 - \frac{x_2}{V_w}\right)$$

$$= -\frac{V_w}{g}(V_{C0} - V_{CT2}) + 2\,F\,(T_0) \quad \ldots\ldots\ldots\ldots (21c)$$

Furthermore, while the reflected wave is proceeding toward the gate, the reverse of Fig. 8, and reaches Point B at the time, T_3, and Point A at the time, T_4, the following equations apply:

$$H_{CT2} - H_{C0} = +\frac{V_w}{g}(V_{C0} - V_{CT2}) + 2\,f\left(T_2 + \frac{x_2}{V_w}\right)$$

$$= +\frac{V_w}{g}(V_{C0} - V_{CT2}) + 2\,f\,(T_4) \quad \ldots\ldots\ldots\ldots (22a)$$

$$H_{BT3} - H_{B0} = +\frac{V_w}{g}(V_{B0} - V_{BT3}) + 2\,f\left(T_3 + \frac{x_1}{V_w}\right)$$

$$= +\frac{V_w}{g}(V_{B0} - V_{BT3}) + 2\,f\,(T_4) \quad \ldots\ldots\ldots\ldots (22b)$$

and,

$$H_{AT4} - H_{A0} = + \frac{V_w}{g}(V_{A0} - V_{AT4}) + 2f(T_4) \text{........} (22c)$$

Generally, at any point, such as Point B, and at any instant, two waves are proceeding: The direct wave toward Point C; and the reflected wave toward Point A. For the direct wave the pressure rise above the initial steady pressure, H_{B0}, at Point B is given by Equation (21*b*), and for the reflected wave it is given by Equation (22*b*). If friction is not being considered, then $H_{A0} = H_{B0} = H_{C0}$ as the water level will be the same in piezometers attached to the pipe at Points A, B, and C; and subtractions of Equations (21) and (22), made in suitable order, give:

$$H_{AT0} - H_{BT1} = + \frac{V_w}{g}(V_{AT0} - V_{BT1}) \text{............} (23a)$$

$$H_{BT1} - H_{CT2} = + \frac{V_w}{g}(V_{BT1} - V_{CT2}) \text{............} (23b)$$

$$H_{CT2} - H_{BT3} = - \frac{V_w}{g}(V_{CT2} - V_{BT3}) \text{............} (23c)$$

and,

$$H_{BT3} - H_{AT4} = - \frac{V_w}{g}(V_{BT3} - V_{AT4}) \text{............} (23d)$$

The case illustrated is that of a pipe discharging from a reservoir through a closing gate. A very common case is also that of a pump discharging through a pipe into a reservoir, as shown in Fig. 6(*c*). The flow is controlled by a gate near the pump, the movement of the gate causing pressure disturbances. It has already been shown that the algebraic signs of the fundamental differential equations are the opposite of those in the case of Fig. 8. This only means that all terms on the right-hand side containing V will change sign, and, hence, for the case of the pump with controlling discharge valve near it, the Equations (23) would be written:

$$H_{AT0} - H_{BT1} = - \frac{V_w}{g}(V_{AT0} - V_{BT1}) \text{............} (24a)$$

and,

$$H_{BT1} - H_{CT2} = - \frac{V_w}{g}(V_{BT1} - V_{CT2}) \text{............} (24b)$$

etc., the letters, A and C, referring to the valve and reservoir ends, respectively, of the pipe, as in the previous case. In Fig. 8, let $x_2 = 10\,000$ ft, in which the length, $x_1 = 4\,500$ ft, and $V_w = 4\,000$ ft per sec. Then, reckoning time from the beginning of closure, it follows that $T_0 = 0$; $T_1 = 0 + \frac{4\,500}{4\,000} = 1.125$ sec; $T_2 = 0 + \frac{10\,000}{4\,000} = 2.5$ sec; $T_3 - T_2 = \frac{10\,000 - 4\,500}{4\,000} = 1.375$ sec; and, $T_4 - T_3 = \frac{5\,000}{4\,000} = 1.125$ sec.

For many problems the series of equations is best left in the form shown in Equations (23) or Equations (24), and some illustrations will be given subsequently in which this method is adopted; but it often happens that the engineer is more interested in the proportional rise in pressure than in the actual pressure and wishes to know the ratio, $\frac{H}{H_0}$, directly. The results are the same whether one obtains H, $H - H_0$, or $\frac{H}{H_0}$; it is purely a matter of convenience for the particular application. If the equations are modified to yield the value of $\frac{H}{H_0}$, Equation (23*a*) may be written:

$$\frac{H_{AT0}}{H_0} - \frac{H_{BT1}}{H_0} = 2\,\frac{V_w\,V_{A0}}{2\,g\,H_0}\left(\frac{V_{AT0}}{V_{A0}} - \frac{V_{BT1}}{V_{A0}}\right) \qquad (25)$$

Ordinarily, small letters are used to designate the ratios; thus: $h_{BT1} = \frac{H_{BT1}}{H_0}$; $v_{BT1} = \frac{V_{BT1}}{V_{B0}}$, etc. Furthermore, the relation, $\frac{V_w\,V_{A0}}{2\,g\,H_0} = \rho$, will be referred to as the pipe-line characteristic, as defined by Allievi. Similar relationships are found for the other equations and Equations (23) then take the form:

$$h_{AT0} - h_{BT1} = +\,2\,\rho\,(v_{AT0} - v_{BT1}) \qquad (26a)$$

$$h_{BT1} - h_{CT2} = +\,2\,\rho\,(v_{BT1} - v_{CT2}) \qquad (26b)$$

$$h_{CT2} - h_{BT3} = -\,2\,\rho\,(v_{CT2} - v_{BT3}) \qquad (26c)$$

and,

$$h_{BT3} - h_{AT4} = -\,2\,\rho\,(v_{BT3} - v_{AT4}) \qquad (26d)$$

which is the form in which they are mostly commonly used in the graphical construction. For the pumping system (see Equations (10)), the algebraic signs of the second terms would be reversed.

The derivation of Equations (23), (24), and (26) has been made for closing movements of the gate only, but they will apply in exactly the same form to opening movements, because in this case, the change of pressure has the opposite sign to the change of velocity as in the case of closure; therefore, the two series (Equations (23) and (26) and the corresponding formulas for the pumping system) apply also to opening movements.

To represent Equations (26) graphically, axes of h and v, Fig. 9, are assumed, the origin having the value, $h = 0$, $v = 0$. The values of h should preferably be plotted on the vertical axis. Continuing still with the problem of closure, as in Fig. 6(*a*), the values at Point A during steady flow before closure begins are $H = H_0$ and $V = V_0$; hence, at the start, $h = 1$ and $v = 1$, corresponding to the time, T_0, in Equations (26). Furthermore, the pressure wave does not reach Point B until the time, T_1, and does not reach Point C until the time, T_2, therefore, up to these respective times the conditions at Points B and C remain unchanged, or $h_{BT1} = h_{CT2} = 1$, and $v_{BT1} = v_{CT2} = 1$. In Fig. 9(*a*), a point A_{T0}, will represent the conditions at Point A at the time,

T_0; similarly, B_{T1} will represent those at Point B at the time, T_1, etc. Therefore, A_{T0}, B_{T1}, and C_{T2} are all at Point $h = 1$, $v = 1$.

For this problem, if it is assumed that the reservoir level is constant, then all values of $h_C = 1$, and hence all points, C, must lie at $h = 1$ on the axis of v; Equations (26*a*) and (26*b*) are then represented by a point. It may be simplest for the moment to proceed at once from Points A to C, thus combining Equations (26*c*) and (26*d*) into:

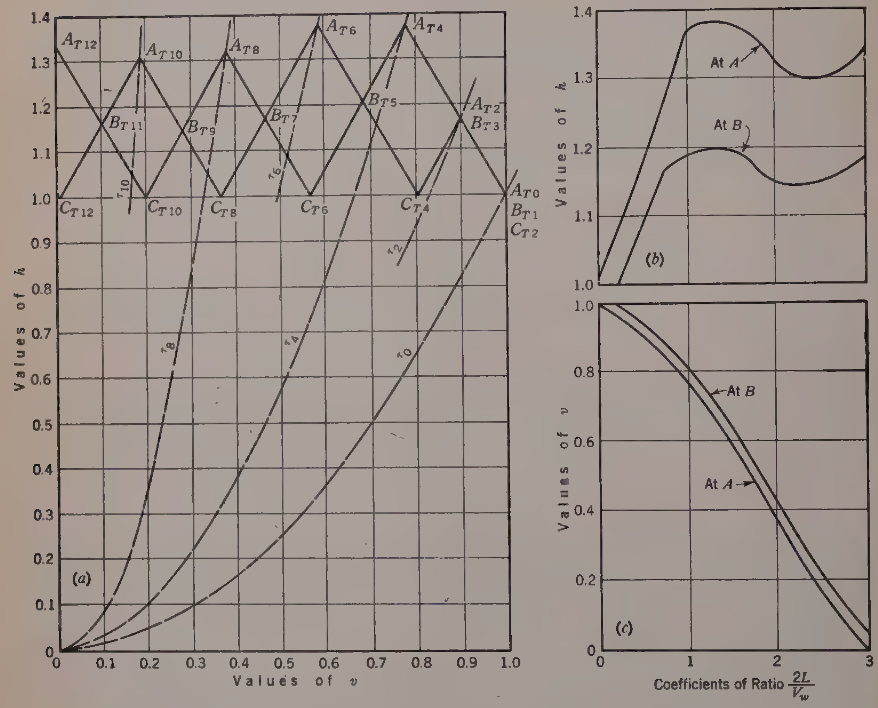

FIG. 9

$$h_{CT2} - h_{AT4} = -2\,\rho\,(v_{CT2} - v_{AT4}) \ldots\ldots\ldots\ldots\ldots (27)$$

This is evidently a line through C_{T2} sloping at the angle, $\tan^{-1} - 2\,\rho$, which defines one line on which A_{T4} lies. It is now necessary to establish the law connecting the head, H_A, and the velocity, V_A, at the gate. No matter what form the gate has, the velocity of discharge from it is,

$$V_A = c_d\,\sqrt{2\,g\,H_A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which c_d is the coefficient of discharge, and in which the discharge is found by multiplying this velocity by A_g, the area of the gate-opening. Hence, the pipe velocity is,

$$V_A = \frac{A_g \times c_d\,\sqrt{2\,g\,H_A}}{A_a} = B\,\sqrt{H_A} \ldots\ldots\ldots\ldots\ldots (29)$$

in which B is a function of the effective gate area, $(c_d\, A_g)$; and, therefore, B is a function of the time. For the initial steady condition:

$$V_{A0} = B_0 \sqrt{H_0} \quad \text{(30a)}$$

and, therefore:

$$\frac{V_A}{V_{A0}} = \frac{B}{B_0}\sqrt{\frac{H_A}{H_0}} = \tau \sqrt{\frac{H_A}{H_0}} \quad \text{(30b)}$$

in which τ is also a function of the time and has a maximum value, $\tau = \tau_0 = 1$, for full gate-opening. Expressing Equation (30b) in the notation adopted:

$$v_A = \tau \sqrt{h_A} \quad \text{(31a)}$$

or, at the time, T_4, at Point A on the pipe:

$$v_{AT4} = \tau_4 \sqrt{h_{AT4}} \quad \text{(31b)}$$

Equation (31b) is evidently the formula for a parabola with its vertical axis coinciding with that of h and with its vertex at $h = 0, v = 0$. Since $\tau = \tau_0 = 1$ for the full open gate, the corresponding curve will pass through $h = 1$, $v = 1$, and is easily plotted. Similarly, the parabola for τ_4 is readily drawn if the law of gate closure is known, because for each instant during gate motion the value of $\frac{B}{B_0} = \tau$ is known.

If a parabola corresponding to τ_4 is drawn in Fig. 9(*a*), it will be the locus of the points, A_{T4}, and where it intersects the other line containing A_{T4}, it will locate the desired point. It is now possible to locate B_{T3} and, for simplicity, B will be assumed at the center of the pipe. Equations (26) show that it lies on the line, $C_{T2} - A_{T4}$, and following the method already adopted, one may write:

$$h_{AT2} - h_{BT3} = +\,2\,\rho\,(v_{AT2} - v_{BT3}) \quad \text{(32)}$$

but A_{T2} lies on the parabola, with the value, τ_2, found as in the case of that for A_{T4} (for linear closure, $\tau_2 = \frac{1}{2}(1 + \tau_4)$), and, therefore, Equation (32) represents a point and B_{T3} coincides with A_{T2}. If one desired to know the conditions at Point A one interval later, it would be necessary to find the point, A_{T8}, for which the following equations would be written:

$$h_{AT4} - h_{CT6} = 2\,\rho\,(v_{AT4} - v_{CT6}) \quad \text{(33a)}$$

and,

$$h_{CT6} - h_{AT8} = -\,2\,\rho\,(v_{CT6} - v_{AT8}) \quad \text{(33b)}$$

Passing through A_{T4}, Equation (33a) locates C_{T6} (for which $h = 1$), and Equation (33b), passing through C_{T6}, gives A_{T8} on the parabola, τ_8, drawn similarly to τ_4. Fig. 9(*a*) has been drawn for $\rho = 0.8$ and for closure in three intervals, conditions at Point A and Point B being shown. The corresponding values of v and h have been plotted on a time base in Fig. 9(*b*) and Fig. 9(*c*). Evidently, in solving such a problem it is necessary to know: (*a*) Complete data on the conditions in the pipe for initial steady flow; (*b*) the law of variation of gate-opening with time; and (*c*) further information about the conditions at some other point such as C. When the pipe is connected to a large open reservoir, or to a large surge tank, the variations of water level in the latter during the water-hammer period are generally negligible, because the water-hammer intervals are very short compared with those required for the mass movement of the water. Practically, however, there is no difficulty in dealing with variations of level at Point C, or with any other condition that may exist there. With water-turbine gates the linear law of movement is used approximately, but for ordinary hand-controlled gates the movement may be quite irregular; once the effective areas are known, however, on a time base, the values of τ for each interval or fraction thereof are known and the problem is easily solved. Of course, the spacing of the τ-lines will not be uniform in the latter case.

GATE-OPENING

For gate-openings, precisely the same set of equations is used, but the value of V to be chosen in computing ρ requires some thought. If the gate is opened from a closed position, the initial pipe velocity is zero and, of course, cannot be used; and if it is moved from one open position to another position with larger area, there are two steady velocities, that at the beginning and that at the end of the movement. An examination of the theory will show that either of these velocities may be used, and v will then be solved for relative to the selected reference velocity. It is always most convenient to use the steady velocity corresponding to the final position of the gate within the limits of the problem, because this gives values of v less than unity on the whole, and such values are easier to work with than when all values of v exceed unity. For that reason this method has been adopted herein, although the same results will be obtained for the other values of V, provided the meanings of ρ and v are kept in mind.

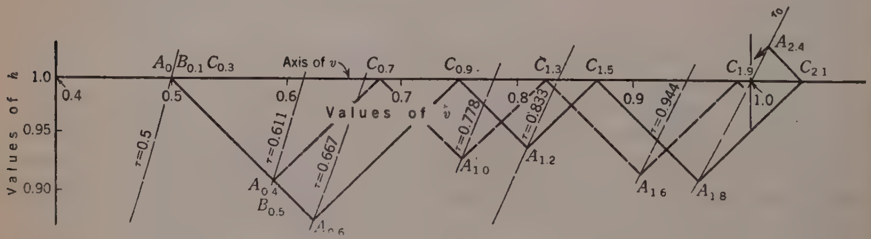

FIG. 10.—OPENING FROM HALF TO FULL GATE IN $T = 3\left(\frac{2L}{V_w}\right) = 18$ SEC

The example shown in Fig. 10 is for a pipe working under a head, $H_0 = 800$ ft, and with a steady velocity, $V_1 = 4$ ft per sec; and the changes of pressure have been determined at Point A and also at Point B ($x_1 = \frac{1}{3}L$). The valve is opened in 1.8 sec to such a position that the final steady velocity will be $V_2 = 8$ ft per sec; and, $V_w = 3\,500$ ft per sec has been assumed. If the pipe is 1 050 ft long, then $\frac{2L}{V_w} = 0.6$ sec; and, hence, $T = 1.8$ sec $= 3\left(\frac{2L}{V_w}\right)$ sec. In other words, opening occupies three intervals. Furthermore, $\rho = \frac{3\,500 \times 8}{2 \times 32.2 \times 800} = 0.543$, so that the lines in Fig. 10 have a slope such that $\tan^{-1} \pm 2 \times 0.543 = \tan^{-1} \pm 1.086$. The equations for the solution of this problem are written from Equations (26). The parabolas, $\tau = 0.5$, $\tau_1 = 0.667$, $\tau_2 = 0.833$, and $\tau_3 = \tau_0 = 1$, are drawn in light lines, but since Point B is at $x_1 = \frac{1}{3} L$, the parabolas one-third the way between the foregoing are also drawn dotted; for example: $\tau = 0.611$, $\tau = 0.778$, and $\tau = 0.944$. The starting point is evidently at Point A_0, Fig. 10, where $v_{A0} = 0.5$; and $h_{A0} = 1$. This point is also $B_{0.1}$ and $C_{0.3}$, and the construction should be followed without difficulty. (In all values such as $B_{0.1}$, etc., the subscript following each letter gives the time, in seconds, at which the values of h and v are represented by the point.) The actual head at the point at the time marked, of course, will be the indicated value of h multiplied by $H_0 = 800$ ft, and the actual velocity is the value of v multiplied by the reference velocity, $V_2 = 8$ ft per sec. All points, $A_{1.8}$ and beyond, are plotted on $\tau_0 = 1$. Since the gate remains in a fixed position from that time on, the pressures along the pipe reach H_0 quickly.

COMPOUND PIPES

The analytical solution for the pressures in compound and branched pipes becomes extremely difficult, if not actually impossible, in many cases; and yet the graphical construction furnishes a very simple method of solving the same problems. The first illustration is the pipe of Fig. 11, consisting of three parts of different lengths and diameters, all necessary data being shown on the inset. In order to clarify the illustration, linear gate closure and convenient ratios between the values of $\frac{L}{V_w}$ have been assumed. Any other data

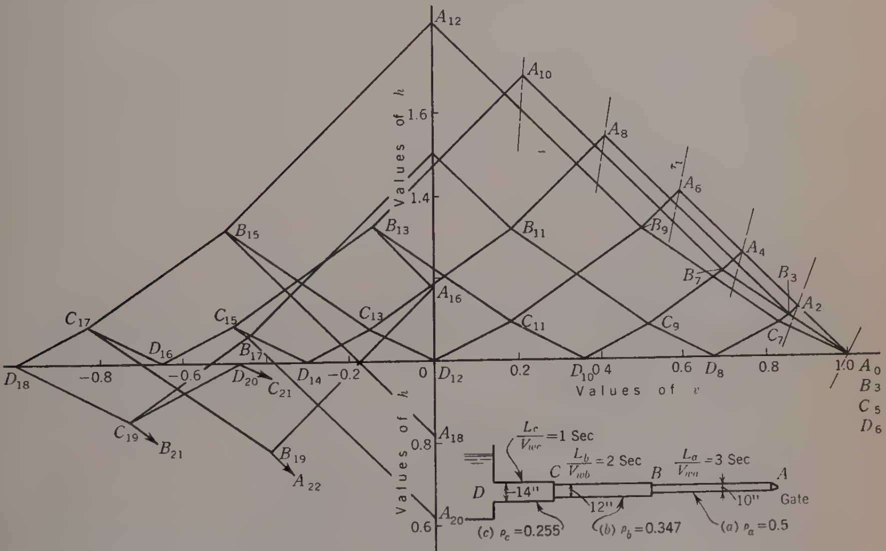

FIG. 11.—LINEAR GATE CLOSURE IN $T = 2\left(\frac{2L}{V_w}\right) = 12$ SEC

may be treated just as easily although the drawing might have more lines on it and, therefore, might become larger. Only the first few equations will be written:

$$h_{A0} - h_{B3} = + 2 \rho_a (v_{A0} - v_{B3}) \qquad (34a)$$

$$h_{B3} - h_{C5} = + 2 \rho_b (v_{B3} - v_{C5}) \qquad (34b)$$

$$h_{C5} - h_{D6} = + 2 \rho_c (v_{C5} - v_{D6}) \qquad (34c)$$

$$h_{B3} - h_{A6} = - 2 \rho_a (v_{B3} - v_{A6}) \qquad (34d)$$

$$h_{A4} - h_{B7} = + 2 \rho_a (v_{A4} - v_{B7}) \qquad (34e)$$

$$h_{C5} - h_{B7} = - 2 \rho_b (v_{C5} - v_{B7}) \qquad (34f)$$

$$h_{A2} - h_{B5} = + 2 \rho_a (v_{A2} - v_{B5}) \qquad (34g)$$

$$h_{B5} - h_{C7} = + 2 \rho_b (v_{B5} - v_{C7}) \qquad (34h)$$

and,

$$h_{D6} - h_{C7} = - 2 \rho_c (v_{D6} - v_{C7}) \qquad (34i)$$

Equations (34*a*), (34*b*), and (34*c*) correspond to the single points, A_0, B_3, C_5, D_6, at $h = 1$, $v = 1$; Equation (34*d*) locates the line, $B_3 A_6$, on $A_0 A_6$, since A_6 is fixed by the value corresponding to the time and consequent gate-opening at the end of one interval for Section $A B$, Fig. 11. Having determined the values of A_0 and A_6, the points, A_2 and A_4, are found by the method described for the simple pipe. Equation (34*e*) with Equation (34*f*) locates Point B_7 and Equation (34*g*) then gives Point B_5; from these, Equations (34*h*) and (34*i*) serve to locate Point C_7. By writing out the successive equations, the points, D_8, C_9, D_{10}, B_9, C_{11}, etc., may be found. All A–B lines have the slope, $\pm 2 \rho_a$, all B–C lines, the slope, $\pm 2 \rho_b$, and all C–D lines, the slope, $\pm 2 \rho_c$. All points, D, are on the line, $h = 1$, since the reservoir level is assumed constant.

Whereas the pressures at Point A rise continually until closure and then fall rapidly, the pressures at Point B rise quickly during the first 9 sec; then they remain nearly constant to the 15th sec, after which they fall. On the other hand, the pressures at Point C remain nearly constant for the first 17 sec, after which they become less than H_0. The velocities may also be read easily, but usually are not desired. A pipe with any number of sections may be investigated just as easily, although those with fractional values of $\frac{L}{V_w}$ may require a lengthy but simple construction, similar to Fig. 11. It is to be noted further that all the values of ρ are connected since they are all referred to the same values of H_0 and, therefore, must be related in the same manner as the pipe areas.

Branched Pipes; The Surge Tank

Pipes with branches form a particularly useful application of the principles under discussion, and these include penstocks with surge tanks, water pipes with a branch parallel to parts of the main line, but connected to the line at both ends of the branch, branches with dead ends, etc. Perhaps the easiest case to begin with is the water-power system with an open-top surge tank (*b*) between the conduit (*c*) and the penstock (*a*), as illustrated in Fig. 12, in which Point A

is the gate (it would be a turbine in this case), Point B, the junction point of the penstock, conduit, and tank, Point D, the water surface in the tank, and Point C is at the conduit entrance. The case of load rejection with non-linear gate closure will be considered.

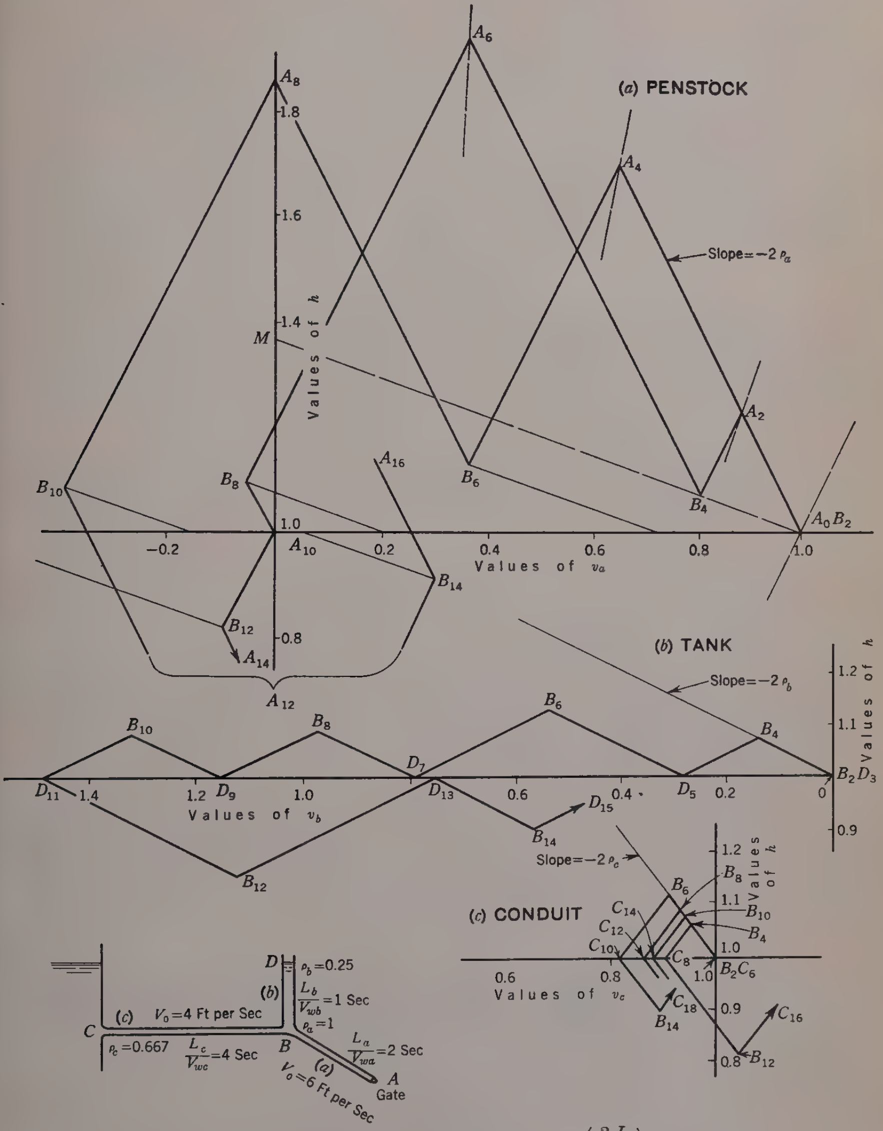

Fig. 12.—Non-Linear Gate Closure in $T = 2\left(\frac{2\,L}{V_w}\right) = 8$ Sec

When the load is rejected, a mass movement of the water takes place up into the tank; but compared to the interval, $2\,\frac{L}{V_w}$, for the penstock or tank, this movement is quite slow, and with tanks of usual proportions the change of level of the tank, say, for five intervals of $\frac{2L}{V_w}$ sec of either the tank or the penstock would be negligible as far as it affected the pressure at Point *B*. On the other hand, the gate movement produces pressure or water-hammer waves, which are very rapid and which, at times, do produce serious pressure changes at Point *B*; but in the first instance, at least, changes of level at Point *C* and Point *D* are not considered. It then follows that in the graphical representation all Points *C* and *D* lie on the line, $h = 1$.

The method of dealing with this case is the same as the others. Friction and velocity head being neglected, the pressure at a given instant at Point *B* is the same whether it is considered as a point in (*a*), (*b*), or in (*c*). Hence,

$$h_{aB} = h_{bB} = h_{cB} = h_B \qquad (35)$$

and during steady flow, $h_{B0} = 1$. In addition, if the positive direction of V_c is toward Point *B*, while those of V_a and V_b are away from Point *B*, the equation of continuity for that point will be written, $Q_c = Q_a + Q_b$; or, $Q_a = Q_c - Q_b$; that is,

$$A_a V_a = A_c V_c - A_b V_b \qquad (36a)$$

For steady flow Equation (36*a*) becomes,

$$A_a V_{a0} = A_c V_{c0} - A_b V_{b0} \qquad (36b)$$

Dividing Equation (36*a*) by $A_a V_{a0}$ gives $\frac{A_a V_a}{A_a V_{a0}} = \frac{A_c V_c}{A_a V_{a0}} - \frac{A_b V_b}{A_a V_{a0}}$; or,

$$\frac{A_a V_a}{A_a V_{a0}} = \frac{A_c V_{c0}}{A_a V_{a0}} \frac{V_c}{V_{c0}} - \frac{A_b}{A_a} \frac{V_{b0}}{V_{a0}} \frac{V_b}{V_{b0}} \qquad (37)$$

In the notation already adopted the left-hand term is written v_a and each of the two right-hand terms has the same form as the left-hand one, thus, the term, $\frac{A_c V_{c0} V_c}{A_a V_{a0} V_{c0}}$, contains a fixed constant multiplied by the velocity ratio, $\frac{V_c}{V_{c0}}$, and the entire term may be written, v_c, by arbitrarily defining V_{c0} so that $A_c V_{c0} = A_a V_{a0}$. A similar statement applies to the second right-hand term, and, therefore, Equation (37) may be written,

$$v_a = v_c - v_b \qquad (38)$$

This method of defining V_{c0} and V_{b0} at once gives a value to the velocities used in finding ρ for the pipes, (*a*), (*b*), and (*c*), and, therefore, these values of ρ will be written:

$$\rho_a = \frac{V_{wa} V_{a0}}{2 g H_0} \qquad (39a)$$

$$\rho_b = \frac{V_{wb} V_{b0}}{2 g H_0} = \frac{A_a}{A_b} \rho_a \qquad (39b)$$

and,

$$\rho_c = \frac{V_{wc} V_{c0}}{2 g H_0} = \frac{A_a}{A_c} \rho_a \qquad (39c)$$

Using Equations (38) and (39) this problem presents no more difficulty than the simple pipe. It is possible to follow the methods described at the beginning of this paper and represent the conditions in the three pipes, (*a*), (*b*), and (*c*), Fig. 12, on a single diagram, but the latter then becomes very complicated and it will be better to draw them separately. The data for the problem are given in Fig. 12, the tank area being assumed uniform and equal to four times the penstock area and the head, $H_0 = 300$ ft. The data assumed give $\rho_a = 1.00$, $\rho_b = 0.25$, and $\rho_c = 0.667$, and closure is assumed in two intervals of the conduit; that is, 8 sec. The lengths of the pipe and tank have been distorted deliberately so that the problem may be solved on a small sheet and still show the principles. In attacking these problems it is best and quickest to have the three diagrams side by side so that all axes of v are in the same straight line; to save space, the figures have been moved from their original positions used when solving the problem.

The diagram for A_2 and A_4 for the penstock, (*a*), should require no explanation, the points, $A_0\,B_2$, $B_2\,C_6$, $B_2\,D_3$, evidently are placed correctly at each starting point and, although v_a and v_c are measured to the right, it is necessary to measure v_b to the left, as it has a negative sign. Having found A_2 and A_4, the formula, $h_{A2} - h_{B4} = +\,2\,\rho_a\,(v_{A2} - v_{B4})$ (from Equations (26)), gives one line on which B_4 lies. The point, B_4, however, must be at the same height in the diagrams for Pipes (*a*), (*b*), and (*c*) from Equation (35) and also, from Equation (38),

$$v_{aB4} = v_{cB4} - v_{bB4} \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

The notation in Equation (40) is complicated but v_{aB4} is written for the velocity in Pipe (*a*) at the point, *B*, and 4 sec after gate motion begins. Equation (40) shows that Point B_4 lies on a line, $A_0\,M$, located as follows: $v_a = 0$ in Equation (40) if $v_b = v_c$, all being taken at the same pressure. This is easily seen to be the pressure at which a line (not shown for lack of space) through the point, $v_c = 0$, $h = 1$ at slope $+\,2\,\rho_b$ on the (*c*)-diagram, intersects a line through the point, $v_c = 1$, $h = 1$, and at the slope, $-\,2\,\rho_c$ on the same diagram. This point, so found on Conduit (*c*), gives the height of the Point *M* so that *M* represents the same pressure; B_4 then lies on $A_0\,M$. This also locates B_4 on the (*b*) and (*c*)-diagrams since,

$$h_{D3} - h_{B4} = -\,2\,\rho_b\,(v_{D3} - v_{B4}) \ldots\ldots\ldots\ldots (41a)$$

and,

$$h_{C6} - h_{B10} = -\,2\,\rho_c\,(v_{C6} - v_{B10}) \ldots\ldots\ldots\ldots (41b)$$

and B_4 clearly lies on $B_2 - B_{10}$, or this line produced.

Next, A_6 is found since it lies on $B_4\,A_6$ from the equation,

$$h_{B4} - h_{A6} = -\,2\,\rho_a\,(v_{B4} - v_{A6}) \ldots\ldots\ldots\ldots (42)$$

It also lies on the parabola corresponding to 6 sec after closure begins. The next point is D_5 which lies on Line $B_4\,D_5$ at Slope $+\,2\,\rho_b$ and at $h = 1$. Next, B_6 lies on a line through D_5 since,

$$h_{D5} - h_{B6} = -\,2\,\rho_c\,(v_{D5} - v_{B6}) \ldots\ldots\ldots\ldots (43)$$

and it also lies on $A_4 B_6$ of the slope, $+ 2 \rho_a$ and, furthermore, it must be located so that $v_{aB6} = v_{cB6} - v_{bB6}$. A little consideration will show that it must then lie on a line parallel to $A_0 M$, but horizontally displaced to the left by a distance, $D_5 D_3$.

The remainder of the construction is similar. To locate such a point as B_{14}, the line, $A_0 M$, is displaced horizontally by the distance, $(D_3 - D_{13}) + (C_6 - C_{10})$, and, similarly, for other points.

It is well to check the method frequently by scaling the drawings and remembering that the continuity equation holds at Point B at any instant and pressure. Thus, the point, B_8, represents the conditions at Point B at 8 sec after closure begins, and scaling from the drawing at this point for each section gives $v_a = -0.05$; $v_b = 0.97$; and $v_c = 0.92$, so that $v_a = v_c - v_b = 0.92 - 0.97 = -0.05$, as it should. At this instant, all the water flowing through the conduit, (*c*) Fig. 12, together with some water from the penstock, (*a*), is flowing into the tank. However, at 10 sec after closure begins the water is coming from the penstock at a high rate, as shown at B_{10} on the diagram for Penstock (*a*). It must not be thought, however, that water is actually flowing into the nozzle from without, for no Point A ever gets to the left of the axis of h, although all points, from A_8 on, lie on this line because the gate is closed.

A little care is needed in the interpretation of the results; for example, at 6 sec, the point, B_6, corresponds to $h = 1.13$ and, therefore, to a rise in pressure at Point B of $0.13 \times H_0 = 39$ ft, and, similarly, for other points.

Although the left-hand scale for Tank (*b*), Fig. 12, is confusing at first, it soon ceases giving any trouble. In dealing with all such problems the points, A, must be chosen close enough together to enable the solution to be made. In most cases the ratio of $\frac{2L}{V_w}$ has the smallest value for the tank, but whether it has or has not, the time unit is selected at least as small as the lowest value of $\frac{2L}{V_w}$ in the system and parabolas are drawn with this time spacing. In this case, the smallest value of this term is 2 sec in the tank and on Fig. 12, therefore, values of A are found for each 2 sec. When the periods of the pipes are not multiples of one another, a sufficient number of values of A must be found to complete the problem, and sometimes the work is prolonged, but gives no trouble of any kind.

The volume of water flowing into the tank due to water hammer is easily computed. At 4 sec, $v_{bB} = 0.14$, and the volume passing into the tank in the first 4 sec, therefore, is:

$$A_a V_{bB} \times \frac{2 L_b}{V_{wb}} = v_{bB} v_{a0} A_a \frac{2 L_b}{V_{wb}} = 0.14 Q_0 \frac{2 L_b}{V_{wb}} \quad \ldots\ldots\ldots (44)$$

in which by definition of V_{b0}, $Q_0 = A_a V_{a0} = A_b B_{b0}$ is the initial steady flow. At the end of 6 sec, the total volume that has passed in is $(v_{bB4} + v_{bB6}) Q_0 \times \frac{2 L_b}{V_{wb}}$ cu ft. The velocities, of course, are all measured on the diagram for Tank (*b*) and, for accuracy, D_5 and D_7 should be raised above $h = 1$ to suit this increased volume; but usually the change of level is so small as to have no practical effect.

WATER MAIN WITH BRANCH; EFFECT OF DEAD END

The next example is the case of the water pipe illustrated in Fig. 13 and Fig. 14. It represents a 15-in. pipe supplied by a reservoir with constant level and discharging through a nozzle or gate, for which it is assumed the law of closure is known. This pipe has an 18-in. branch connected to the main at Points B and C in Fig. 13, but only at Point B in Fig. 14, the free end of the 18-in. pipe being plugged in this latter case. The pressures in the pipe are sought in the two cases, and, in that way, a comparison is possible between the closed circuit and the one with "dead end" under exactly similar conditions.

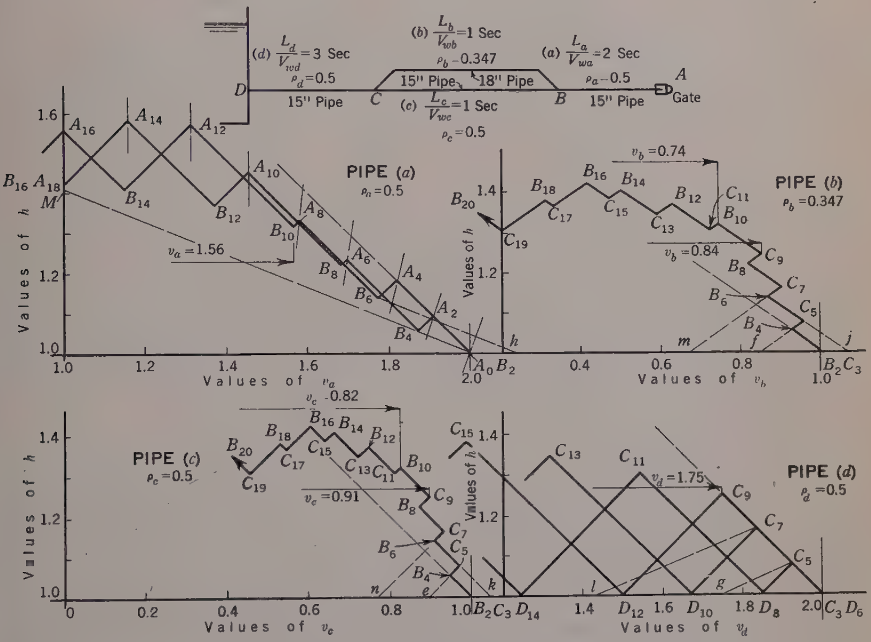

FIG. 13.—LINEAR GATE CLOSURE IN 4 $\left(\frac{L_a}{V_{wa}}\right)$ = 16 SEC; 18-INCH BRANCH CONNECTED TO MAIN AT TWO POINTS

Starting with the first case (Fig. 13), separate diagrams will be made for the four parts, (a), (b), (c), and (d), designated in the drawing. The data are selected to give values of $\frac{L}{V_w}$ in the four parts as 2 sec, 1 sec, 1 sec, and 3 sec, respectively, and, also, $\rho_a = 0.5$, from which the previous discussion shows that this is also the value of ρ_c and ρ_d, and that $\rho_b = \frac{A_c}{A_b}\rho_a = 0.347$.

Furthermore, assuming that all velocities have their positive directions toward Point A, the continuity equation at Point B is $v_a = v_b + v_c$, and at Point C, it is $v_d = v_b + v_c$. Therefore, the point, A_0, in Fig. 13(a), corresponds to $v_{a0} = 2$; in Fig. 13(b) and Fig. 13(c) both v_{b0} and v_{c0} are unity, whereas, in Fig. 13(d), the value is $v_{d0} = 2$. The initial starting points on the four diagrams are designated $A_0 B_2$, $B_2 C_3$, $B_2 C_3$, and $C_3 D_6$, in accordance with the notation already adopted, and the slopes of the principal lines on these diagrams are $\pm 2\rho_a$ etc.; furthermore, in Fig. 13(*a*), parabolas for 2 sec, 4 sec, 6 sec, etc., after closure begins, are drawn and the points, A_2 and A_4, are found in the usual manner. Point B_4 is located at the same elevation in Fig. 13(*a*), Fig. 13(*b*), and Fig. 13(*c*), and also, for this point, $v_a = v_b + v_c$; and in order to carry out this condition, an auxiliary line, $A_0 M$, is drawn, exactly as in Fig. 12, such that at any pressure the velocity shown by $A_0 M$ is the sum of the velocities in Fig. 13(*b*) and Fig. 13(*c*) at the same pressure. Then,

$$h_{A2} - h_{B4} = +2\,\rho_a\,(v_{A2} - v_{B4}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

gives the line, A_2-B_4, and, consequently, the point, B_4, and this point is then projected over to Fig. 13(*b*) and Fig. 13(*c*).

Next Point C_5 is to be found and the equations by which it is located are, for Fig. 13(*b*):

$$h_{B4} - h_{C5} = +2\,\rho_b\,(v_{B4} - v_{C5}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (46a)$$

and for Fig. 13(*c*):

$$h_{B4} - h_{C5} = +2\,\rho_c\,(v_{B4} - c_{C5}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (46b)$$

so that lines through Point B_4 representing these equations are drawn on the two diagrams. The continuity equation also gives $v_b + v_c = v_d$ so that Point C_5 will also lie on a line, $g\,C_5$ (Pipe *d*), having the same inclination as $A_0 M$, but with its tangent of opposite sign, the point, *g*, being located so that $B_2 e + B_2 f = D_6 g$. (This is quite easy to demonstrate by geometry or by actual measurement, as the reader desires.) Again, the equation,

$$h_{D6} - h_{C9} = -2\,\rho_d\,(v_{D6} - v_{C9}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

shows that Point C_5 is located at a point on the line through Point D_6 and at a slope, $-2\,\rho_d$, so that the location of Point C_5 is determined in Fig. 13(*d*), and then projected horizontally to Fig. 13(*b*) and Fig. 13(*c*) as indicated. The next point, B_6, is similarly found and, in this case, $A_0 h = B_2 j + B_2 k$; and the line, $h\,B_6$, is parallel to $A_0 M$. Point B_6 is also located on $\overline{A_4 B_6}$ at the slope, $+2\,\rho_a$. Another point, C_7, is obtained by making Line $D_6 l = \overline{B_2 n} + \overline{B_2 m}$, and drawing Line $l\,C_7$ parallel to Line $g\,C_5$.

The process is then carried on to the final solution, it being remembered that all *A–B* lines have a slope, $\pm 2\,\rho_a$, all *C–D* lines, a slope, $\pm 2\,\rho_d$, and all *B–C* lines, a slope, $\pm 2\,\rho_b$, or $\pm 2\,\rho_c$, depending on whether they belong to Pipe (*b*) or to Pipe (*c*). The dimensions of one point, *B*, and one point, *C*, are shown, and the drawing made by the method described gives, for B_{10}: $v_{bB10} = 0.74$, $v_{cB10} = 0.82$, and $v_{aB10} = 0.74 + 0.82 = 1.56$; and, for C_9: $v_{bC9} = 0.84$, $v_{cC9} = 0.91$, and $v_{dC9} = 0.84 + 0.91 = 1.75$. In constructing the diagram it is well to check the results frequently by actually measuring the values of *v* and adding, as in Fig. 13.

Although the pressures at Points *A*, *B*, and *C*, have been found (Points *D*, of course, lie on the line, $h = 1$), for the first 20 sec, only curves of pressures at Points *A* and *C* have been plotted on the separate pressure diagram, Fig. 14, and it shows that Point *A* reaches the highest pressure at about 12 sec, and the pressure at Point *C* its maximum value at about 15 sec.

Fig. 14 represents precisely the same piping system as Fig. 13, with the valve at Point A closed in the same manner and time as the former case, but the connection at Point C has been cut off and Pipe (b) plugged. As this case is quite similar to the surge-tank problem, very little explanation will be necessary. In the diagram for Pipe (b) all points, C, must lie on the vertical line, $v_b = 0$, and the pressures at Point C, therefore, rise rapidly; furthermore, the velocities at Point B in Fig. 14(b) change only as much as the elasticity of the water and pipe walls permit, which naturally is not very much. It is like a very small surge tank. In this case, as with the surge tank, $v_{a0} = 1$, $v_{b0} = 0$, and $v_{c0} = 1$. A comparison with the previous case shows that the pressure rise at Points A and C, plotted on Fig. 14, is considerably more in the latter case than in the former, and this agrees with Joukowsky's statement that dead ends aggravate the surge pressures and show the exact amount of such pressures. Fig. 14 is easy to allow for pressure variations at Point D, provided the law of such variation is known, but in that case, of course, the several points, D, would not lie on the one horizontal line. Pressures at any intermediate point in the system are found as with the uniform pipe.

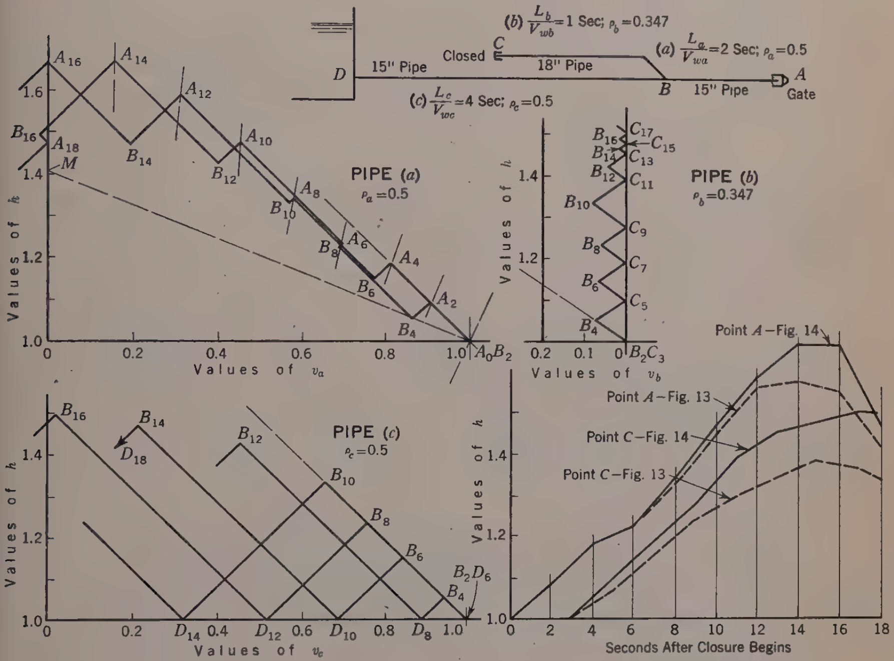

Fig. 14.—Linear Gate Closure in 4 $\left(\frac{2L_a}{V_{wa}}\right)$ = 16 Sec; 18-Inch Branch Connected to Main at Only One Point

Pipe with Two Branches Each Discharging Water

Another interesting case is shown in Fig. 15 in which a pipe, $C\,B$, designated by (c) issuing from a fixed level reservoir, has two branches: $B\,A$, marked (*a*); and, *B D*, denoted by (*b*), and discharge is occurring at both Points *A* and *D*. Without going into the details of the pipe dimensions, values have been selected to give a clear drawing, and $\frac{L}{V_w}$ is taken at 1.5 sec for Branch (*a*), 1 sec for Branch (*b*), and 0.5 sec for Line (*c*), whereas the selected values of ρ are 0.5 for Branch (*a*), 0.4 for Branch (*b*), and 0.6 for Line (*c*). It is assumed that the gate at Point *A* is closed by a known law in 6 sec, or two intervals for Pipe (*a*); also that the law of variation between pressure and discharge at Point *D* is known. For example, a turbine may be connected to Point *D* in which the power produced is constant; or some other specified condition may be imposed. In the problem, the law of variation has been assumed as represented by the dotted line on the drawing for Branch (*b*).

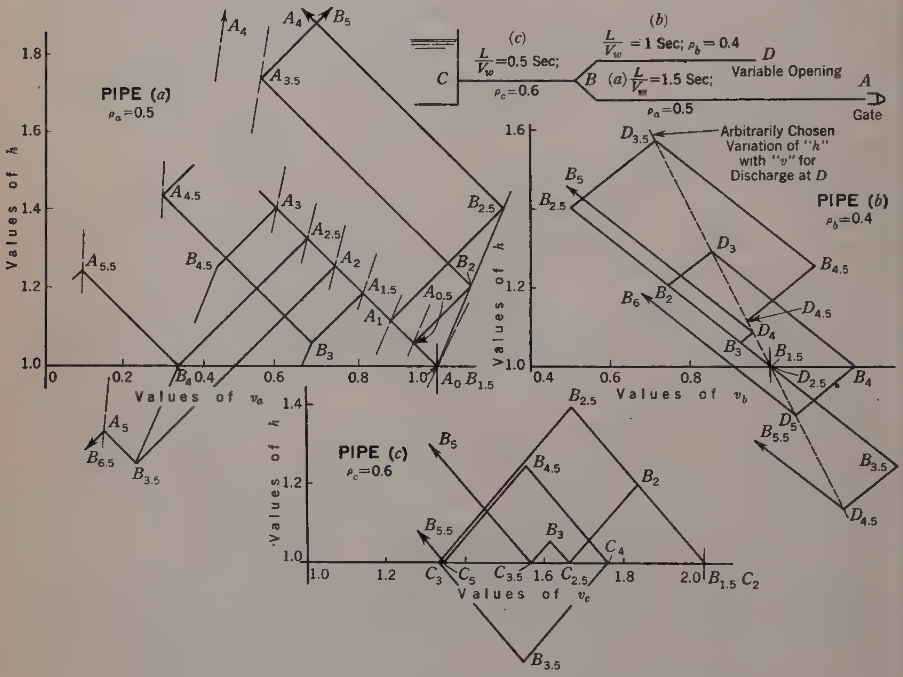

FIG. 15.—GATE *A* HAS LINEAR CLOSURE IN 2 $\left(\frac{2\,L_a}{V_{wa}}\right)$ = 6 SEC

This case is dealt with in a manner similar to the others discussed, and, here, $v_{a0} = 1$, $v_{b0} = 1$, and $v_{c0} = 2$. All points, *D*, fall on the dotted line, and the construction need not be explained. The line through A_0 locating Point B_2 has a positive tangent with the data used. The pressure variations at Points *A*, *B*, and *D* are so large and so erratic that it is evidently impossible to use the system in practice. A second case has been developed and illustrated in Fig. 16 for the same system and rate and time of closure of *A*, and the same control at Point *D*, but with the addition of a surge tank at Junction *B*.

This, of course, introduces an extra branch, making four pipes in all, and if the surge tank is denoted by (*d*), then,

$$v_c = v_a + v_b + v_d \ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

and $v_{c0} = 2$; $v_{a0} = 1$; $v_{b0} = 1$; and, $v_{d0} = 0$. Using Equation (48) in a method exactly similar to the foregoing, the line, through A_0, is found on the penstock drawing in such a way as to locate Point B_2 from Point $A_{0.5}$, and all points, *B*, on this penstock drawing will lie on lines parallel to Line $A_0 B_2$. Point B_2 is then transferred to the drawings for Pipes (*b*), (*c*), and (*d*) by making them all represent the same pressure and, therefore, Point B_2 is given on each diagram. From Point B_2, Point D_3 is located on Fig. 16(*b*), Point $C_{2.5}$ on Fig. 16(*c*), and Point $E_{2.5}$ on Fig. 16(*d*). It is also evident that Point $B_{2.5}$ lies in each diagram on the same line as Point B_2. Furthermore, at any time (such as 4.5 sec after closure begins) the value of v_c indicated by Point $B_{4.5}$ on Fig. 16(*c*) is the sum of v_a indicated by Point $B_{4.5}$ on Fig. 16(*a*) plus v_b, as shown at Point $B_{4.5}$ on Fig. 16(*b*) plus v_d as given by Point $B_{4.5}$ on Fig. 16(*d*); this is easily checked by measurement.

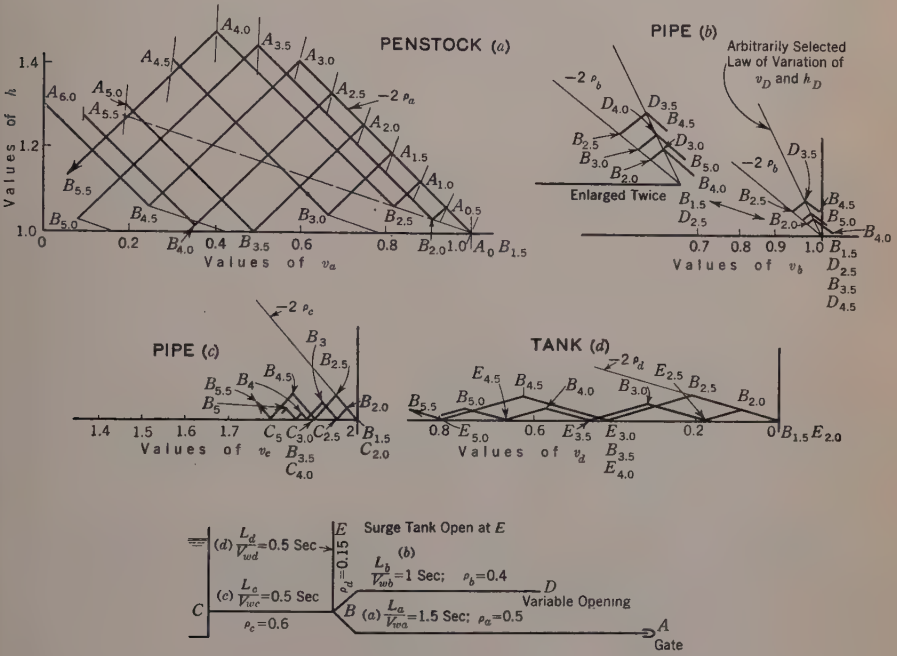

FIG. 16.—GATE *A* HAS LINEAR CLOSURE IN $2\left(\frac{2L_a}{V_{wa}}\right) = 6$ SEC WITH SURGE TANK

Fig. 16 affords an interesting illustration of the effect of the surge tank. In this case the tank has a very small area, being only $\frac{\rho_c}{\rho_a}$, or four times that of Pipe (*c*), and yet it renders the system of Fig. 15 quite practicable as far as the investigation has gone. With no tank, the pressure at Point B after 5 sec is 2.2 times the original pressure, whereas with the tank it is only about 3% above the original level. With no surge tank the fluctuations at Point D up to 5 sec vary from 64% to 158% of normal, a condition that would render any governing mechanism at Point D useless; on the other hand, the surge tank keeps the pressure at Point D within 9% of normal during the first 5 sec. At Point A, the variation in the same time with no surge tank is from 83% to 210% of H_0, whereas the surge tank reduces this variation to 30% of H_0, although at 4 sec it was 48%; and, of course, a larger tank will reduce the variations still further.

This is only a further example of how these difficult problems may be solved simply and accurately, on the drawing board. Although the proportions chosen in the example are most unusual, they serve to make the reduced scale drawings clear, and simplify the method of attack.

PENSTOCK, TURBINE, AND DRAFT-TUBE

A reaction turbine installation, for any but very low heads, consists of a penstock, a turbine with a distributor, and a draft-tube. Water hammer will be produced for every load change, because each change is accompanied by a movement of the distributor gates causing, in turn, an increase or decrease in the volume of water per second being delivered to the turbine; so that the velocities in the penstock and draft-tube change very frequently. It is quite usual to have the gates complete the full opening or closing movement in 2 sec, which corresponds roughly to two intervals in which the penstock is about 1 600 ft long. As a matter of fact, it would correspond to one interval if the turbine happened to be operating at half load, which it suddenly rejected. Governing of such turbines, therefore, may cause high pressures, and the study of this problem will be solved by the graphical method.

Usually, both penstock and draft-tube have a taper, and the method of dealing with compound pipes has already been discussed; but in order to make the diagram simpler, both the penstock and the tube will be assumed to be uniform in size, the draft-tube being the larger and also the shorter. (A method used by Allievi to avoid working with tapering pipes is to assume an equivalent uniform value of V_w from $L = V_w \sum \frac{l_x}{V_{wx}}$. Dr. O. Schnyder has also proposed that an equivalent area may be determined from $A\,L = \sum A_x\,l_x$. In both formulas, l_x, A_x, and V_{wx} refer to the data for the uniform pieces of pipe composing the entire line.[11, 4] Friction and velocity head are not taken into account. The turbine distributor gates serve to cause the varying velocities according to the load demand. In the problem assumed herewith, the dimensions have had to be distorted for clarity, but the diagram to suit any case is easily constructed.

The system is shown on Fig. 17 in which the head, $H_0 = 450$ ft; Penstock (b) is joined to the forebay at Point C and to the turbine at Point B''; and Draft-Tube (a) carries the water from Turbine B' to the tail-water at Point A; the two points, B' and B'', are assumed close enough together so that the volume of water between them has no effect on the water-hammer pressures. Without more definite data the value of V_w is taken as 3 220 ft per sec in both Draft-Tube (*a*) and Penstock (*b*); the steady velocity, V_0, in the penstock is 4.5 ft per sec; and that in the draft-tube is 3.4 ft per sec. These data give $\rho_a = 0.375$ and $\rho_b = 0.5$, and if the penstock length is 1 290 ft the value of $\frac{L_b}{V_{wb}} = 0.4$ sec. Furthermore, for the purpose of clarity $\frac{L_a}{V_{wa}} = 0.2$ sec (a length of draft-tube which could not be tolerated in practice).

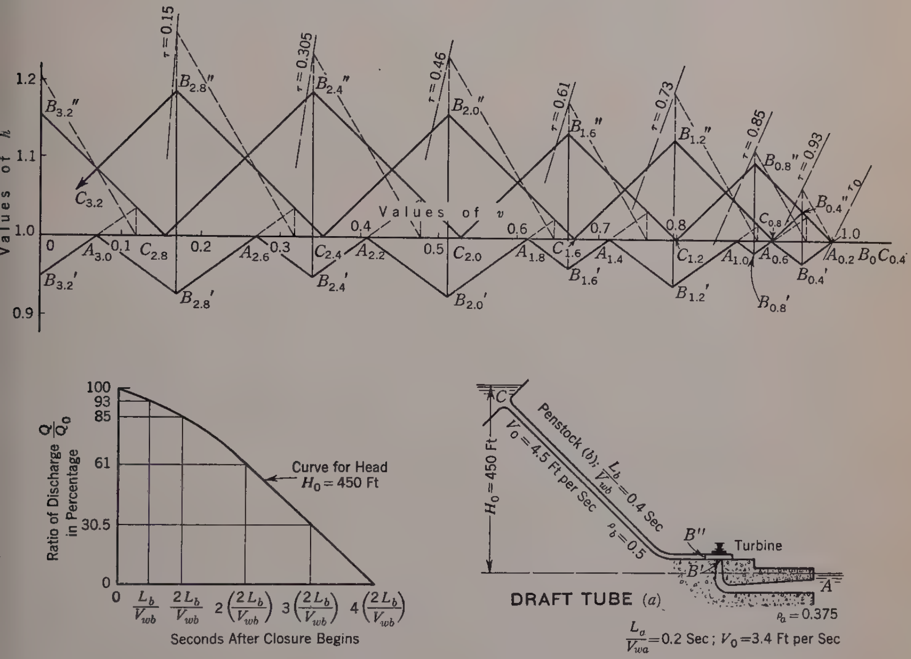

FIG. 17.—GATE CLOSURE IN $4\left(\frac{2\,L_b}{V_{wb}}\right) = 3.2$ SEC

From tests on a large turbine, the curve plotted in Fig. 17, showing the relation between gate position and discharge at normal constant head, has been obtained and there can be little error in assuming that the gate position is a linear function of the time, which has been done in this case; the curve of discharge on a gate-position base, therefore, is also taken to be a curve of discharge on a time base. It is further assumed that closure is effected in 3.2 sec, or in $4\left(\frac{2\,L_b}{V_{wb}}\right)$ sec, which is certainly not unduly short. The time interval in this case must be 0.4 sec, corresponding to the draft-tube value, $\frac{2\,L_a}{V_{wa}} = 0.4$ sec.

Fig. 17 shows the axes of h and v with the point, $h = 1$, $v = 1$, marked $B_0\ A_{0.2}\ C_{0.4}$, since in this case the controlling mechanism is at Point B. Only a few of the equations to be applied will be written (see Equations (26)):

$$h_{C0.4} - h_{B''0.8} = -2\,\rho_b\,(v_{C0.4} - v_{B''0.8}) \ldots\ldots\ldots\ldots (49a)$$

$$h_{B''0.8} - h_{C1.2} = +2\,\rho_b\,(v_{B''0.8} - v_{C1.2}) \ldots\ldots\ldots\ldots (49b)$$

and,

$$h_{B''0.4} - h_{C0.8} = +2\,\rho_b\,(v_{B''0.4} - v_{C0.8}) \ldots\ldots\ldots\ldots (49c)$$

Furthermore, from the corresponding formulas for the pump series:

$$h_{A0.20} - h_{B'0.4} = +2\,\rho_a\,(v_{A0.20} - v_{B'0.4}) \ldots\ldots\ldots\ldots (50a)$$

and,

$$h_{B'0.4} - h_{A0.6} = -2\,\rho_a\,(v_{B'0.4} - v_{A0.6}) \ldots\ldots\ldots\ldots (50b)$$

etc., so that the lines on which Points $B'_{0.4}$ and $B''_{0.4}$ lie are readily found, but their position is not determined. The discharge curve for the turbine, however, shows that at 0.4 sec $= \dfrac{L_b}{V_{wb}}$, the proportional discharge at the head, H_0, is 0.93, and experience with the action of turbines proves that the flow at a given gate-setting varies with $\sqrt{H_{B''} - H_{B'}}$; that is, with $\sqrt{h_{B''} - h_{B'}}$. A parabola, therefore, is drawn in Fig. 17 with the value, $\tau = 0.93$, and the problem then is to locate $B'_{0.4}$ and $B''_{0.4}$ on the same vertical line (since the value of v is the same at both points at this instant), and having a vertical spacing equal to the height of the parabola, above the axis of v at this same velocity. The reason for this is that $h_{B''0.4} - h_{B'0.4}$ will be the same as the pressure rise to which this velocity corresponds. Schnyder[11] has devised a simple graphical solution to a similar problem by drawing through Point B_0 the dotted line such that its tangent is $2\,(\rho_a + \rho_b) = 1.75$, and where it intersects the parabola, $\tau = 0.93$ is directly above Point $B'_{0.4}$ and Point $B''_{0.4}$. In this manner, these two points are determined and Point $A_{0.6}$ and Point $C_{0.8}$ are found without difficulty (accidentally, they nearly coincide in Fig. 17) since all points, A and C, lie on the line, $h = 1$, if the forebay and tail-water levels do not vary.

The construction for the points, $B'_{0.8}$ and $B''_{0.8}$, is arrived at precisely as before by the aid of the parabola, $\tau = 0.85$, corresponding to the time, $\dfrac{2\,L_b}{V_{wb}}$, the Schnyder construction being shown by short dots.

Although no definite conclusion can be drawn from the numerical results obtained in this problem (because of the distortion of the dimensions), some comment on the results will be of value. Taking as an example the condition at the turbine 2 sec after closure begins, the values are $h_{B''2} = 1.165$ and $h_{B'2} = 0.93$, which mean that the pressure at the lower end of the penstock has risen $0.165\,H_0$, or 74 ft above normal, whereas at the top of the draft-tube it has fallen $(1 - 0.93) = 0.07 \times H_0$, or 31.5 ft below normal. Of course, with ordinary plants, the column in the draft-tube would separate with a slightly greater pressure drop than this shows, and the condition in the tube could then be investigated as in one of the problems solved in this paper.

BREACH OF WATER COLUMN DUE TO LOW PRESSURE

A case that has recently come to the writer's attention is illustrated on Figs. 18 and 19, in which a pump discharges through a long pipe line laid with the profile shown. The first part of the pipe slopes somewhat steeply from Valve A at the pump to a point, B, whereas the latter part has a more gentle slope up to the reservoir. The characteristic curve for the pump at its normal speed of $n = 750$ rpm is shown, and the efficiency curve for the machine is also known, as well as the $W\,R^2$-value of the rotating parts (W = the equivalent weight of the rotating parts and R = its radius). The pump is assumed to be delivering 30 000 000 U. S. gal per day in a pipe in which it produces a velocity of 4 ft per sec. Friction is not considered.

Suppose, now, that the power is suddenly cut off; then the pump begins to slow down, its deceleration depending on the value of $W\,R^2$ for the rotating parts and on its output; and δn, the decrease in speed, in revolutions per minute in the time, δT, is given by the easily derived formula,[13]

$$\delta n = 183\,540 \frac{Q\,H\,\delta T}{W\,R^2\,e\,n} \qquad \text{(51)}$$

in which feet and pounds are the units, and e is the efficiency of the pump.

The method of plotting the characteristic of a pump at any speed, from the curve obtained at some other speed, is well known and need not be explained, and, therefore, the speed corresponding to the characteristic curve passing through any point is readily found. The speed of the pump at any time after disconnecting the power supply can be computed by Equation (51).

The slowing down of the pump causes the pressure to decrease all along the line, and at Point A its value is fixed by the characteristic curve at the reduced pump speed. At very slow speeds, the pressure at Point A is nearly atmospheric unless the pump reverses; the valve at A is assumed to close at the instant the flow reverses. At Point B the low pressure at Point A may produce a pressure much less than the atmosphere, but in this problem a vacuum valve is installed at Point B in such a manner as to prevent the pressure from falling to that extent; that is, the hydraulic gradient at Point B cannot fall below 160 ft above the pump.

Referring to Fig. 18, which has axes of H and V, the starting point is $A_0\,B_1\,C_6$, as marked, and according to Equations (24) the water-hammer line will be drawn through this point, and with a slope, $+\frac{V_w}{g}$. The pump characteristics passing through the point where the water-hammer line intersects the pressure, H = 160 ft, corresponds to a speed of 590 rpm and Equation (51) shows that if average values of Q, H, and e, for the period are used, this speed will be reached 0.60 sec after the power is cut off; therefore, Point $A_{0.60}$, which is also Point $B_{1.60}$, is located. In 1.60 sec air begins to enter at Point B and the column parts at that point.

The admission of air and its subsequent release at Point B maintains atmospheric pressure there until the columns re-unite; the original column separates into the two parts, Line $A\ B'$ and Line $B''\ C$, the values of $\frac{L}{V_w}$ for which are, respectively, 1 sec and 5 sec, and it follows, therefore, that Point $B_{1.60}$ also represents the conditions of the upper column for 5 sec after the point, C_6; that is, the velocity at B'' of the upper column remains constant from 1.6 sec to 11 sec at $V_{B1.6}$. This is readily seen by applying Equations (24) to this case, which give $H_{C6} - H_{B''11} = V_g\,(V_{C6} - V_{B''11})$, so that Point $B''11$ lies on Line $C_6\ A_{0.6}$, and as its pressure is atmospheric, it is fixed at $B'_{1.6} = B''_{1.6}$. The construction already adopted enables one to find Points $C_{6.6-16}$, $B''_{11.6-21}$, etc., and these points are marked on Fig. 18; the location of Points B_2', B_3', etc., give no trouble.

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[13] "Water Hammer in Pipes, Including Those Supplied by Centrifugal Pumps," by R. W. Angus, *Proceedings*, Inst. Mech. Engrs., 1937.

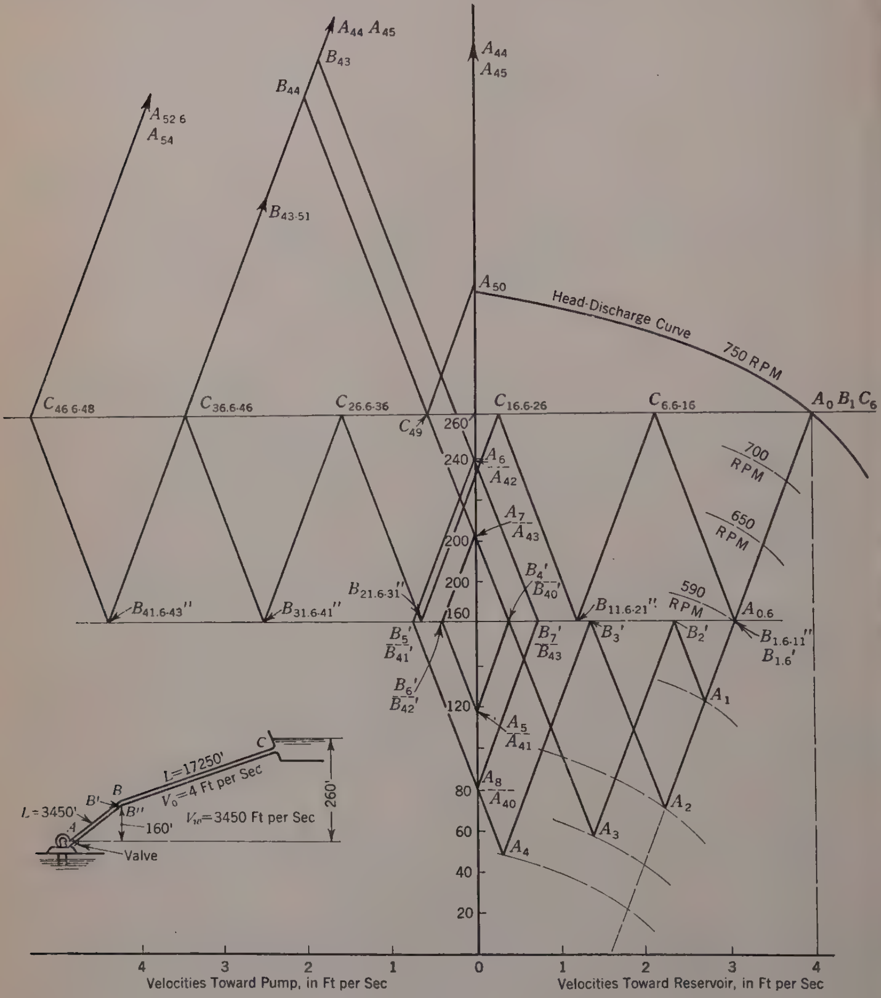

FIG. 18

If the valve at Point A is closed in slightly less than 5 sec, the flow through the pump will not reverse and all subsequent values of Point A will lie on the axis of H. On Fig. 19, the velocities at Point B (that is, for Point B' and Point B'') are plotted against time, and it is evident that the hatched area, M, representing an integral of a velocity time diagram gives the linear distance between Point B' and Point B'' at any instant; the columns reach a maximum separation at the time, 21.6 sec, where the two curves cross. After that they approach one another and finally come together again at the time where Area N = Area M, which in this case is at 43 sec. At the time of re-uniting, the velocity of the upper column is 4.4 ft per sec toward the pump, and that of the lower column is about 0.9 ft per sec in the opposite direction, so that the pressure rise will be caused by the sum of these two velocities, and may be determined as shown. It is assumed that the air admitted to the pipe is released before the columns meet again.

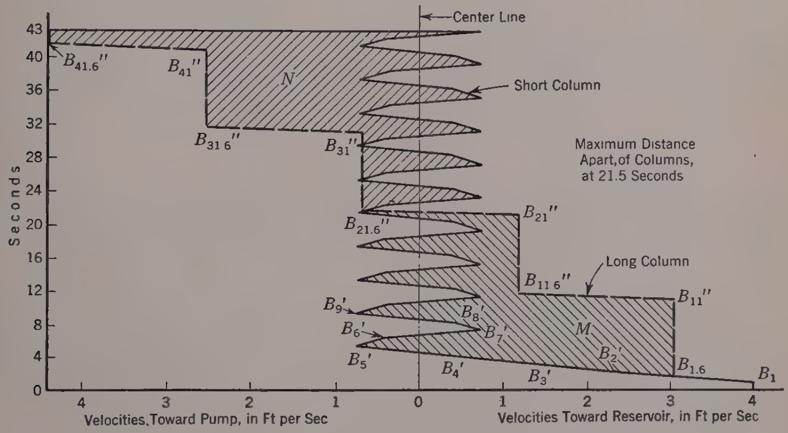

FIG. 19

Evidently, the study may be made with equal ease for any other time of closure of the valve at Point A, or for any control desired at Point B. The only error arises from the fact that the values of $\frac{L}{V_w}$ are assumed to remain constant during the study, whereas the columns actually decrease in length after separation; but in most cases this change produces a very small effect. In this particular problem the maximum distance between the columns is approximately 37 ft.

The odd form of the velocity curve for Point B' is interesting but, of course, the average velocity there could be taken as zero after closure, without affecting the answer. The pressure rise at Point A would be very high and that at Point B_{43} is easily scaled.

In computing values of δn the time intervals must be taken very short and Fig. 19 shows only a few of the points used in the calculation, as the complete set of lines would complicate a small scale drawing.

EFFECT OF FRICTION AND VELOCITY HEAD

The equations necessary to allow for friction and velocity head have been already given and, although the results obtained from them are not exact, they enable closely approximate answers to be obtained in the few cases where these factors are important. In general, it may be said that the friction decreases the bad effects due to water hammer, and computations made by neglecting it are usually on the safe side. In dealing with the problem, it has been frequently assumed that friction loss varies directly with the square of the velocity and, although the calculation may readily be made without this modification, the work is somewhat simplified by it. With this assumption the two quantities, velocity head and friction loss, may be combined into a single term, thus:

$$\text{Velocity head} + \text{friction loss} = \frac{V^2}{2g} + f\frac{L}{D}\frac{V^2}{2g} = V^2\left(\frac{1}{2g}\right) + \frac{f}{2g}\frac{L}{D} = b\,V^2 \quad \text{..(52)}$$

in which the coefficient, b, varies slightly, as f is the only variable in it. To apply this to the h, v-diagram, it must be divided by H_0 and is then written:

$$\frac{b\,V^2}{H_0} = \frac{b\,V_0^2}{H_0}\left(\frac{V}{V_0}\right)^2 = \frac{b\,V_0^2}{H_0}\,v^2 \quad \text{................(53)}$$

which gives the proportional effect on h for each value of v. Thus far, the only known method of allowing for friction loss has been to assume that the total loss is concentrated at certain points along the pipe. In Fig. 20, for example, it has been assumed to be concentrated at two points, B and C; that is, if the friction loss at Velocity V is 12 ft, it is assumed that in each of the obstructions

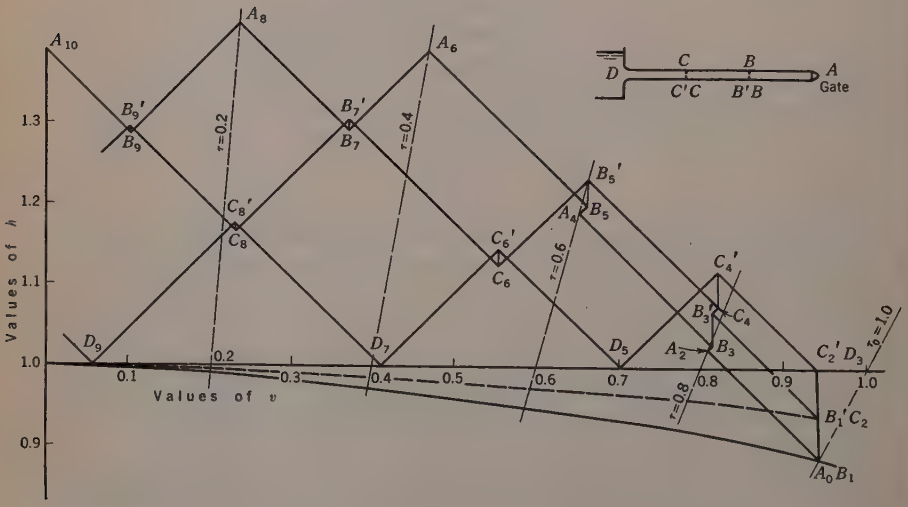

FIG. 20

at Point B and Point C, the loss of head is 6 ft and there is no loss in Pipes $A\ B$, $B'\ C$, and $C'\ D$, each having a length of $\frac{L}{3}$ ft.

With this assumption, each of the parts may be dealt with exactly as in the earlier discussion, but, at any given velocity, Points B and B' must be separated vertically by $b\,\frac{V_0^2}{H_0}\,v^2$, and they must be at the same velocity at the same instant. In Fig. 20 the values $\frac{L}{V_w}$ for each section are the same and are taken as 1 sec $\left(\text{for the entire pipe, } \frac{L}{V_w} = 3 \text{ sec}\right)$; $\rho = 0.5$; and closure is effected in 10 sec, or five intervals of $\frac{2\,L}{V_w}$ sec for the part, $A\ B$. The construction is shown well enough to need little explanation; the curve through $v = 0$, $h = 1$, and A_0 is separated from the axis of v by $\frac{b\,V_0^2\,v^2}{H_0}$; and the dotted line, which represents the loss at each obstruction, is half-way between the axis of v and the curve. Each pair of points, $B\ B'$, $C\ C'$, is separated vertically by the distance from the axis of v to the dotted curve corresponding to the velocity, v.

CONCLUSION

The variety of the illustrations shows that the method is not only quick, but is easily applied to problems met in practice. Knowing the starting point on the diagram and the law of operation of the valve or other device causing the pressure disturbance, one can solve the problems easily. A variable reservoir level introduces no trouble and merely means that such points as D (Fig. 11) no longer lie on the line $h = 1$ or H_0, but at distances above and below this line, corresponding to the variation from the original level. When the friction head is large compared with H_0 it is best to use axes of H and V rather than h and v, and to choose scales that make distinct intersections of the lines; but a little practice gives one much guidance in the plotting.

The method avoids the tedious tracing of the various pressure waves, and the complicated study of various reflection factors, as these have all been included automatically in the construction. Such experimental studies as have been made available, check the accuracy of the construction, but it is hoped that many more experiments will be conducted by those who are able to do so, and that the results of these will be available for comparison. The writer expresses his obligation to Dr. Schnyder,[11] and Professor Bergeron,[12] for their papers on this work and for the illustrations they have given. He is also indebted to many other authors and to the excellent paper on "High Head Penstock Design," by Messrs. A. W. K. Billings, O. H. Dodkin, F. Knapp, and Adolpho Santos, presented on June 30, 1933, under the joint auspices of the Hydraulic Division, A. S. M. E., and the Power Division, Am. Soc. C. E.[14]

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[14] Not published.

APPENDIX

NOTATION

The following symbols, as defined in the paper where they first appear, conform essentially with "Symbols for Hydraulics" compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1929:[15]

A = area of a pipe, a subscript designating the pipe referred to; A_g = area of a gate-opening; $A_{4.5}$ = a point on the diagram representing both the pressure and velocity at Point A on the pipe at 4.5 sec after water hammer begins;

$B = \frac{c_d A_g \sqrt{2g}}{A_a}$ and is a function of effective gate area $= c_d A_g$; B_0 = the value of B for the initial, steady flow condition;

b = a coefficient (see Equation (52));

c_d = coefficient of discharge;

D = inside diameter of pipe;

E = Young's modulus for the material in the pipe;

e = efficiency of pump;

F = short form for $F\left(T - \frac{x}{V_w}\right)$, is the magnitude of a direct wave at time, T, and at x ft from gate; F_n = magnitude of a wave in its nth movement; F_1 = magnitude of a wave at the end of the first movement;

f = short form for $f\left(T + \frac{x}{V_w}\right)$, is the magnitude of a reflected wave at time, T, and at x ft from the gate;

g = acceleration due to gravity;

H = general term for pressure head at a given time and place on the pipe; H_0 = initial pressure head at the gate under steady flow conditions; H_{BT_2}, etc. = head at Point B on the pipe (Fig. 8) at the time, T_2 sec after gate movement begins; H_A = head at Point A, usually at the point of disturbance;

h = pressure head change; δh = small pressure head change; also, h_{BT_2} etc. = ratio $\frac{H_{BT_2}}{H_0}$, etc.; h_f = friction loss in feet;

K = bulk modulus of elasticity of the liquid;

L = length of pipe, or the distance traveled by a wave;

l = short length of uniform diameter in a tapering pipe;

m = mass of the water changed in velocity;

n = speed of the rotating part of a centrifugal pump and motor, in revolutions per minute; δn = a decrease in speed;

p = pressure intensity at any point in a pipe line, due to water;

Q = discharge; Q_0 = a uniform steady discharge;

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[15] A. S. A.—Z10b—1929.

R = effective radius of the rotating parts of a centrifugal pump and motor;

T = time; δT = short interval of time; also, δT = time required for a decrease in speed of δn;

t = thickness of a pipe;

V = average velocity; V_0 = steady, uniform velocity; V_w = velocity of pressure wave; δV = velocity extinguished; V_1 = velocity reached at the end of the first interval; V_2 = velocity reached at the end of the second interval; V_{BT2} = velocity at the Point B on the pipe at the time, T_2, after disturbance begins; V_A = velocity at Point A; v_{BT2} = a ratio, $\frac{V_{BT2}}{V_{B0}}$, etc.;

W = equivalent weight of the rotating parts of a centrifugal pump and motor;

w = weight of 1 cu ft of water;

x = variable distance along a pipe measured from the gate;

y = piezometer pressure reading at Point x during water hammer; y_0 = piezometer pressure reading at Point x under steady flow conditions;

z = height of any point, x, above the gate;

α = slope angle of a pipe;

ρ = pipe line characteristic $= \frac{V_w V_{A0}}{2 g H_0}$;

τ = a function of time (see Equation (30b)) $= \frac{v_A}{\sqrt{h_A}} = \frac{B}{B_0}$.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ANALYSIS OF STRESSES IN SUBAQUEOUS TUNNEL TUBES

Discussion

BY A. A. EREMIN, ASSOC. M. AM. SOC. C. E.

A. A. EREMIN,[8] ASSOC. M. AM. SOC. C. E. (by letter).[8a]—The favorable response to the method of analysis presented in this paper is gratifying. The discussion contains much valuable information and constructive criticism.

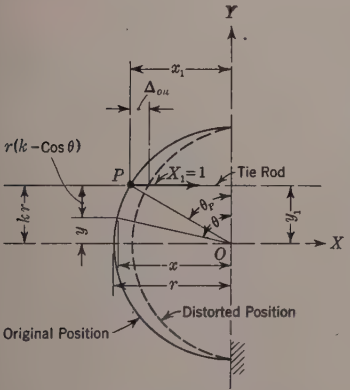

FIG. 7.—DIAGRAM OF HORIZONTAL DISTORTION

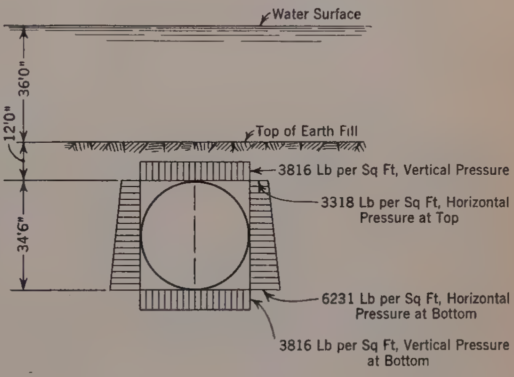
Water Surface
36'0"
12'0"
Top of Earth Fill
3816 Lb per Sq Ft, Vertical Pressure
3318 Lb per Sq Ft, Horizontal Pressure at Top
34'6"
6231 Lb per Sq Ft, Horizontal Pressure at Bottom
3816 Lb per Sq Ft, Vertical Pressure at Bottom

FIG. 8.—TYPICAL TUBE LOADING

Equations (28) to (30) for horizontal displacement at Point x, y (Fig. 6), as presented by Mr. Peery, are correct. However, the computations of the horizontal displacements of the tube may be greatly simplified. For convenience of reference the left half of the tube, with Point P defined by

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NOTE.—The paper by A. A. Eremin, Assoc. M. Am. Soc. C. E., was published in December, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1937, by David J. Peery, Jun. Am. Soc. C. E.; and November, 1937, by Orrin L. Brodie, M. Am. Soc. C. E.

[8] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[8a] Received by the Secretary December 13, 1937.

the co-ordinates, x_1 and y_1, is shown in Fig. 7. The elastic center of the tube and the origin of the co-ordinates, x and y, are at the center. The vertical distance of the point, P, to the horizontal diameter of the tube, y_1, expressed as a proportion of the radius, is:

$$y_1 = k\,r \qquad (33)$$

Therefore, the vertical distance from some point, x, y, on the shell to Point P, Fig. 7, is,

$$(y_1 - y) = r\,(k - \cos\theta) \qquad (34)$$

and the horizontal displacement at Point P on the left half of the tube fixed at the invert, Fig. 7, is,

$$\Delta_{0u} = r\int M\,(k - \cos\theta)\,dw + \int H\cos^2\theta\,dv + \int V\sin\theta\cos\theta\,dv$$
$$- H_0\left[r^2\int (k - \cos\theta)\cos\theta\,dw + \int \cos^2\theta\,dv\right] - M_0\,r\int (k - \cos\theta)\,dw. \qquad (35)$$

For practical purposes in computing horizontal displacements the influence of direct and shear stresses may be neglected without serious error. Therefore, Equation (35) may be written,

$$\Delta_{0u} = r\int M\,(k - \cos\theta)\,dw - H_0\,r^2\int (k - \cos\theta)\cos\theta\,dw$$
$$- M_0\,r\int (k - \cos\theta)\,dw \qquad (36)$$

Horizontal displacements can be computed more quickly by Equation (36) than by Equation (29). Furthermore Equation (36) involves integral terms similar to those in formulas for computing the redundant forces at the elastic center of a tunnel tube. The horizontal displacement of the left half of a tube, loaded with a force equal to unity ($X_1 = 1$) and acting along the upper tie-rod (see Fig. 7), or with any other loading, may be determined by Equation (36). The bending moment at any point, x, y, on the left half of a tube loaded with a force, $X_1 = 1$ (Fig. 7), is,

$$M = r\,(k - \cos\theta) \qquad (37)$$

In computing the horizontal displacement of the tube at Point P by Equation (36), each term containing an integral may be computed separately. Substituting the bending moment from Equation (37) in Equation (36), the first term in the latter is,

$$B_1 = r^2\int_{\pi}^{\theta_p} (k - \cos\theta)^2\,dw \qquad (38)$$

Integrating and simplifying,

$$B_1 = \frac{r^2}{E\,I}\left[k\,\pi + \frac{\pi}{2} - k^2\,\theta_p + 2\,k\sin\theta_p - \frac{1}{2}(\theta_p + \sin\theta_p\cos\theta_p)\right] \qquad (39)$$

Integrating and simplifying the second term of Equation (36), B_2 is:

$$B_2 = H_0\, r^2 \int_{\pi}^{\theta_p} (k - \cos\theta) \cos\theta \, dw$$

$$= \frac{H_0\, r^3}{E\, I} \left[-\frac{\pi}{2} - k \sin\theta + \frac{1}{2} (\theta_p + \sin\theta_p \cos\theta_p) \right] \ldots\ldots\ldots (40)$$

Integrating and simplifying the third term of Equation (36), B_3 is:

$$B_3 = M_0\, r \int_{\pi}^{\theta_p} (k - \cos\theta) \, dw = \frac{M\, r^2}{E\, I} (k\, \pi - k\, \theta_p + \sin\theta_p) \ldots\ldots (41)$$

Therefore, the horizontal displacement at Point P, Fig. 7, as computed by Equations (36) to (41) is,

$$\Delta_{0u} = B_1 + B_2 + B_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Information concerning mud pressures on subaqueous tunnels, as given by Mr. Brodie, is most interesting. It is true, as Mr. Brodie states, that the method of considering the distribution of external pressures in subaqueous tunnels "forms the other half of the picture."

In Example 1, Fig. 5, the loadings are assumed. It may be interesting to compare them with the specified external loading used in the design of the Posey Tunnel. Water pressure on that structure was considered as acting (independent of earth pressure) radially over the entire circumference. The vertical pressure on the top was considered equal to the total weight of water, at 64 lb per cu ft, plus the weight of the submerged earth at 62 lb per cu ft. The intensity of the lateral pressure at any section was assumed equal to total vertical pressure, excluding water pressure, multiplied by 33%, plus the intensity of water pressure. This also agrees with lateral pressure computed with Rankine's formula for wet sand with an angle of repose of 30° recommended by the late Milo S. Ketchum, Hon. M. Am. Soc. C. E.[9]

Assume the tunnel in Example 1 to be placed 48 ft below the water surface, as shown in Fig. 8. The top of the earth fill is 12 ft above the top of the tunnel. Therefore, the vertical pressure on top of the tube is $64 \times 48 + 62 \times 12 = 3\,816$ lb per sq ft. The side pressure at the top of the tube is $64 \times 48 + 62 \times 12 \times 0.33 = 3\,318$ lb per sq ft. The side pressure at the bottom is $64 \times 82.5 + 62 \times 46.5 \times 0.33 = 6\,231$ lb per sq ft.

Assuming the weight of tube to be equal to the buoyant force of the submerged tube, the vertical pressure at the invert will be the same as that at the top. If the tube is filled with water, and if the earth fill is placed directly on top of the tube, the vertical pressure on the invert will be greater than that on top by an amount equal to the weight of the submerged tube. To simplify computations in Example 1 the loading on the tube was as shown in Fig. 5. Evidently, this loading gives greater stress in the tie-rods than that in Fig. 8. The exact loading on subaqueous tunnels varies with the method of construction, the supporting foundation beneath the finished tunnel tube in place, the variation of the water level, and the plastic flow in the earth and the mud fill on top of the tube. Interesting effects of the plastic flow of mud on the external pressures and deformations of the Hudson and Manhattan Railroad tubes were observed during the construction of the cast-steel rings. Perfectly circular steel rings, when shoved forward under pressure by means of jacks, were changed in form by the lengthening of their vertical diameters and the shortening of their horizontal diameters. After the plastic flow of mud had occurred the shape was reversed; the vertical diameter decreased, and the horizontal diameter increased. By tightening the steel tie-rods the tube rings may be returned to their original form. Equation (31) gives the value of the maximum bending moment in the circular tunnel tube without tie-rods. This bending moment may occur at the crown, at the invert, or at the haunch section as stated by Mr. Brodie. It is to be regretted that Mr. Brodie did not give sufficient information to permit one to check Equation (31). The need for a more determinate method of computing bending moments at desired sections is self-evident. The redundant force, H_0, and the bending moment, M_0, at the elastic center of the tunnel tube without tie-rods may be computed by Equations (25) and (26), respectively. The bending moment, M_1, at any point on the tube without tie-rods, by Equation (16), is:

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[9] "The Design of Walls, Bins, and Grain Elevators," by Milo S. Ketchum, Third Edition, p. 73.

$$M_1 = -M + H_0 y + M_0 \qquad \text{.................. (43)}$$

The maximum bending moment may be determined algebraically by Equation (43), with the external loading expressed by a continuous function or by plotting a force polygon. In practice, however, with a few trial computations, the maximum bending moment in a tube may be determined without much effort.

A method of computing stresses in subaqueous tunnel tubes without tie-rods,[10] has recently been advanced by M. Alfano, who assumes the origin of the co-ordinates at the invert instead of that at the center of the tube. However, he failed to show any advantages in computing stresses with the origin of co-ordinates selected in this manner.

In conclusion, the writer wishes to express his appreciation to those who contributed to the discussion and thus increased the value of his paper.

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[10] *Annales des Ponts et Chaussees,* February, 1937, p. 197.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ECONOMICS OF HIGHWAY-BRIDGE FLOORINGS OF VARIOUS UNIT WEIGHTS

Discussion

BY J. A. L. WADDELL, HON. M. AM. SOC. C. E.

J. A. L. WADDELL,[12] HON. M. AM. SOC. C. E. (by letter).[12a]—The discussion of this paper, although not as extensive as the writer had hoped, has served to emphasize several important features. The points raised by Messrs. Jones, Tammen, and Franklin are of especial interest.

Messrs. Tammen and Fowler have asked for a separation of the substructure and superstructure costs used in preparing the curves of Figs. 1 to 8. On the average, the substructure for the open-grate flooring was found to cost about $14 per lin ft of bridge less than that for the "standard" flooring for a roadway width of 45 ft, and about $7 less for a 20-ft roadway. Cost differences for intermediate types can be interpolated with sufficient accuracy. For the case mentioned in the second paragraph of Mr. Jones' discussion, the substructure differential should be subtracted from the difference found from the curves of Fig. 1 or Fig. 5.

Mr. Jones emphasizes the value of "the method of approach and analysis" which the paper sets forth, but questions whether the findings are sufficiently general for the curves to be used in actual comparisons of cost. He states: "From such considerations comes the writer's feeling that, although the method herein given is invaluable, the most exact data should be applied to each particular case, and the author's data should be considered as illustrative."

In making his criticism, Mr. Jones has apparently overlooked the fact that Figs. 1 to 8 give merely the combined costs of substructure and superstructure to carry floorings of various weights, and that the costs of the floorings themselves are to be determined and added separately. Tables 1 and 2 give data regarding the costs of various floorings that are fairly representative of present-day light-weight floors; but it is assumed that an engineer, in dealing with a

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NOTE.—The paper by J. A. L. Waddell, Hon. M. Am. Soc. C. E., was published in February, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: April, 1937, by Jonathan Jones, M. Am. Soc. C. E.; June, 1937, by Messrs. Henry C. Tammen, Miller McClintock, Joseph G. Shryock, and C. Calor Mota; and October, 1937, by Messrs. W. G. Fowler, and Philip A. Franklin.

[12] Cons. Engr., New York, N. Y.

[12a] Received by the Secretary December 8, 1937.

specific problem, will determine the weights and costs of the actual floorings he is comparing, and will use his results rather than those of Tables 1 and 2. Ample provision is made, therefore, for variation in flooring costs and for the future development of new types.

Table 3 of Mr. Jones' discussion, showing the effect of a variation in floor costs, is correct; but this is no argument against the accuracy of the results found from Figs. 1 to 8, for the reasons just stated.

Messrs. Jones, Tammen, and Fowler have referred to factors affecting the accuracy of the diagrams themselves, such as variations in prices and in the substructure. They have, however, over-estimated the relative importance of such variations. Suppose, for instance, a silicon-steel highway bridge of 400-ft simple spans, carrying a 45-ft roadway, is being considered, and that comparison is to be made for the "standard" flooring, a 60-lb flooring, and an open-grate floor. From Fig. 2 and Table 2, the values in Table 4 are obtained, the separate items for substructure and superstructure being taken from the computations on which the diagrams were based.

TABLE 4.—COMPARISON OF COSTS

Item No.	Description (1)	TYPE OF FLOORING		
		Standard (2)	Sixty pound (3)	Open grate (4)
1	Superstructure	$311.40	$282.30	$263.00
2	Substructure	111.00	103.50	97.30
3	Cost, without deck	$422.40	$385.80	$360.30
4	Cost of deck	39.60	67.80	88.60
5	Total	$462.00	$453.60	$448.90
6	Excess cost over open-grate type	+$13.10	+$4.70	0

If a 10% change in superstructure unit prices is assumed, the differential between the "standard" and open-grate floors would be changed by $4.84, and that between the 60-lb and open-grate floors by $1.93. A considerable change in the substructure conditions would mean merely a change in the differential between the "standard" floor and the open-grate flooring ($13.10), or in the differential between the 60-lb floor and the open-grate floor ($6.20). In most instances, such changes in costs would not be great enough to upset the relative economics of the floors being considered; and where the relative standings were affected, it would merely be because the costs for the types being compared were so nearly alike that it would make little difference, as far as cost is concerned, which type was used. In the case under consideration, a difference of $5 per ft would mean only a 1% variation, and even $10 per ft would amount to only 2 per cent. Such differences are well within the ordinary errors of estimating. Furthermore, changes in the general price level can be allowed for by multiplying the results taken from the diagrams by a correction factor, and a major difference in substructure conditions can be allowed for by increasing or reducing the substructure differentials given in the second paragraph of this closure.

Since this paper was published the writer has had occasion to prepare several estimates affording an opportunity for a check on the curves in Figs. 1 to 8, and has found them absolutely reliable.[13] A further study, made in connection with the preparation of this closure, shows that a considerable variation in pier heights makes no appreciable difference in the relative costs of different types of floors.

Mr. Jones is correct in calling attention to the fact that lightening the floor of a suspension bridge will require an increase in the weight of the stiffening trusses. In most cases, however, this means simply a reduction in the saving to be realized by the use of a lighter floor. For a comparatively short span, where the saving in other parts of the bridge is not large, the increase in the stiffening trusses will be small; and, for a long span, on the other hand, where the increase in stiffening truss weight will be considerable, the saving in other parts—floor-system, suspenders, cables, towers, and anchorages—will also be large.

Referring to cantilever spans, there would be no objection to using a heavy floor on the anchor spans of a three-span Type *A* cantilever, and a light floor on the central span. For a Type *C* cantilever or a long bridge consisting of an alternation of anchor and cantilever spans, it would be unsatisfactory, from the standpoint of both appearance and traffic, to use one type on the anchor spans and another on the cantilever spans.

Mr. Tammen mentions the fact that different engineers will reach different results for the same flooring, due to variations in design procedure, details, minimum thickness of metal, and allowances for maintenance and amortization. Such considerations do not affect the usefulness of the paper in any way, because it is assumed that each engineer will determine for himself the total costs and allowable span lengths of the floorings he is comparing. Allowances for maintenance and amortization are difficult to determine, since relative costs and life for light-weight floors and older types of heavy floors are not yet known.

Mr. Tammen criticizes the use of thinner metal in the flooring rather than in other parts of the superstructure—and possibly his comment is truly sound; but it must not be forgotten that, if the light-weight flooring should either rust out or wear out, it could easily be repaired or replaced, whereas thin metal in the superstructure proper could not. It is either the manufacturers or the patentees of such floors who have determined the thickness of their products; and if these sections are computed properly for strength, experience will ultimately tell whether the use of the thin sections is truly economical.

Mr. Tammen's data relative to the savings resulting from the use of light-weight floors on movable spans are interesting and valuable, and the writer endorses them heartily.

It is difficult to agree with Mr. Shryock's conclusion that floor weights can be reduced to 50% of the weight of the standard type "without undue sacrifice of stiffness and rigidity," but that a reduction to 25% "could scarcely be secured without a sacrifice of these important requirements," because both the percentages quoted are purely arbitrary. The 540-ft spans of the Marine Parkway Bridge over Rockaway Inlet in New York City, which are paved with open-grating, are perfectly stiff and rigid under traffic. It should also be noted, as stated by Mr. Franklin, that an open-grate floor, when properly detailed and thoroughly connected to the supporting steelwork, forms an effective horizontal girder.

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[13] "Quick Estimating of Economic-Bridge Costs," by J. A. L. Waddell, Hon. M. Am. Soc. C. E., and Shortridge Hardesty, M. Am. Soc. C. E., N. Y., John Wiley & Sons. (Publication pending.)

To Mr. McClintock and Professor C. Calor Mota, the writer desires to express his deep appreciation of their hearty endorsements of his technical efforts. Mr. Fowler should not forget that the sole object of the paper is to compare the economics of different types of flooring and not to provide the data for making cost estimates of bridges.

Mr. Franklin's discussion is most important. The writer endorses unequivocally all the points he raises, and specially commends the division of the conditions into "tangibles and intangibles"; moreover he concedes that the latter are, and should be universally considered by far the more important. In selecting the type of flooring for a highway bridge, the prime requisite certainly is safety; and economy in first cost should always be a subordinate consideration. Safety from accidents through skidding should always be a primary consideration, because human life and human welfare are endangered by such accidents.

Mr. Franklin has prepared his discussion very skilfully, in that he has handled the subject of "intangibles" without any reference to patented floorings or special interests. Nevertheless, he states:

> "* * * The consideration of this larger problem of the 'Ultimate Economics of Highway-Bridge Floors' is quite worthy of the efforts of the profession. It should not be hampered by the limitations of mathematics in the discussion, but the merits and shortcomings of all the various types of floors in common use should be reviewed; the factors which lead to desirable features, as well as those which produce undesirable results, should be listed; and conclusions should be drawn concerning the attributes of the perfect floor."

Such a general open discussion would be of truly great value.

The objection to open-grid floors that they "require a secondary system of supports and complicate the original detailing and shopwork" is an economic one, and, therefore, pertains to the "tangible" group of factors; consequently, it is taken care of by the cost-curves and tables of the paper. From the standpoint of safety, the principal advantage of the open floor is its elimination of lateral skidding, even during times of snow and sleet.

The Marine Parkway Bridge, previously mentioned, will provide an excellent measure of the safety, the durability, and the cost of maintenance for open flooring, as compared with the "standard" solid flooring; because, on the three long main spans, open-grate flooring is used, whereas all the approach spans have concrete flooring. Experience during one winter may settle the question of the comparative safety of the two types, but years will be required in order to determine the comparative maintenance costs.

In the writer's opinion, the open flooring has come to stay, and is destined to be one of the standard highway-bridge floors of the future.

In closing this paper, the writer desires again to thank all the engineers who have so kindly discussed it, especially Mr. Jones and Mr. Tammen, whose skepticism concerning accuracy of results has enabled him to show that such skepticism is unfounded, and Mr. Franklin who indicated the existence of "intangible" as well as economic factors in the comparison of bridge floorings.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

NATIONAL ASPECTS OF FLOOD CONTROL

A SYMPOSIUM

Discussion

By RALPH W. POWELL, M. AM. SOC. C. E.

RALPH W. POWELL,[70] M. AM. SOC. C. E. (by letter).[70a]—It is scarcely possible to exaggerate the value of this Symposium. Between the time the papers were presented orally (October, 1936) and their appearance in *Proceedings* (March, 1937) another flood occurred on the Lower Ohio River, in most respects more serious than any described in the Symposium. Therefore, some of the statements in the Symposium were already "out of date" when printed, but the importance of the subject was still further emphasized. The fact that it is now a national problem is clear from the wording of the Flood Control Act of 1936.

With the exception of those few non-navigable streams that discharge directly into the ocean or into the Great Lakes, the control of floods on any stream in the United States is now "a proper activity of the Federal Government in co-operation with States, their political subdivisions, and localities thereof."

One minor correction might be offered to Mr. Jacobs' valuable paper. Under the heading, "Engineering Aspects," he states that three of the Muskingum reservoirs are automatic retention basins. Probably the three reservoirs meant are the Mohawk, Mohicanville, and Bolivar. These are "dry reservoirs," but the outlets are gate-controlled. Of course, with the gates open, the reservoirs would still give a certain volume of automatic storage just as they did in the January, 1937, flood when the gates were kept open due to legal reasons; but, normally, the gates will be closed as

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NOTE.—This Symposium was presented at the Fall Meeting of the Society and at the meeting of the Waterways Division, Pittsburgh, Pa., October 13 and 14, 1936, and published in March, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: June, 1937, by Messrs. F. C. Scobey, Howard T. Critchlow, T. T. Knappen, M. C. Tyler, Gordon R. Williams, Arthur T. Safford, W. G. Hoyt, J. D. Arthur, Jr., John H. Meursinge, H. K. Barrows, E. D. Hendricks, and Edward W. Bush; September, 1937, by Messrs. H. K. Barrows, Ivan E. Houk, and John E. Field; October, 1937, by Messrs. C. S. Jarvis and Joseph Jacobs; and December, 1937, by Messrs. W. M. Dawley, and Howard M. Turner.

[70] Assoc. Prof. of Mechanics, Ohio State Univ., Columbus, Ohio; formerly, Hydr. Engr., Muskingum Watershed Conservancy Dist., New Philadelphia, Ohio.

[70a] Received by the Secretary December 9, 1937.

soon as it appears that the safe capacity of the channel down stream would otherwise be exceeded.

The Pleasant Hill Reservoir has an automatic outlet (orifices in an intake tower), but it is not a simple retention basin, as it has 13 500 acre-ft of permanent storage that can be released through two 3.5 by 7-ft sluicegates. Since these gates are normally under a head of about 45 ft, they could, if it were desired, afford an appreciable controlled discharge in addition to the automatic discharge through the orifices. The original plans called for gates in the orifices also, but this was ruled out by the consulting board as one more complication in an already complicated design.

Of the plotting of discharge per square mile against drainage area (as in Fig. 3, presented by Mr. Uhl, and in Fig. 13, presented by Messrs. Harrington and Johnston) there is no end, and the graphs are always interesting. The latest to come to the writer's attention is by Victor H. Cochrane,[71] M. Am. Soc. C. E. It would seem to the writer, however, that they would be much more valuable if the data were limited to the maximum annual floods over the entire period of record of a large number of streams. Then a line or a smooth curve which had 1% of the points outside it might be thought of as representing a 100-yr probability; one with 5% of the points outside it, a 20-yr probability; etc. It must always be remembered, however, that drainage area is only one of the factors determining probable peak discharge. The arrangement of tributaries, whether fan-shaped and converging on the point considered, as at Pittsburgh, Pa., and Dayton, Ohio, or with various concentration periods, as in the Mississippi River at New Orleans, La., plays an extremely important part. As C. R. Pettis, M. Am. Soc. C. E., has shown,[72] the average width of the drainage area is probably a better criterion than the total area, and a plotting of peak flow against average width might be the best way to represent the data.

In spite of its obviousness, furthermore, attention should be called to the fact that it is only in the design of channel improvements, levees, bridge openings, etc., that the unmodified peak flow is important. If the flow is to be controlled by reservoirs, the essential item is the volume of the entire flood hydrograph. Fortunately this can be estimated much more accurately than the peak flow, since for safety it must be taken as equal to, or nearly equal to, the entire rainfall over the drainage area; and the data on rainfall are much more extensive than those on run-off.

In this connection the writer feels that Messrs. Morse and Thomas have erred in their treatment of the effects of forests. The run-off from light and medium rains is greatly affected by the cover, and a good forest should definitely reduce the average annual flood. For the great floods upon which the design of adequate flood control must be based, however, it is doubtful whether the retention would be a large enough part of the entire precipitation to affect greatly the design. For example, the January, 1937, flood at Louisville, Ky., would probably have been almost as great if the same rainfall had fallen 200 yr earlier. Certainly accurate quantitative data on the subject are greatly to be desired.

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[71] *Engineering News-Record,* November 25, 1937, p. 867.

[72] "Floods in the United States," U. S. Geological Survey, *Water Supply Paper No. 771,* p. 36.

With the remarks of Messrs. Morse and Thomas on "flood routing" and on the importance of the experiments at the Carnegie Institute of Technology, at Pittsburgh, the writer is in hearty accord. Some have felt that the Burns-Harkness-McCarthy method is a satisfactory solution of the problem, but the writer's experience is that the ratio of storage increment to the corresponding weighted flow increment is not the same for great floods as it is for medium-sized floods, and that it is not the same for winter floods as for summer floods. To be able to compute the effect at all points down stream, of storing or discharging at given rates from each reservoir, is so necessary for flood forecasting as well as for the design of control systems, that the problem is probably the most important one confronting flood control engineers. Fortunately, sufficiently complicated systems are now in existence on such rivers as the Tennessee and Muskingum, to try out, by full-scale experiment with small floods, any theory or method of computation proposed. It is hoped that such experiments will be made.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

PRACTICAL USE OF HORIZONTAL GEODETIC CONTROL

Discussion

BY JULIUS L. SPEERT, JUN. AM. SOC. C. E.

JULIUS L. SPEERT,[11] JUN. AM. SOC. C. E. (by letter)[11a]—The author gives voice to a complaint that undoubtedly has been felt by many engineers, but has seldom been mentioned. It is unfortunate that the relative simplicity of many kinds of surveying computations should be masked by complicated explanations or instructions. This condition is probably due to the fact that, as far as the writer knows, the theory of surveying computations is to be found, in general, in only two types of publications; namely, college textbooks, and Government publications. By its very nature, a college textbook must approach its subject gradually, emphasizing the method of approach rather than the ultimate conclusion. The voluminous explanation necessary in this type of treatment makes the book unsuitable as a practical working manual. Similarly, Government publications on surveying, as a rule, are scholarly treatises, written by, and of value to, scholars and mathematicians, but too frequently unintelligible to the average surveyor, whose knowledge of higher mathematics is somewhat limited and who has little desire to become involved in its ramifications.

If there were available a concise manual of instructions in which the method of performing the computations was clearly explained and the significance of each step was briefly stated, there would be less tendency for the young engineer and the practical surveyor to shun precise methods of computing and recording surveys in favor of the simpler methods of plane surveying. The mathematical justification for the various steps should be included in the appendix to this manual, where it would be available to the mathematically minded but would not confuse and bewilder those less gifted.

The author's proposed remedy is to simplify the use of geodetic control

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NOTE.—The paper by R. C. Sheldon, Assoc. M. Am. Soc. C. E., was published in April, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1937, by Messrs. Philip Kissam, Ralph Z. Kirkpatrick, E. B. Roberts, H. W. Hemple, J. C. Carpenter, and George D. Whitmore; and November, 1937, by R. M. Wilson, M. Am. Soc. C. E.

[11] Asst. Topographic Engr., U. S. Geological Survey, Washington, D. C.

[11a] Received by the Secretary November 26, 1937.

by substituting for the more rigorous computations what amounts almost to the methods used in plane co-ordinate computation. For the Canal Zone, where the maximum convergence of meridians amounts to not more than 5′, this method appears to be satisfactory. However, examination of Table 3(*b*) and Table 4(*b*) shows that for latitudes in the United States excessive errors might readily be introduced into surveys of any appreciable extent. The author offers no simple method for evaluating these errors.

There can be little doubt that the development of the State plane co-ordinate systems, published by the U. S. Coast and Geodetic Survey in *Special Publication No. 193* (11)[2] constitutes the greatest advance in recent years in the simplification of surveying computations. These systems permit computations as simple as those of plane surveying and yet make it possible to tie any located point rigidly to the national geodetic-control net as well as to every other point tied thereto. Although a true, undistorted representation of the earth's surface is impossible on a plane, the State systems permit a precise evaluation of the error involved in the computed position of any point, thereby, in effect, eliminating that error by making possible a compensation for it.

To make the geodetic-control net more readily available for the use of plane co-ordinates, the U. S. Geological Survey has recently published, in pamphlet form, a set of formulas and tables by means of which the conversion of geodetic co-ordinates to plane co-ordinates on the State systems may be greatly simplified.[12] These formulas and tables are designed to be used with a computing machine and natural functions, thereby dispensing with cumbersome logarithms, and greatly reducing the labor and time required for the conversion. Complete detailed instructions are included in the pamphlet and, for those interested in the theory, the derivation of the formulas is explained in a short appendix.

With all the facilities now available for tying any survey to the national geodetic control net, there should be little excuse for an isolated, uncontrolled survey, and there is no longer any need for a part-way compromise between geodetic control and its practical application.

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[2] For reference to figures in parentheses, see "Bibliography," in Appendix, *Proceedings*, Am. Soc. C. E., April, 1937, p. 668.

[12] "Formulas and Tables for the Transformation of Geodetic to Plane Co-Ordinates on the Lambert and Transverse Mercator Projections," by J. L. Speert, U. S. Dept. of the Interior, Geological Survey, 13 pp.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

EFFECT OF DOWEL-BAR MISALIGNMENT ACROSS CONCRETE PAVEMENT JOINTS

Discussion

By L. E. Grinter, Assoc. M. Am. Soc. C. E.

L. E. Grinter,[14] Assoc. M. Am. Soc. C. E. (by letter).[14a]—This excellent paper reports the results of tests made to determine a second influence on the action of dowels that may be quite as important as their action in resisting vertical shear caused by wheel loads. Any attempt to design dowels to resist, safely, the shear produced as a wheel load moves across the joint evidently will fail in its major objective if misalignment can produce failure anyway. Strangely enough, the stresses produced by wheel loads and the stresses produced by misalignment are essentially alike, and this statement holds true both for the stresses in the bar and for the stresses that the bar produces in the concrete slab. The result of excessive stresses in either case would be a spalling failure of the concrete as illustrated so clearly in Fig. 9. It must be the purpose of good design to prevent such failure from the action of the various possible load conditions, and it is the purpose of good construction to prevent such possibility from misalignment.

Internal Deformations.—An understanding of the kinds of strains produced by misalignment might aid one in an interpretation of the author's conclusions. It will be evident from Fig. 12 that the influence of a wheel load or of a heaving sub-grade, as indicated in Fig. 1(*a*), is identical with the influence of misalignment, as shown in Fig. 1(*b*). In either case the dowel across the joint acts as a short beam, restrained at its two ends, which undergoes differential settlement. If its end conditions are identical, this beam will deflect in a symmetrical reversed curve and will show a point of contra-flexure midway between the slabs. However, since the end conditions are unlike, one end being bonded and the other end being unbonded (and, therefore, slightly

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Note.—The paper by Arthur R. Smith, M. Am. Soc. C. E., and Sanford W. Benham, Assoc. M. Am. Soc. C. E., was published in June, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1937, by Messrs. L. W. Teller, David J. Peery, and L. J. Mensch; and December, 1937, by Messrs. W. O. Fremont, and George A. Smith.

[14] Director of Civ. Eng., and Dean of the Graduate Div., Armour Inst. of Technology, Chicago, Ill.

[14a] Received by the Secretary December 4, 1937.

loose in the concrete), it is qualitatively justifiable to assume that the moment at the sliding end is only 50% or less of the moment at the bonded end.

The general load condition, then, of a misaligned dowel bar is similar in most respects to the problem of any bar or beam of indefinite length, supported on an elastic foundation, as shown in Fig. 13. This problem has been analyzed by Professor S. Timoshenko[15] who found the deflection at the end of the bar to be:

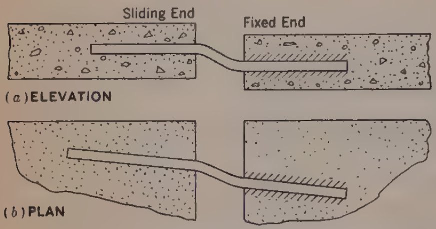

FIG. 12.—DISTORTION FROM SETTLEMENT AND MISALIGNMENT

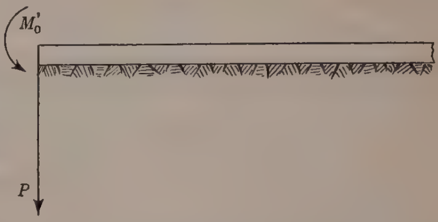

FIG. 13.—BAR ON AN ELASTIC FOUNDATION

$$y_0 = \frac{P + \beta\, M_0}{2\, \beta^3\, E\, I} \qquad (18)$$

in which,

$$\beta = \sqrt[4]{\frac{K\, b}{4\, E\, I}} \qquad (19)$$

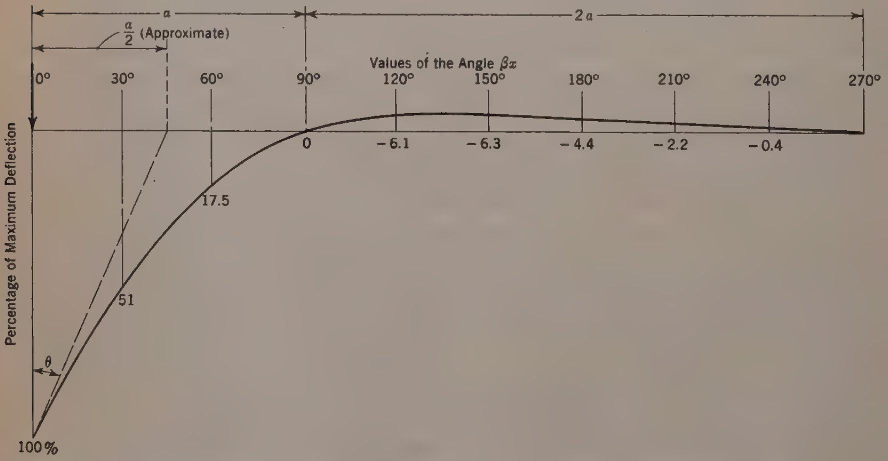

FIG. 14.—DEFLECTION DIAGRAM FOR A BAR ON AN ELASTIC SUPPORTING MEDIUM

In Equations (18) and (19), E, I, and b are, respectively, the modulus of elasticity, the moment of inertia, and the breadth or diameter of the bar; K is the foundation modulus, in pounds per square inch per inch of settlement. Hence, K is essentially an indeterminate factor in the case of a loaded dowel bar since it should combine the resistance of the slab with that of the sub-grade. For a dowel bar misaligned in the horizontal plane, K apparently becomes the same as the modulus of elasticity of the concrete.

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[15] "Applied Elasticity," by S. Timoshenko and J. M. Lessels, Westinghouse Technical Night School Press, pp. 133–153.

It is possible to avoid a discussion of the factor, K, in one study by investigating the general equation for the deflection of the bar,

$$y = \frac{e^{-\beta x}}{2 \beta^3 E I} \left[P \cos \beta x + \beta M_0 (\cos \beta x - \sin \beta x)\right] \ldots\ldots (20a)$$

which will be simplified temporarily by dropping the term involving M_0,

$$y = \frac{e^{-\beta x}}{2 \beta^3 E I} (P \cos \beta x) \ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

Equation (20b) is plotted in Fig. 14 in terms of the angle, βx. The important considerations are that the positive or downward deflection under the applied load exists over a length, a, which is one-third the length to the second point of zero deflection.

From a study of Fig. 14 it seems reasonably evident that any dowel bar must act at some stage of its life in such a manner that only bearing pressure under about one-third its embedded length is available to resist the dowel shear, and that over this length the bearing pressure reduces in nearly a parabolic line from a maximum value to zero. The dowel moment, M_0, will act to increase the severity of the dowel pressure, but, in so far as the general shape of the deflection curve is concerned, the influence of M_0 will not be important.

Example of Misalignment Study.—If the concrete of the slab has an ultimate compressive strength of 3 000 lb per sq in., and if the dowels are $\frac{3}{4}$-in. round bars, 24 in. long, the ultimate shearing capacity per dowel before the concrete begins to spall slightly, probably, is not greater than $V = \frac{3\,000}{3}\left(\frac{12}{3} \times \frac{3}{4}\right) = 3\,000$ lb. This conclusion is based upon the form of the deflection curve of Fig. 14. The authors found that the alignment error in a 6-in. pavement could be 1 in. without causing spalling even when the joint was opened to a width of 0.75 in. An error of 1 in. in 22 in. amounts to 0.03 in. in a length of 0.75 in.

If the misalignment to be considered is in the horizontal plane of the pavement, the elastic compression of the concrete due to the bar pressure can be computed from Equation (18) when the value of K is the modulus of elasticity of the concrete. This modulus is assumed herein at the low value of 1 500 000 to account in a crude manner for deformations at stresses near the ultimate. Hence, by Equation (19) $\beta = \sqrt[4]{\frac{1\,500\,000 \times 0.75}{4 \times 30\,000\,000 \times 0.049 \times 0.75^4}} = 0.88$. In order to obtain an average condition (joint open 0.75 in.), M_0 may be taken at $0.375 \times 3\,000$; and, by Equation (18),

$$y_0 = \frac{3\,000 + 3\,000 \times 0.375 \times 0.88}{2 \times 0.88^3 \times 30\,000\,000 \times 0.049 \times 0.75^4} = 0.0062 \text{ in.}$$

As shown by the exaggerated picture, Fig. 15, the misalignment is accounted for by three distortions:

(*a*) The bending of the bar as a fixed-end beam in the gap between slabs (this contribution is small, amounting to only 0.0002 in.);

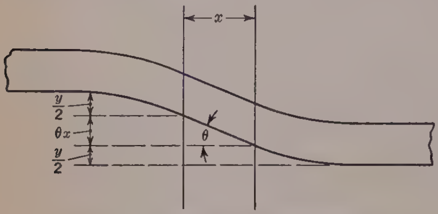

FIG. 15.—DOWEL BAR DEFORMATION EXAGGERATED

(*b*) The movement, y, permitted by the elastic and inelastic deformation of the concrete; and,

(*c*) The movement, $\theta\, x$, that is permitted by the slope taken by the bar across the open joint.

Contribution (*b*) evidently is twice the value of 0.0062 computed by Equation (18), or 0.0124 in. Angle θ can be approximated from Fig. 14 as the deflection divided by one-sixth of the embedded length of bar of $0.0062 \div 2 = 0.0031$ radian. Contribution (*c*), therefore, will be $0.0031 \times 0.75 = 0.0023$ in. when the gap across the joint is $\frac{3}{4}$ in. in width. Hence, the total misalignment deformation would become $0.0124 + 0.0023 + 0.0002 = 0.015$ in.

Conclusion.—Since this deflection is only about one-half the movement that the authors consider permissible, it appears that the action of the test slabs could be explained on one of the following bases:

(1) That the bars did not fit tightly into the slabs and, therefore, were able to "give" slightly (such dowels are of questionable value in transferring load across a joint, which is their major function since an unsupported interior edge is a line of great weakness);

(2) There may have been some undetected crushing or even a slight spalling which would prove a source of ultimate failure if slippage should be repeated many times; and,

(3) Looseness or "give" at one slab would produce a possible maximum moment in the bar of $3\,000 \times 0.75 = 2\,270$ in-lb. This moment would stress a $\frac{3}{4}$-in. round bar to 55 000 lb per sq in. If the bars used are of ordinary mild steel, they may, therefore, be deformed permanently.

The writer calls attention to the approximations involved in the computations presented herein. Much can be learned by such calculations, but they should be taken as significant qualitatively rather than quantitatively. From them the writer draws the general conclusion that the authors' specifications for permissible misalignment may be overly generous if proper dowel action is expected during the entire life of the pavement. Since a practical construction procedure could be devised to eliminate inaccuracy in excess of $\frac{1}{4}$ in. in alignment, this value could be set as a practical standard of workmanship. The importance of this requirement will be evident when it is understood that the stresses accompanying dowel action induced by loads, by heaving of the sub-grade, and by misalignment, may be additive.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

WATER TRANSPORTATION VERSUS RAIL TRANSPORTATION

A SYMPOSIUM

Discussion

BY C. D. BORDELON, ESQ.

C. D. BORDELON,[12] ESQ. (by letter).[12a]—The authors of the papers comprising the Symposium—otherwise of generally divergent views—agree that inland waterway transportation was essential to the very life of the Middle West and the Ohio Valley at least until about the 1860's. Before that time there was virtually no other means of mass transportation of goods and people.

Even at the middle of the Nineteenth Century there were no railroads west of the Mississippi River; St. Louis, Mo., Cairo, Ill., Chicago, Ill., and New Orleans, La., had no lines of railroad whatever until about 1852, and the important cities of New Orleans, St. Louis, Chicago, Cincinnati, Ohio, Pittsburgh, Pa., and the "Twin Cities" (Minneapolis and St. Paul, Minn.) were connected only by watercourses. It is small wonder then, that the inland waterways were regarded at that time as of such great value to the life and development of the nation.

The authors are also in agreement that the subsequent extension of rail lines to include all the vast territory tributary to these main rivers brought about the general demise of river transportation. These rivers were no longer indispensable to economic existence.

The essential differences between Major Putnam and Mr. Wonson may fairly be said to be whether there is at this time any economic justification for the further expenditure of public funds to create and maintain inland waterways whose use is free to private purposes as well as to operations for profit; and whether inland waterway costs should include interest on funds spent in creating and maintaining the channels. An appraisal of these differing points

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NOTE.—The Symposium on Water Transportation Versus Rail Transportation was presented at the meeting of the Waterways Division, Little Rock, Ark., April 25, 1936, and published in September, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: October, 1937, by George Hartley, Esq.; and December, 1937, by W. D. Faucette and J. E. Willoughby, Members, Am. Soc. C. E.

[12] Asst. Freight Traffic Mgr., Mo. Pac. Lines, St. Louis, Mo.

[12a] Received by the Secretary November 13, 1937.

of view, as well as of governmental policy in respect to the inland waterways, may be aided by a review of certain facts probably, in the main, forgotten.

Beginning this brief discussion with the period in which the railways had virtually supplanted inland water carriers, and at which time also the capacity of the rail lines was occasionally over-taxed, it is found that the organized movement for a revival of inland water transportation began in the early 1890's and crystallized in 1895. During that year the International Waterways Convention was held in Cleveland, Ohio, followed five or six years later by the First National Rivers and Harbors Congress, in Baltimore, Md. The assembling of these two conventions, joined with the work of various commercial associations, stimulated new interest in waterway transportation. In 1902, Congress passed a Rivers and Harbors Act providing for the establishment of a Board of Army Engineers to review, from an engineering standpoint, all recommendations for waterway projects. In 1903, the people of New York voted $101 000 000 for the improvement of the Erie Canal.

Many conventions and congresses were held in 1907, and in that year, acting in part on the belief he had voiced in his Memphis, Tenn., speech and as a part of his general conservation program, President Theodore Roosevelt appointed the Inland Waterways Commission with the then Senator Theodore E. Burton, of Ohio, as Chairman. In the preliminary report of that Committee, in 1908, it was stated:

> "While the railways of mainland United States have been notably efficient in extending and promoting the production and commerce of the country, it is clear that at seasons recurring with increasing frequency they are unable to keep pace with production or to meet the requirements of transportation."

The Commission further declared that, although navigation of the inland waterways had declined with an increase in rail transportation during the latter decades of the Nineteenth Century, it was clear that the time had arrived for restoring and developing such inland navigation and water transportation as upon expert examination appeared to confer a benefit commensurate with the cost, to be utilized as a necessary adjunct to rail transportation.

> "While trustworthy estimates cannot be made without further data," said the Committee, "it is reasonable to anticipate that congestion of interstate commerce can be obviated in a large measure by judicious improvement of waterways adapted to barge and boat traffic, at a figure much less than estimated by competent authorities for so increasing railway facilities as to meet present needs."

The Commission recommended that the President, with the advice and consent of the Senate, appoint a National Waterways Commission to bring into co-ordination the Corps of Engineers of the Army, the Bureau of Soils, the Forest Service, and other branches of public service in so far as their work related to inland waterways, for the purpose of making further investigation of various kinds.

The National Waterways Commission, composed of twelve members of the Senate and House of Representatives, came into being as the result of an Act of Congress, dated March 3, 1909. The duty imposed upon this Commission by statute was to investigate questions pertaining to water transportation and the improvement of waterways, and to make recommendation to Congress.

In its preliminary report in 1910 this Commission declared that although there had been a tendency to improve waterways for the purpose of lowering railway rates, it could not endorse this as a desirable policy to adopt; that such a policy rested, in the first instance, upon the transparent fallacy that the railways constitute an intrenched and uncontrollable monopoly which could not be reached by legislation or other orderly legal methods; and that the only way in which to compel them to lower rates is by the expenditure of large sums of money.

It recommended that a definite policy be adopted regarding the relations between railways and waterways. Such a policy, the Commission stated, should be based on the idea that the purpose of improving waterways was to secure additional means of transportation, and not to reduce rates or regulate the railways. Experience had demonstrated, according to the Commission's report, that waterways could not be relied upon as the great cheapeners and regulators of railway rates, which they once were supposed to be. The report also contains the statement that no European country used its waterways to control railway rates, that function being accomplished by the proper administrative bureau of the Government, and that the waterways were improved with the idea of securing additional means of transportation.

In its final report, in 1912, it was said that some of the economic factors theretofore unfavorable to the development of water transportation were gradually changing. Population was increasing rapidly. The average increase for the decade preceding 1912 had been more than 1 590 000 per year. Industrial development of the country was also making rapid strides, according to the report, and the value of agricultural products had doubled in the preceding ten years.

The Commission found that the movement of traffic had increased entirely out of proportion to the increase in transportation facilities. During the period, 1900 to 1910, according to the report, the railway mileage increased about 36%; the number of locomotives increased 56%; the number of freight cars, 56%; while the ton-miles of service performed increased 80 per cent. Previous to that time, the development of transportation facilities had increased more rapidly than the demand for them, and the railways made special efforts to secure traffic of any kind for their idle equipment, even on very low rates. With the rapid expansion which took place between 1904 and 1907, however, all the surplus capacity of the railways was soon exhausted, and a traffic congestion ensued.

The car shortages of 1906–1907 caused widespread complaint and dissatisfaction, and increased the belief that the railroads were inadequate to meet the needs of the country's commerce. Herbert Quick,[13] one of the most ardent advocates of inland waterway development, declared that the railroads could not handle the nation's traffic. He referred to the car shortage in 1906 and 1907 as establishing this fact, and stated:

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[13] "American Inland Waterways," by Herbert Quick (1909), p. 75.

> "The best railway description of the situation is that we have been trying to force a three-inch stream of commerce through a two-inch pipe of railways; that we need over 75,000 to 120,000 miles of new track, and so many new cars and engines that on the whole there is not enough iron in the country to meet their needs, not labor enough to make and install the new equipment and track, and not enough money to pay for the transactions."

This thought persisted during the years prior to the World War.

In the "Census of Transportation by Water," a special report of the Census Bureau for 1916, it is stated that, although the development of inland waterways had not continued in the preceding decade to the extent desirable, further development was necessary to meet the demands of existing commerce, to prevent the recurrence of congestion at the ports and to permit water-borne commerce among the States.

The World War brought the situation to a head. The movement of traffic which it caused undoubtedly over-taxed the then existing railroad plant. Water transportation was partly taken over by the Government as a means of transport supply and to supplement existing transportation services in sections where abnormal production prevailed.

It is of more than passing interest to observe that the entry of the Federal Government into the water-transportation business was due to the exigencies of war. In the Annual Report of the Hon. William G. McAdoo, Director General of Railroads, to the President in 1918, the former stated that he had appointed a committee to make a prompt investigation and to suggest a definite plan for the additional use of internal waterways so as to relieve or supplement the railways under the conditions caused by war.

The Annual Report of the Chief of Inland Coastwise Waterways Service to the Secretary of War for the fiscal year 1920 stated that the particular purpose of the Committee on Inland Water Transportation, and that to which its efforts were mainly addressed, was the study of the feasibility of utilizing the waterways for the relief of war-time freight congestion of the railroads, which was then beginning to be keenly felt; and, that the necessities of the circumstances required the adoption of a plan that would give immediate relief to shippers and the public by moving a fixed quantity of water-borne freight to the relief of other facilities. He stated further that as rail-transportation facilities became yearly less able to handle, alone, the entire traffic business of the Mississippi Valley, and as annual periods of rolling-stock shortage brought increasing losses to the mercantile communities of the Central West, active interest began to be directed to the interior river system.

Corroborating this thought, Maj. Gen. T. Q. Ashburn, in his testimony in March, 1935, before the Joint Committee of the Senate and House considering the bill[14] to regulate water carriers, stated in part: "Please remember that the rehabilitation of our interior waterway commerce began with the crying need of our interior communicative system for more and better means of supplying the needs of our troops at home and overseas."

The experience of the war led to the deep-seated belief in many quarters that the railroads would never be adequate for the transportation needs of the country and that they would have to be supplemented by some other agency, which later was considered to be the waterways. This belief was repeatedly expressed by those who investigated inland waterway development or who championed the inland waterway movement. For example, James E. Smith, President of the Mississippi Valley Waterways Association, stated in 1919:[15]

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[14] S. Doc. No. 1632, H. R. Doc. No. 5379, 74th Cong., 1st Session, March, 1935.

> "The time has come when the people of our whole country are insisting upon the Government adopting a broad, comprehensive plan for the improvement and use of all of our important navigable rivers as transportation highways.
>
> " * * * The people of our country know that our railroads have not kept pace with the growth of our commerce, and that its further growth and expansion and the full development of our country's resources are absolutely dependent upon the use of our navigable waterways * * *."

At the same time Jackson Johnson, President of the St. Louis Chamber of Commerce, made the following statement:

> "Heretofore the Mississippi Valley has not had sufficient outlet to encourage intensive production. Agricultural, industrial, commercial and mining resources of the Valley are so varied and so unlimited, that it was not surprising that commerce, growing out of these resources, was expanding beyond the capacity of the railroads to handle, even before the war added to their burdens; and now with the return to peace conditions, the transportation outlet by rail is not encouraging. It cannot be for several decades to come, even at inestimable expense. There has been no railroad development at all in the last two years. In fact, there is less trackage now than there was two years ago, and yet there is an industrial condition that demands a greater outlet. We must find a market for the overplus of manufacture and agriculture, which was developed by the intensified war production."

The statement by A. W. Mackie, Manager of the Mississippi Section, Mississippi-Warrior Waterways, follows:

> "Wonderful as has been the development of railroad transportation in the United States, commercial development employing that transportation has been equally marvellous, and we now face a situation more serious to business progress than is generally recognized. That situation is one of continued rapid development of industry and production without adequate expansion and development of rail facilities. Proof of this statement is offered every time there is a heavy crop movement on, as in the recent war emergency, a heavy demand upon transportation. The railroads alone cannot provide the necessary transportation facilities and service. We must, then, employ the proven capacity of our improved inland waters."

During the consideration by the House of Representatives of the original Inland Waterways Act, Mr. E. E. Denison, who was on the Committee, and more or less in charge of the bill, emphasized the supposed inability of the railroads to carry the nation's commerce.[16] To the same effect are the following conclusions in the 1922 report of the International Joint Commission on the St. Lawrence Waterway:

(1) The then existing means of transportation between the tributary area in the United States and the seaboard were altogether inadequate, and the railroads had not kept pace with the needs of the country.

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[15] *National Waterways Magazine,* March, 1919.

[16] *Congressional Record,* Vol. 65, Pt. 9, p. 8720.

(2) Although war conditions had something to do with the dislocation of railway traffic in the United States and, although various other factors should be taken into account, such as the congestion of traffic at certain critical points between the West and the Atlantic Seaboard, and the abnormal demand for cars at certain times of the year, the fundamental difficulty lay rather in the phenomenal growth of the population and industry throughout the Middle Western and Western States, a growth with which the railroads had failed to keep pace.

(3) The solution of the problem called for the utilization of every practicable means of communication, and particularly of the natural waterway extending from the Atlantic into the very heart of the continent.

That the belief of the Department of Commerce that the further and continued improvement of inland waterways was justified was based at least in part on inadequate rail transportation facilities, and this is clearly demonstrated in the statement of the Director of the Bureau of Foreign and Domestic Commerce in submitting, in June, 1923, to the then Secretary of Commerce, a survey of inland water transportation in the United States. The Director stated that, although interest in the development of internal water routes had never been entirely lacking in recent years, particularly those during and immediately following the World War, there had been a definite focusing of attention on rivers and canals as a means of supplementing the periodically inadequate transport system. He said that it was the opinion in many quarters that the railroads would be unable in the future to increase their capacity to the extent demanded by a rapidly expanding commerce, and that it would be necessary to rely to a greater extent than formerly on water routes for the carriage of both bulk and package freight.

Even after the inauguration of the program for the rehabilitation and expansion of the national railroad system, the notion of the permanent inadequacy of the railroads continued. Expressions of this so-called inadequacy are found in the hearings before the Committee on Interstate and Foreign Commerce, House of Representatives.[17] The so-called Denison Act, which resulted, gave rise to the Court proceedings relating to the Commission's power to prescribe, without a hearing, through routes and joint rates between the railways and inland water carriers.

Later, and in 1930, Mr. Theodore Brent, in an address before the Thirty-Sixth Annual Convention of the Ohio Valley Improvement Association, said that during the last 75 or 80 yr, the American public had given its attention almost exclusively to the development of railroads as a means of transportation. "Since the beginning of the Twentieth Century, however, there had been rude awakenings," said Mr. Brent. Furthermore, one need but read the elaborate and emergency provisions of the Interstate Commerce Law, dealing with congestions and break-downs of service, and remember the conditions that caused their enactment, to realize that the railroads had marked limitations, and that these reflections had brought studious people to renew their pleas for the development of the inland water resources as auxiliary means of transportation.

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[17] H. R. Doc. No. 10710 as amended by H. R. Doc. No. 13512, 70th Cong., 2d Session.

Senator Joseph E. Ransdell, in an address at the same Convention, stated that shortly after the National Rivers and Harbors Congress was formed in 1906, he traveled throughout the United States, everywhere talking waterways, but that his voice was like one in the wilderness; that no interest was taken in waterways, but that gradually, as the result of educational work of the Rivers and Harbors Congress, of the annual conventions which were being held, and of the many speeches made in the United States by the field directors and agents, some public sentiment was developed.

The divergence of views of these two waterways spokesmen is remarkable. Mr. Brent referred to the pleas for inland waterway development as an auxiliary means of transportation, and Senator Ransdell deplored the lack of public sentiment which manifested itself, in some degree, only after many years of intensive educational work. It may be said that the need for new and additional transportation facilities lent great strength to public demand and served largely to secure the appropriations of generous sums of money for the development of the inland waterways of the United States.

It is proper to conclude, therefore, that the periods of greatest interest in inland waterway transportation can be traced generally to those of railway congestion. The great majority of the people, apparently, have been indifferent to the question, except on those occasions in the past when railroad facilities and service have proved inadequate.

No one will deny that the existing railway plant is capable of handling much more tonnage than is available, or in reasonable prospect. The peak traffic of 1929, amounting to more than 447 billion revenue ton-miles, was handled with no car shortage. This compares with the traffic volume of 410 billion revenue ton-miles in 1920 during which year there was a reported maximum shortage of 147 309 cars.

The potential capacity of the railways is only the result of a plan of expansion adopted shortly after the end of Federal control. Early in 1923, railway executives agreed upon this program, looking toward the rehabilitation of facilities, and to an increase in efficiency and economy in operation. The entire program rested upon the basis of investment of large sums of new money in the industry. During the succeeding eight years, until the end of 1930, the program of heavy annual expenditures for additions and betterments to plants was consistently advanced. During that 8-yr period nearly 6.75 billions of dollars were so expended. The program had a twofold purpose: First, to supply the country with adequate rail transportation; and, second, to assure the public that the transportation would be efficiently and economically rendered, from the standpoint both of the user and the producer of the service. The program still continues.

In the report of Federal Co-Ordinator of Transportation, Joseph B. Eastman,[18] of March 10, 1934, the statement is made that from 1920 to 1932 the net total expenditures for new transportation facilities amounted to $19 110 000 000, or substantially the same as the railways' investment in 1920. Included in that sum is an investment of $423 613 000 in pipe lines for the transportation of petroleum products; $12 500 000 000 for highways and streets; and $604 000 000 of Federal money for the improvement of rivers and harbors.

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[18] Senate Doc. No. 152, 73d Cong., 2d Session.

With this great increase in transportation facilities and the fact that the physical expansion of the nation is largely accomplished, it is apparent that there is little if any justification for the continued spending of millions of dollars yearly on the inland waterways as supplements to land transportation facilities.

Despite the conviction that water transportation was necessary to supplement the railways, it has generally been considered by responsible authorities that the cost necessary for the creation and maintenance of waterways is a factor to be given consideration. The report of the Windom-Select Committee on Transportation Routes to the Seaboard, in 1874, stated that, after a most careful consideration of the merits of various proposed improvements, taking into account the cost and other factors, the Committee came to the conclusion that certain water routes would be feasible and advantageous. The following is quoted from the report: "The Committee believes that the water route suggested should constitute free highways of commerce, subject only to such tolls as may be necessary for maintenance and repairs." To quote further from the report: "If, however, Congress shall deem it expedient to require them to provide interest on the cost of construction and the means for ultimate redemption of the principal, the whole improvement will involve only a loan of Government credit."

The preliminary report of the Inland Waterways Commission, in 1908, contains the statement that inland waterway transportation should be restored wherever, upon expert examination, it appeared that a benefit commensurate with the cost would result; and, that the practicability of any waterway project depended not only on local and general demands of commerce, but upon various factors, including physical and economic considerations, entering into or tending to counterbalance the cost.

"The cost of facilities for carrying freight," quoting from the Commission's report, "whether borne by the Federal Government or by private capital is a burden upon the resources of the country. While the tendency of water improvements to lower freight rates is an important element to be considered, the fundamental criterion should be whether a railway or a waterway, whether constructed or improved, will be a profitable investment of capital." In its final report, submitted in 1912, the National Waterways Commission reached the conclusion that if inland waterways cannot afford cheaper transportation than railroads, the sums spent for navigation could be more profitably utilized for increasing railway mileage and efficiency, and that in determining the cost of transportation by water each ton should be charged with its proportion of the cost of improvement even if, in actual practice, this expense is borne by the Government.

The Commission found that a frequent method of demonstrating that water transportation is cheaper than rail is by comparing the average cost per ton-mile on some waterway with that for all the railways in the United States but that no such exact comparisons as these can be made for the reason that the rates compared do not include the same elements of cost. It made the definite statement that any comparison of transportation cost by rail and by water would be of value only when the two rates include similar elements of expense. The Commission held that whether water traffic should be charged with all the expenses incurred for aiding navigation, or whether these should be met, in whole or in part, by taxation, is a question of public policy.

As to differentials in favor of the water carriers, the National Waterways Commission declared that the main problem is whether the cost of transportation by a water route is sufficiently cheaper than that by rail to warrant the granting of a certain differential in its favor.

The Department of Commerce takes the position that, although the Government has authorized the expenditure of many millions of dollars to provide the most improved type of equipment for the experiment it has undertaken, and the benefits of its experience will be available to private capital, it may be that the experiment will not prove remunerative if all factors of cost are taken into consideration, including interest on the sums spent in constructing the channels; and, furthermore, that for services on channels already made navigable, or on canals already built, the returns need cover only current operating expenses and necessary maintenance of channel, terminals, and floating equipment. The Department further stated that, although the expenses for these waterways are borne by the Government, they are a drain on the economic resources of the country.

Recognition that the "savings" in transportation costs accruing to the users of barge service are made up by the taxes paid by the general public, is found in the Annual Report of the Chief of Inland and Coastwise Waterways Service, for 1923, wherein it is stated that there should be a free interchange of freight with the railroads, on the ground that otherwise it would be unfair discrimination "to tax the public at large" for the benefit of a small section of the country.

In an address at Louisville, Ky., on October 23, 1929, President Herbert Hoover, Hon. M. Am. Soc. C. E., stated that "as a general and broad policy, I favor modernizing of every part of our waterways which will show economic justification."

The Board of Engineers for Rivers and Harbors, of the U. S. War Department, in co-operation with the United States Shipping Board, deals with the subject in these words:[19]

> "The primary measure of the economic value of any waterway system is the savings in transportation costs which it affords. Usually, this saving is shown by comparing the water transportation cost with the rates for moving the same goods between the same points by rail. Included in this cost of water transportation should be the charges for interest on capital investment in waterway improvements plus annual cost of operation and maintenance."

In an address before the Thirty-Sixth Annual Convention of the Ohio Valley Improvement Association, in 1930, C. O. Sherrill, M. Am. Soc. C. E., in discussing certain data relating to Ohio River operations, stated that they were based on the real cost of transportation by water, including the interest charges on the costs of the improvement, as well as operation and maintenance costs.

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[19] "Transportation on the Ohio River System" (Interim Report), Library of Congress Card 27–26771.

The United States Chamber of Commerce submitted to the so-called Shannon Committee, investigating Government competition with private enterprise, a number of principles, among which is found No. IV, reading as follows:

> "Grants of public money, whether as gifts or as loans and regardless of the conditions attached, to particular forms of business enterprise, or for the special advantage of particular forms of business enterprise in their competition with other lawful forms, cause unfair damages to private citizens in their lawful pursuits and this damage cannot be offset either by pretended or realized benefits to other citizens or classes of citizens."

The report of the Shannon Committee,[20] states, among other things, that one of the facts which has been demonstrated in the investigation is that transportation by barges of the Inland Waterways Corporation is not cheap transportation. To quote the words of the Committee: "When a shipper ships by railroad, the rate which is charged by the railroad is the total charge for the transportation of material." The report further stated that the Barge Line pays nothing for taxes, makes no attempt to earn a return on its investment, and pays nothing for the use of the stream or to reimburse the Government for its expenditures for construction and maintenance. It also declared that part of the transportation cost represented by the maintenance of roadway, interest on investment, and taxes, which is paid by the shipper when he ships by rail, s borne by the taxpayer when the materials move by the Federal Barge Line, and that this burden which is borne by the taxpayer is much greater than the difference between the railroad freight rate and the Barge Line freight rate.

The National Transportation Committee, in its report of February 13, 1933, reached the conclusion that Government assumption of all or part of the costs of inefficient competing transport as a defense against monopoly is no longer warranted and should be abandoned, and that, as a general principle, inland waterways should bear all costs of amortization, interest; maintenance, and operation of the facilities for their navigation. The report concluded that the studies of the Committee showed no economic benefits to the country commensurate with the money spent on inland waterways; that these costs bore heavily on the taxpayer as a direct burden. The Committee found it difficult to justify the wasteful outpouring of hundreds of millions of dollars for results so barren of economic returns.

It will be recalled that the National Transportation Committee was composed of the Hon. Calvin Coolidge, the Hon. Alfred E. Smith, and Messrs. Bernard M. Baruch, Clark Howell, and Alexander Legge.

In an address before the National Rivers and Harbors Congress, at Washington, D. C., on April 30, 1934, Joseph B. Eastman, Federal Co-Ordinator of Transportation, stated:

> "The point is that the cost of construction of the new waterway and the annual cost of its maintenance are as much a part of the real cost to the public of the new transportation facilities as are the cost of construction and the cost

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[20] H. R. Doc. No. 1985, 72d Cong., 2d Session.

of maintenance of the steamship or barges which operate over the waterway. The water carriers do not bear the expense of constructing or maintaining the waterway under our present practice; nevertheless, it is there—it cannot be escaped and, in one way or another, it is a burden upon the people of the country."

Similar views were expressed by Mr. Eastman in an address before the American Life Convention, at Chicago, on October 10, 1934, and before the Convention of the Mississippi Valley Association, at St. Louis, on November 26, 1934.

The report of the Mississippi Valley Committee of the Public Works Administration, which Committee was composed, according to the Department of the Interior, of a group of the nation's leading scientists and technicians, had the following to say on this point:

> "A system of accounting is also required through which the capital investment, as well as maintenance and operating costs, on any given going navigation project can be currently determined * * *. The nation undoubtedly needs more definite and disinterested data to show the relative costs, taking in all factors, of transporting commodities by inland waterways in comparison with other media of transportation. Factual data are needed on this subject to establish justification or non-justification for either the building or continuation of any waterways program whether it be extensive or limited."

To summarize: (*a*) As recently as ten years ago justification for inland waterway development at public expense rested almost entirely on the supposed inadequacy of the railways; (*b*) the railway plant is now, and has been for many years, capable of handling much more tonnage than is reasonably in sight—much more than was moved in the peak year of 1929, and without car shortage or delay; (*c*) overwhelming expert opinion holds that the cost of providing and maintaining these waterways is a drain on the resources of the country; and (*d*), there has been a tremendous expansion in the transportation facilities of all kinds since 1920, so that to-day there are far more ways to move goods than there are goods to be moved.

In presenting this discussion, the writer does not have in mind so much a rebuttal of the conclusions or theories presented in the Symposium as the need for emphasizing, in this connection, the background and history of waterway development in the United States. In the earlier days, there was an absolute necessity for waterways and in the consideration of new projects it was the very definite view of various authorities, including governmental bodies, that costs incurred in creating and maintaining inland waterways were a fixed burden upon the people—one that must be borne by them through taxation.

If the writer has interpreted Major Putnam's theory correctly (especially as given in the text following Table 5) he has not given much weight to this fact. In common with all advocates of further inland waterway developments he ignores entirely the undisputed fact that land transportation (that is, rail, motor, or pipe line) is now fully capable of handling all the commerce of this country. This is a condition, of course, that did not exist even as late as 75 yr ago. In other words, the reasons that prompted and justified the creation of waterways at public expense, to move the Nation's goods, have disappeared completely.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

DESIGN OF REINFORCED CONCRETE IN TORSION

Discussion

BY MESSRS. BRUCE G. JOHNSTON, AND DEAN PEABODY, JR.

BRUCE G. JOHNSTON,[10] ASSOC. M. AM. SOC. C. E. (by letter).[10a]—Nearly all structural frames of the so-called "rigid" type are space frames; few are analyzed as such. In many cases there is little interaction between the various planes of the structure, and the analysis may be satisfactorily divided into separate plane problems. In frames of the type represented by the author, however, the three-dimensional analysis is essential because of the torsion introduced. In actual design direct torsional loads should be avoided wherever practicable.

Three general types of deflection occur in a member of a "rigid" frame—it can be bent, twisted, or deformed longitudinally. A complete analysis of such a statically indeterminate structure would include the effects of these three types of deflection. Only in "hybrid"[11] structures, or in structures of unusual proportions, does it become necessary to consider the effect on the analysis of the secondary interaction of two, or possibly all three, types of deflection. The effect on the design, or total deflection, may be another matter. The author has presented, in an orderly manner, the application of the Cross method of moment distribution to the analysis of the combined bending and twisting problem.

The writer wishes to point out certain variations which occur in extending the author's method to beams of non-uniform cross-section, or to cases in which structural steel beams are used instead of reinforced concrete. It is first necessary to discuss the basic torsion constant, denoted by the symbol, T, in the paper. In 1855, Saint Venant presented the solution of the torsion of a prism of rectangular cross-section.[12] In Fig. 7 the writer has shown the deviation of Equation (2) for the torsion constant from the solution of Saint Venant. It is

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NOTE.—The paper by Paul Andersen, Assoc. M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1937, by Messrs. C. W. Deans, and L. E. Grinter.

[10] Instr. in Civ. Eng., Columbia Univ., New York, N. Y.

[10a] Received by the Secretary December 1, 1937.

[11] See "The Relation of Analysis to Structural Design," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1931.

[12] "Torsion des Prismes," by Saint-Venant, Paris, 1855.

seen that this formula is most nearly correct for the square section, and that an error of only 6% or less is introduced for ratios of long to short side as great as 4. If the ratio of the long to the short side is 1.6, or greater, as is the usual case for beams designed primarily for bending, another simple formula gives a closer approximation than Equation (2):

$$T = \frac{b^3\,d}{3}\left(1 - 0.63\,\frac{b}{d}\right) \qquad (16)$$

The deviation of Equation (16) from the correct solution is also shown in Fig. 7. The greatest error is for the square section (about 12.3%) but it decreases rapidly to a negligible quantity as the ratio of side lengths increases. The writer introduces Equation (16) principally because it is in a form which is easily integrated for determining the torsional distribution factors for the important case of the haunched beam. The use of the exact solution is impractical since it is in the form of a series and, although Equation (2) can be integrated, it yields a very complicated expression.

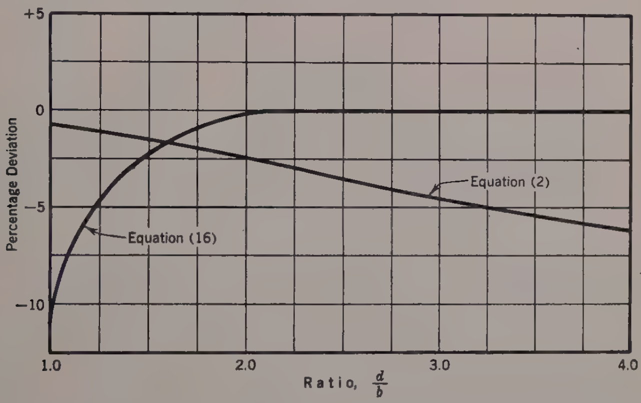

FIG. 7.—DEVIATION OF APPROXIMATE FORMULAS FOR TORSION FACTOR OF RECTANGULAR SECTION FROM EXACT SOLUTION

If a torsional moment is applied at any point of a straight beam, the author in Equations (3*a*) and (3*b*), has shown that the distribution to each end will be inversely proportional to the length of the segment. Such will not be the case for a beam haunched at one or both ends, and this case will now be studied for the straight haunch. The additional notation required is given in Fig. 8, in which d_1 = least depth of haunch; d_2 = depth at wall; and, l_H = length of haunch. The angle of twist caused by a constant torsional moment, M, in an elemental length of haunched section is given by,

$$d\phi = \frac{M}{T\,E_s}\,dx \qquad (17)$$

For the total length of the haunched element, therefore, the total angular twist equals,

$$\phi = \frac{M}{E_s}\int_0^{l_H}\frac{dx}{T} \qquad (18)$$

Assume that the breadth, b, is constant and that the torsion factor, T, at any point in the haunch is given by Equation (16). The depth, d, of the haunch at any point will be:

$$d = d_1 + \left(\frac{d_2 - d_1}{l_H} \right) x \quad \text{.....................(19)}$$

in which x is the distance from the shallow end of the haunch. Making this substitution in Equations (16) and (18) the integration gives, for the average twist per unit length:

$$\frac{\phi}{L_H} = \frac{3\,M}{b^3\,E_s\,(d_2 - d_1)} \log \left(\frac{d_2 - 0.63\,b}{d_1 - 0.63\,b} \right) \quad \text{.............(20)}$$

Designating by T_H the effective or average torsion constant over the length of the haunch,

$$T_H = \frac{M\,L_H}{\phi\,E_s} = \frac{b^3\,(d_2 - d_1)}{\log \left(\dfrac{d_2 - 0.63\,b}{d_1 - 0.63\,b} \right)} \quad \text{.................(21)}$$

or,

$$T_H = C_H\,d_1^4 \quad \text{.........................(22)}$$

in which C_H is a factor depending on two ratios, $\frac{d_1}{b}$ and $\frac{d_2}{d_1}$, with the restriction that $\frac{d_1}{b} > 1.2 \frac{d_2}{d_1} > 1.0$. Fig. 8 gives values of C_H for different ratios of $\frac{d_1}{b}$ and $\frac{d_2}{d_1}$.

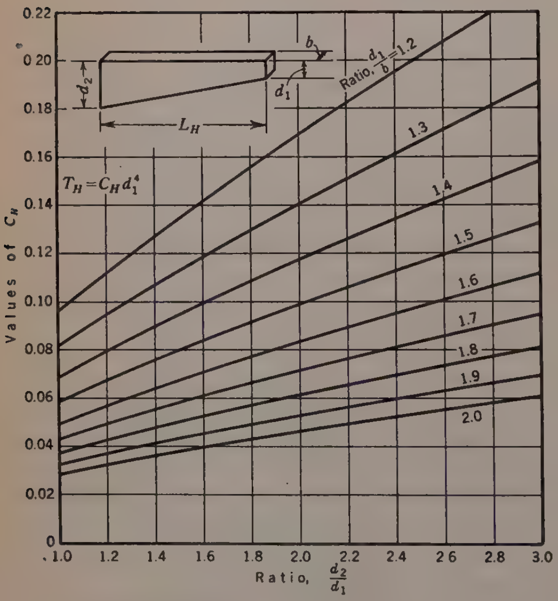

FIG. 8.—TORSION FACTOR FOR HAUNCH

The total angular twist between any point on the uniform section of a haunched beam and the end will be:

$$\phi = \frac{M}{E_s} \left(\frac{L_H}{T_H} + \frac{L_u}{T_u} \right) \quad \text{..(23)}$$

in which L_u and T_u are the length and torsion factors for the uniform segment.

The torsional stiffness for the combined segment will be,

$$K = \frac{1}{\dfrac{L_H}{T_H} + \dfrac{L_u}{T_u}} \quad \text{.........................(24)}$$

and the torsional moment will be distributed to either end in proportion to the torsional stiffness of that particular segment.

The application to an actual case will now be illustrated. Assume a concrete beam 12 in. wide and 20 ft long unsymmetrically haunched as shown in Fig. 9, with a torsional moment, M, applied 5 ft from the left end. For the haunch at the left end: $\frac{d_1}{b} = \frac{18}{12} = 1.5$; $\frac{d_2}{d_1} = \frac{36}{18} = 2.0$; and (see Equation (22)), $T_H = (0.985)\,(18)^4 = 10\,330$ in.^{4} Similarly, for the right haunch, $T_H = 7\,660$ in.^{4} For the uniform section, by Equation (16), $T_u = \frac{18 \times 12^3}{3}\left(1 - \frac{0.63 \times 12}{18}\right) = 6\,010$ in.^{4} Substituting in Equation (24), the torsional stiffnesses of the two parts of the beam are obtained: At the left, $K_L = \dfrac{1}{\dfrac{48}{10\,330} + \dfrac{12}{6\,010}} = 150.8$; and at the right, $K_R = \dfrac{1}{\dfrac{144}{6\,010} + \dfrac{36}{7\,660}} = 34.8$. From these values are obtained the distribution factors to give the fixed-end torsional moments, M_f, at the left and at the right ends: At the left end $M_f = \frac{150.8}{150.8 + 34.8} M = 0.812\,M$; and at the right end, $M_f = \frac{34.8}{150.8 + 34.8} M = 0.188\,M$, instead of 0.75 and 0.25 for a beam of uniform cross-section.

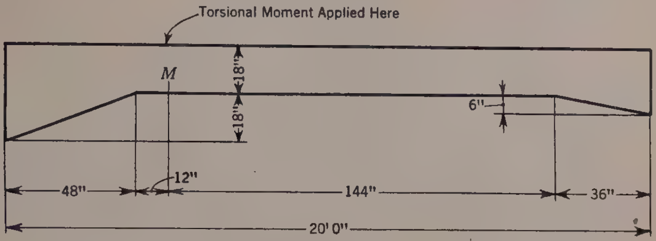

FIG. 9

The torsional stiffness of the entire beam may be obtained by similar computations or directly from K_L and K_R as follows:

$$K = \frac{K_L\,K_R}{K_L + K_R} = \frac{150.8 \times 34.8}{150.8 + 34.8} = 28.3$$

The torsional carry-over factor will always be 1 regardless of haunches, but the bending-moment carry-over and distribution factors will need to be computed by use of the column analogy, from the generalized slope deflection equations, or by reference to previously published diagrams. The analysis then proceeds in a manner analogous to that of the author.

In analyzing similar structures in which the members are steel I-beams instead of concrete, another factor affects the torsional distribution constants appreciably. The usual torsion factor is calculated on the assumptions that end sections are free to warp.[13] If the end sections are restrained from warping, there is a localized increase in torsional stiffness. This increase is insignificant in the case of a rectangular section, but in the case of the I-beam the prevention of the warping of the section at a fixed end has a considerable effect. P. W. Werner, Assoc. M. Am. Soc. C. E., has discussed[14] the general problem of a steel beam fixed at each end and subjected to a twisting moment at any point, which gives, directly, the torsional distribution factors for such a case.

Since these formulas are developed elsewhere, and require considerable routine calculations, only the results of an actual case will be given here. Assume an 8 by 8-in. wide-flanged beam at 67 lb per ft, 16 ft long, fixed at each end, with torsional moment, M, applied at the quarter-point. It is found that the distribution to the shorter segment is 0.818 M and to the longer segment, 0.182 M. The net effect, therefore, is similar to the haunching of a concrete beam, although for an entirely different reason.

The writer has discussed two cases in which the torsional distribution factors cannot be treated as simply as in the case of the rectangular beam of uniform cross-section which was used in the author's example. In some cases, the resulting variations will be negligible, but in other instances they will appreciably affect the analysis.

DEAN PEABODY, JR.,[15] M. AM. SOC. C. E. (by letter).[15a]—This paper marks a distinct advance into one of the unexplored areas of reinforced concrete design. The discussion of torsional stiffness and the distribution of the torsional moment in a space frame is of value to the designer.

For the stress analysis at a given section the author makes torsional computations independent of bending computations for fiber stress, shear stress, or diagonal tension. His design equation for the square section is based on the same line of reasoning used for diagonal tension computations and thus makes for consistent design. Equation (13) gives the useful result, $\frac{A_s}{p}$, which enables one to choose the spiral reinforcement immediately.

For a square section the maximum shear stresses occur at the center of each face. The illustrative computations for the problem of Fig. 3 apply to Particle A of a square section (Fig. 10), and the torsion reinforcement should be supplied at Point A for this section. At Particle C on the opposite side the shear stress, v_B, due to the vertical shear force, V, and the torsional shear stress, v_T, due to the torsion couple, M_T, are opposite in direction. For the beam of Fig. 3 the net shear stress at Point C equals 3 lb per sq in. (upward). Therefore, at Point C, no diagonal tension or torsion steel is needed but it is impractical, of course, for the steel to be omitted at C and supplied at A. Particle D has the same stress, v_T, as at A and Fig. 3 shows that an equal or greater percentage of spiral steel is supplied. Due to the slope of the spirals less torsion reinforcement may be supplied at Particle *B* than at Particle *A* but, since there is no shear stress, v_B, the full allowable shear stress of 90 lb per sq in. is permissible, although the spiral steel at *A* was computed for an allowable stress of 46 lb per sq in. The reduced number of spirals at *B* will be safe, except at sections where the shear force, *V*, is small. Maximum torsion usually occurs near the support where the shear forces are large. It would appear, then, that the torsional design for square sections is usually adequate when only the particle with maximum total shear stress is considered.

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[13] "Structural Beams in Torsion," by Inge Lyse, M. Am. Soc. C. E., and Bruce G. Johnston, Jun. Am. Soc. C. E. *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 857.

[14] *Loc. cit.*, p. 912.

[15] Assoc. Prof. of Structural Design, Mass. Inst. Tech., Cambridge, Mass.

[15a] Received by the Secretary December 8, 1937.

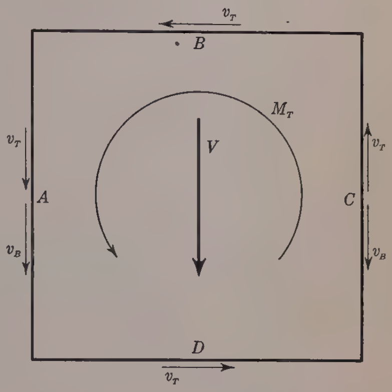

FIG. 10

No recommendation has been made for the allowable shear stresses, but the writer assumes that the customary values for diagonal tension will be used for the total allowable shear stress at Particle *A*.

The author apparently uses hoops or ties for the spiral steel. It is difficult to wire inclined hoops securely, and the writer believes the added cost of welding these hoops to the longitudinal steel is justified.

Square cross-sections will undoubtedly be chosen for members with large torsional moments, as the author states, but many wall beams and spandrel girders of rectangular shape are subjected to torsional moments. Still more are poured integrally with the slab and the design section is angle or tee-shaped. An authoritative discussion of the design of such sections must wait a comprehensive program of tests and inspection of existing structures.[15b]

[15b] Correction for *Transactions:* In Table 1(*a*), change the fixed-end moments to 667 and − 337 instead of 167 and − 83, respectively.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

SOLUTION OF TRANSMISSION PROBLEMS OF A WATER SYSTEM

Discussion

BY MESSRS. LYNN PERRY, CHARLES M. MOWER, JR., AND THOMAS R. CAMP

LYNN PERRY,[5] M. AM. SOC. C. E. (by letter).[5a]—The thorough manner in which the author has presented this subject certainly should be appreciated by the membership of this Society. For many years, the writer has solved problems of the nature herein presented by the time-worn analytic method. On many occasions, during this time, he has urged graphic methods for the solution of various types of problems. Such methods eliminate many errors, save time, and give a result sufficiently accurate for all practical purposes.

It appears that the upper parts of Figs. 2 and 3 show curves of parabolic form on rectilinear cross-section paper. If so, these curves would be straight lines when plotted on logarithmic cross-section paper and it seems as if some advantage might be gained from its use.

Frequently an old municipality having an inadequate water supply enlarges its supply by acquiring another, more abundant, at a higher elevation, and from an entirely new direction and source. A larger water main is likely to be installed, leading the new supply to the existing distributing system, sometimes under the direction of an engineer. In most such cases that have been brought to the writer's attention, the new line supplies the ordinary demand and overflows into the old intake. Only at periods of excessive demand is the old supply likely to be drawn down into the distributing system.

CHARLES M. MOWER,[6] JR., M. AM. SOC. C. E. (by letter).[6a]—The graphical approach to the problems of flow and loss of head in the elements of transmission and distribution systems is a method of analysis readily adaptable to a variety

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NOTE.—The paper by Ellwood H. Aldrich, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[5] Asst. Prof., Hydr. and San. Eng., Lafayette Coll., Easton, Pa.

[5a] Received by the Secretary November 4, 1937.

[6] Asst. Engr., The Pitometer Co., New York, N. Y.

[6a] Received by the Secretary November 16, 1937.

of problems. Mr. Aldrich has shown how the fundamental Freeman curves may be expanded to cope with the more complicated problems in water distribution which have too often been solved by guess.

With this, as with other office methods, however, there is the danger that the mathematical accuracy of the theory may hide inaccuracies in fundamental assumptions. Coefficients of carrying capacity, rates of flow, and rates of consumption, are often assumed, and the behavior of a pipe system is then analyzed by a flawless process of reasoning. However, the end result is in error because quantities that could, and should, be measured in the field were assumed.

Although one of the most usual assumptions is that $C = 100$ in the Williams-Hazen formula, a few loss-of-head tests on representative mains in a distribution system will show how far this assumption may be from the truth. Similarly, assumptions of flow, take-offs, or consumption may be too far from the truth to be of value, because these quantities are influenced by too many unknown factors. It is essential, therefore, to make actual field measurements instead of assumptions if the design of additions to a distribution system is to have a sound foundation.

The graphical method so clearly presented by Mr. Aldrich is of great value in determining how new mains and new storage facilities will affect a distribution or transmission system; but the analysis of flows and carrying capacities in the existing system should first be made by instrumental methods.

THOMAS R. CAMP,[7] M. AM. SOC. C. E. (by letter).[7a]—The adaptation of the graphical method of determining "equivalent pipes" to the solution of flow-distribution problems in complex networks, as presented in this paper, is not new. W. E. Howland,[8] Assoc. M. Am. Soc. C. E., has demonstrated that the Freeman method may be used for cross-overs and for simple networks similar to those in Fig. 6. The author seems to be the first, however, to demonstrate that accurate solutions may be obtained by this method for somewhat complicated water-distribution systems.

Three methods have now been shown to be practical for the analysis of flow in networks of a similar order of complexity to that of the network shown in Fig. 7. These methods are as follows:

(1) The "electric network analyzer" method[9];
(2) The Hardy Cross method[10]; and,
(3) The Freeman graphical method.

It should be pointed out that all these methods are "trial-and-error" methods of solving a large number of simultaneous equations. These equations are of three types, arising from three laws which may be stated as follows: (*a*) The algebraic sum of the rates of discharge toward any junction point is zero; (*b*) the algebraic sum of the head losses around any closed circuit is zero; and (*c*) for any pipe or system of pipes the head loss is directly proportional to some power of the discharge. The number of equations involved is so great that their direct solution algebraically is impracticable, even for simple networks. For example, in the single cross-over net of Fig. 5(*b*), eleven simultaneous equations are involved, five of which are exponential and the remainder linear.

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[7] Associate Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[7a] Received by the Secretary November 23, 1937.

[8] "Expansion of the Freeman Method for the Solution of Pipe Flow Problems," by W. E. Howland, *Journal*, New England Water Works Assoc., December, 1934, p. 408.

[9] "Hydraulic Analysis of Water Distribution Systems by Means of an Electric Network Analyzer," by Thomas R. Camp, M. Am. Soc. C. E., and H. L. Hazen, *Journal*, New England Water Works Assoc., December, 1934, p. 383.

[10] "Analysis of Flow in Networks of Conduits or Conductors," by Hardy Cross, M. Am. Soc. C. E., *Bulletin No. 286*, Univ. of Illinois Eng. Experiment Station, November 13, 1936.

Methods (1) and (2) may be classified as "controlled" methods; that is, the errors produced by successive trials become smaller. The Freeman method (Method (3)), however, appears not to be controlled. The solution is obtained by "juggling" the position of the curves that represent the elements in the system. The writer is at a loss to understand how one is to know in which direction to shift a curve when a large number of curves are involved. In the simple net of Fig. 6, consisting of two bays in each direction, four sets of curves must be adjusted. The distribution system represented by Fig. 7, although it involves a large number of elements, is less difficult to solve by the graphical method than the net of Fig. 6, because no stage of the solution of this system involves nets of more than one cross-over. It appears to the writer that a grid system consisting of numerous bays in each direction would not yield to solution by the graphical method except by chance.

In Methods (2) and (3), the process consists of assuming a division of the flow at junctions and computing the corresponding head losses (or, conversely, of assuming the head losses and computing the corresponding flows). If the corresponding head losses do not conform to Law (*b*), an adjustment in the assumed flows is made and the process is repeated until the heads do conform reasonably well with this "head-loss summation" law. In Method (1), both Laws (*a*) and (*b*) are taken care of automatically. Since Ohm's electrical resistance law is not analogous to the hydraulic resistance law, adjustments must be made in the resistors representing the elements until the head-discharge (that is, voltage-current) relation conforms to Law (*c*). The number of corrections or adjustments required by both Method (1) and Method (2) will seldom exceed three. In both methods, with each adjustment, the errors in the assumed flows converge toward zero for nearly all the elements in a system. In the graphical method (3), one shift in the position of all the curves corresponds with one adjustment in the other two methods. A shift in the position of the curves, of course, is much more readily accomplished than is an adjustment by the other two methods; but a great many more than three shifts may be necessary and there appears to be no means of knowing that the shifts are being made in the right direction for a solution.

In comparing the three methods, the water-works engineer is interested in the relative lengths of time and amounts of equipment required. In the electric analyzer method, each element is represented by a resistance box. In preparing to solve a problem, the resistors must be connected to one another in a pattern similar to the water distribution system. "Put-ins" and "take-outs" are represented by leads from and to a battery or other source of electricity. Flow is represented by electric current and head loss by voltage drop. Hence, a scale ratio must be selected for conversion between the electric and hydraulic systems, and the proper range of resistance estimated for each element before the actual solution of a problem can be started. This preliminary work in Method (1) corresponds with the preparation of the curves representing the elements in Method (3). The analogous work in Method (2) consists only of computing a constant for each element which represents the relation of flow to lost head, all of which may be done on a slide-rule.

The solution of a problem by the electric analyzer method (1) consists of reading voltage and current for each element, and of adjusting the resistance so that the head (voltage) bears the proper relation with discharge (current). The readings and adjustments are quickly made. A solution by the Hardy Cross method (2) consists of assuming a division of flow at each junction, computing the corresponding heads, and then correcting the flows by an amount computed from the observed error in the heads. No equipment other than a slide-rule is necessary.

The time required to set up the electric analyzer for the problem represented by Fig. 7 was about 10 hr, and about 8 hr was required for the solution. The same problem solved completely by Method (2) occupied the writer for about 6 hr. J. J. Doland,[11] M. Am. Soc. C. E., states that in his solution of the same problem by the Hardy Cross method (2) only about 4 hr was required. It will be helpful if the author would give an estimate of the time required to solve this problem by the graphical method (3).

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[11] *Engineering News-Record,* October 1, 1936, p. 475.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

MULTIPLE-STAGE SEWAGE SLUDGE DIGESTION

Discussion

BY EDWARD W. MOORE, ESQ.

EDWARD W. MOORE,[12] ESQ. (by letter).[12a]—In that it not only treats of an interesting development in the technique of sludge digestion, but also pioneers in the application of certain principles of physical chemistry to the study of the behavior of digesting sewage sludge, this paper appeals to the writer as a particularly valuable contribution to the literature of sludge digestion. The value of the concepts and methods of physical chemistry to the practising engineer is as yet recognized too infrequently.

Another physio-chemical concept which yields fruitful results when applied to the decomposition of organic matter both under aerobic and anaerobic conditions is that of reaction velocity. The course of the decomposition of organic matter under both aerobic and anaerobic conditions can be formulated approximately as a uni-molecular reaction, the rate of which is given by a constant, k, known as the reaction velocity constant. For aerobic decomposition at 20° C, k has a value of 0.1, as shown by the work of Professor Earl B. Phelps, Victor L. Streeter, Jun. Am. Soc. C. E., E. J. Theriault, and others. The writer, in collaboration with G. M. Fair, M. Am. Soc. C. E., has been engaged in collecting data from which the value of k for anaerobic decomposition at various temperatures may be computed. For 85° F the value of k obtained from the data thus far collected, is 0.066 for digestion of sludge seeded with material previously digested at this temperature. From the bottle experiments made by the authors, the ultimate total gas production of the Los Angeles sludge may be assumed to be about 625 cu cm per gram of volatile matter, and the volume produced (say, in 10 days' detention time) is 485 cu cm. From these data and assumptions a value of $k = 0.065$ is indicated.

NOTE.—The paper by A. M. Rawn, M. Am. Soc. C. E., A. Perry Banta, Assoc. M. Am. Soc. C. E., and Richard Pomeroy, Esq., was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 16, 1936, and published in November, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[12] Instructor in San. Chemistry, Harvard Graduate School of Eng., Cambridge, Mass.

[12a] Received by the Secretary December 21, 1937.

It is of interest to the writer to note that, in sludge digestion studies, two questions perpetually recur, namely, the relationship between the weight of the gas produced and the weight of the volatile matter destroyed, and the evolution or absorption of heat by the reactions of digestion. The authors find that the weight of the gas produced is 7% more than the weight of the volatile matter destroyed, despite the fact that their total gas production per unit of volatile matter added to the tanks is very low (485 cu cm per gram of volatile matter added). The work of A. M. Buswell and his associates,[13] and that of Professor Fair, and the writer[14] has indicated that the weight of the gas may exceed the weight of the volatile matter destroyed by as much as 28 per cent. This increase in the weight of the decomposition products over the weight of the original material is explained by Buswell as due to the fact that water enters into the reactions by which the gas is formed. Willem Rudolfs, M. Am. Soc. C. E.,[15] has held that the weight of the gas does not exceed that of the volatile matter destroyed.

Sludge digestion consists of a complex of reactions, some of which are exothermic (heat-evolving) and others endothermic (heat-absorbing). The over-all heat effect (that is, the actually observed heat absorption or evolution during the course of digestion) is the sum of the effects of the individual reactions. The work of Messrs. Keefer and Kratz showing the over-all process to be endothermic was based on a few experiments in which gas production per unit of volatile matter was relatively high. Prior work by Professor Fair, and the writer had indicated on theoretical grounds that since the endothermic reactions of the digestion process were concerned with dissolution and evolution of the gas, the over-all process would be expected to be endothermic when gas production per unit of volatile matter added was high, and exothermic when it was low. The results reported in the paper would be expected to be exothermic rather than endothermic on account of the low gas yield.

Despite the layer of lime scale formed on the heating pipes of the digestion tank, the value of the coefficient of heat transfer from water to sludge is about 13 Btu per hr, per sq ft of pipe surface per degree of temperature differential, which is as good as most values reported in practice. It would be of interest to know the average temperature of the hot water entering the coils, since this is believed to control the degree of incrustation of the pipes. It would also be interesting to calculate the heat transfer coefficient from the tank to its surroundings from the data following Table 7 and the surface area of the tank, since the matter of the rate of heat loss from digestion tanks still requires considerable study.

Some sludge digestion studies have indicated the presence of hydrogen and carbon monoxide in the gas produced. No dependable evidence of the presence of these gases in significant quantities has been obtained in the Harvard University laboratories, although hundreds of analyses have been made. The very delicate methods used by the authors show only traces of these gases, and, as the authors state, do not definitely establish their presence. It is possible that much of the hydrogen and carbon monoxide reported in sludge digestion studies is due to errors in the determination of methane by combustion. The determinations of hydrogen sulfide are of practical as well as theoretical interest, since the corrosiveness of the combustion products of the gas is determined largely by the quantity of hydrogen sulfide it contains. The writer believes that some of the sulfur present in the gas evolved from digesting sewage sludge is present in the form of organic homologues of hydrogen sulfide, particularly in the case of thermophilic digestion. Some of the foul odors carried by these gases cannot be accounted for by the presence of such small percentages of hydrogen sulfide.

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[13] *Sewage Works Journal, No. 4,* p. 454 (1932).

[14] *Loc. cit., No. 4,* p. 756 (1932).

[15] *Loc. cit., No. 4,* p. 444 (1932).

The fact that none of the nitrogen contained in the digesting sludge appears in the gas as elemental nitrogen is confirmed also by the results of the Harvard studies on sludge digestion. This conclusion, however, cannot be extended to the decomposition of other organic deposits, such as those found on the bottoms of lakes and streams. Studies of the decomposition of such deposits by Allgeier, Peterson, Juday, and Birge[16] and by Professor Fair and the writer,[17] indicate the production of substantial quantities of nitrogen gas. Since these deposits are similar to partly digested sludges, the radical difference in the behavior of the two materials has not as yet been explained.

In Table 9, the analyses of the sludge liquor for phosphate and sulfate appear to be confused, and the text associated with the table refers to the second test period as being both in October and September. The disappearance of phosphate from the sludge liquor during the second test period might possibly be accounted for by absorption of the phosphate by organisms carried down with the solid matter of the sludge. Bio-chemists often use the decrease in soluble phosphate as a measure of the growth of organisms in a medium.

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[16] *International Revue der Gesamten Hydrobiologie und Hydrographie,* Vol. 26, p. 444 (1931–32).

[17] *Civil Engineering,* December, 1937, p. 846.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

PRACTICAL APPLICATION OF SOIL MECHANICS
A SYMPOSIUM

Discussion

BY MESSRS. RICHARDS M. STROHL, WILLIAM P. CREAGER, JACOB FELD, AND Y. L. CHANG

RICHARDS M. STROHL,[53] M. AM. SOC. C. E. (by letter).[53a]—The application of soil mechanics to levee design, as described by Mr. Buchanan, is thoroughly helpful to those interested in levee problems.

In discussing the importance of seepage through the levee, reference is made to Fig. 10, in which the path of percolation is $\sqrt{29^2 + 217^2} \times 30.48 = 6\,675$ cm long. The time required for water to seep this distance at the rate, $K = 5 \times 10^6$ cm per sec, will be 15 450 days. As Mr. Buchanan states that the maximum high-water stages do not extend over a period of more than 20 days, it would appear that seepage could not be developed through such a levee during any flood of record.

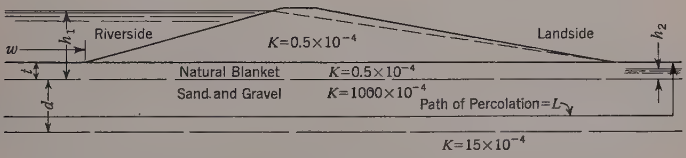

FIG. 61

Necessity for constructing a levee on a foundation similar to that shown in Fig. 61 frequently arises. Water from the river side readily finds access to the pervious substratum. The quantity of percolation is found approximately by

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NOTE.—This Symposium was presented at the meeting of the Soils Mechanics and Foundations Division, at Boston, Mass., October 7, 1937, and published in September, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1937, by the members of the Committee of the Society on Earths and Foundations; November, 1937, by Messrs. S. C. Hollister, T. T. Knappen, and L. F. Harza; and December, 1937, by Edward Adams Richardson, Esq.

[53] Engr., U. S. Engr. Office, Memphis, Tenn.

[53a] Received by the Secretary November 24, 1937.

means of Professor Terzaghi's expression[54]:

$$Q = \frac{K\,h}{0.88 + \frac{L}{d}} \qquad \text{(17a)}$$

which is the equivalent of Darcy's formula (published in Paris, 1858):

$$Q = K\,A\,i = K\,A\,\frac{(h_1 - h_2)}{L} \qquad \text{(17b)}$$

The height to which water would rise above the pervious stratum on the land side for conditions of free flow is,

$$h_2 = h_1 - \frac{h\,L}{0.88\,d + L} \qquad \text{(18)}$$

whereas, for static conditions, the uplift on the natural blanket is practically equal to h_1. Where it is possible to permit nominal flooding on the land side of the levee, the flow of water upward through the blanket can be controlled. By permitting discharge on the surface of the blanket, h_2 becomes equal to t, and the flow at the ground surface becomes,

$$Q = k\,d\,\frac{(h_1 - t)}{L} \qquad \text{(19)}$$

per linear foot of the levee. This quantity will be rather small, and it appears more practical to control this flow than to try to prevent it.

The width, w, of the blanket on the river side of the levee is of importance in reducing percolation under the levee, Q being inversely proportional to L. It is the practice in levee building to secure the embankment material from borrow-pits in front of the levee, leaving a berm with a minimum width of 40 ft. Where the depth of the borrow-pit materially reduces the thickness, t, of the material blanket, it appears worth while to increase the berm width.

William P. Creager,[55] M. Am. Soc. C. E. (by letter).[55a]—The theories of soil mechanics, as applied to the design of dams, are frequently quite complex. Mr. Buchanan has made an excellent summary of the fundamental principles involved; but, on account of the limitations in space for a paper of this kind, he has not been able to do more than give a hint of some of the extent of the procedures necessary for many cases. The only criticism which the writer can give regarding this paper is the lack of indication that, in some instances, the procedure involved is not as simple and direct as shown by the examples given. As examples of lack of simplicity in designs the writer gives two cases as follows:

(1) The design of the slopes of any dam is not complete without the inclusion of seepage forces. This has been mentioned only briefly in this paper, and the

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[54] Unpublished analysis of seepage, by Charles Terzaghi, M. Am. Soc. C. E.

[55] Cons. Engr., Buffalo, N. Y.

[55a] Received by the Secretary November 10 and 29, 1937.

impression has been given that the consideration of such forces must wait on "further research." A number of excellent papers, involving the consideration of such forces, were presented at the Second Congress on Large Dams held in Washington, D. C., in 1936. Mr. Buchanan's example is lacking in respect to this force.

(2) The stability of the foundations, as used in the example (see heading, "The Design of a Levee Unit"), is correct without modification, on the assumption that the foundation drains quickly. If, due to insufficient drainage time, the foundation is not completely consolidated, some of the weight is carried by the water in the foundation and part of the ultimate frictional resistance to shear is lacking. Therefore, the measured shearing strength as given by Item (*c*) of the example should be determined experimentally on the basis of the correct amount of partial consolidation when the dam is first completed.

Attention is also called to the following comments on the paper by Mr. Hough, regarding the necessary modification to foundation stability equations due to this condition.

Mr. Hough has presented an excellent paper which, among other interesting items, describes a proposed method of construction that is unique in the annals of dams. He has mentioned one limitation to photo-elastic model studies. The writer would like to mention another limitation.

In the case of a saturated foundation which has consolidated only partly during the construction of the dam, a part of the weight of the dam is carried by a direct loading of the soil of the foundation and the remainder is carried by the hydrostatic pressure of the pore water in the foundation. The hydrostatic pressure in the foundation, which is built up by, and equals, that part of the loading carried by the water in the foundation, acts in all directions, of course, and forces the surplus water to drain off in all directions with velocities proportional to the lengths of the various paths of percolation to the nearest point of relief.

Fig. 62(*a*) represents a pervious sand dam resting on a layer of silty clay which, in turn, is underlaid by pervious sand. The water in the silty clay layer drains almost directly upward and downward. Decrease the perviousness of the dam and the underlying sand layer, and some of the water in the silty clay layer begins to drain sidewise as well as upward and downward, as shown in Fig. 62(*b*). Should the dam and the underlying layer be absolutely impervious, all the drainage would be horizontal and diagonally upward to the valley bottom adjacent to the dam, as shown in Fig. 62(*c*).

Fig. 62(*d*) is an actual case of a concrete dam resting on a deep, lightly consolidated shale bed. Between the dam and the shale is to be placed an impervious coating of asphaltum to prevent drying out and slaking of the shale during construction. The drainage is downward at the center and horizontal or inclined upward at the edges of the base.

It will be noticed that, in all practical cases, some part of the drainage is horizontal and upward toward the sides of the foundation in the direction of failure if the foundation should prove weak.

The entire hydrostatic pressure in the foundation is balanced by the friction of the water in the direction of flow, the pressure decreasing to zero at the outlet of the seepage. Thus, the loading carried by the hydrostatic pressure is transferred to the soil, not as a vertical loading which can be simulated by the photo-elastic method, but in an infinite number of directions which cannot be simulated by present methods.

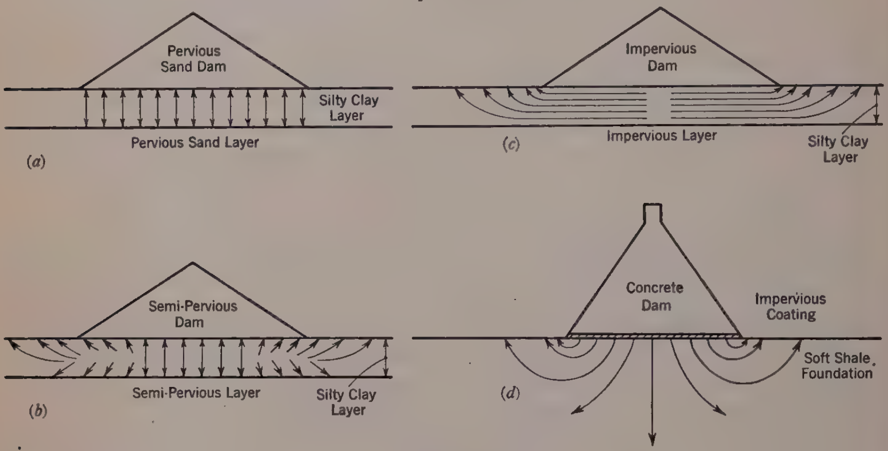

FIG. 62

Very obviously, a greater loading can be placed on a soil in a vertical downward direction than in a horizontal or diagonally upward direction, in the direction of failure. For this reason, the photo-elastic method and, incidentally, the commonly accepted theories and formulas are not applicable for the condition of partial consolidation of the foundation. Unfortunately, the writer does not have at present a solution for this special case. He presents it for the purpose of stimulating interest in the problem and as a thought to be considered in the interpretation of usual methods.

JACOB FELD,[56] M. AM. SOC. C. E. (by letter).[56a]—Seldom in engineering work is there an opportunity for full-scale research such as is available to the U. S. Waterways Experiment Station. Results cannot be expected immediately, but only from a continuous study over many years. The main problems are the determination of settlement and seepage, using variations of material combinations, not only at the end of and during construction operations, but the effect of age, receding flood levels, precipitation intensities, and unusually high and low temperatures. The problems are not simple, yet because of the fairly limited number of soils available for use, solutions covering a considerable range of time and weather conditions can be expected from a program of not more than 10 yr.

It would be sound engineering as well as sound economics to design and contract for a series of test sections, using controlled soil mixtures and construction methods. In such a test group, attempts should be made to study the use of colloidal material from the river bottom as a temporary surface seal, to give added resistance to seepage until such time as the settlement of the fill reduces the seepage rate through the full section.

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[56] Cons. Engr., New York, N. Y.

[56a] Received by the Secretary November 29, 1937.

Systematic laboratory research can be of great aid in lessening the cost of full-scale tests. As far as seepage is concerned, such work as that of B. A. Bakhmeteff, M. Am. Soc. C. E., and N. V. Feodoroff[57] gives a clear picture of the problem as well as a mathematical solution of a number of conditions encountered in levee and dam design.

Little has been learned of the impervious region at the contact zone between layers of different grain size and shape. The contact lens or skin is denser, less pervious, and of greater bearing value than either of the adjacent materials. Such zones are often found in river beds, just below the shifting layer of sediment. Below this may be much more pervious strata. Cutting through this zone, whether on the land side or on the river side of a levee, in a borrow-pit, exposes the subsoil under the levee to excessive pressure. If the borrow-pit is on the land side, the added weight of the levee and the opening of the porous strata permit a ready squeezing out of the subsoil moisture with consequent rapid settlement and slumping of the fill. If the borrow-pit is on the river side, high water causes a sudden increase in upward pressure, which cannot be dissipated because of the impervious continuous layer on the land side.

In the example of design which Mr. Buchanan illustrates by Figs. 8, 9, 10, and Table 3, the assumption of a cylindrical surface of rupture starting at the top of the slope is not necessarily the true failure picture. More often, failure occurs by slippage of a dome section, with a vertical surface intersecting the original slope. In the design of the foundations, the assumption is made that the added weight is distributed uniformly, no allowance being made for the lateral flow of the subsoil. As has been proved by experiments, the maximum added pressure is not under the highest point of the fill. Item (*e*), (see heading, "Foundations") giving the necessary base width (348 ft), assumes that a uniform added base pressure results from the levee load over the entire width of the base, that there is no lateral distribution of loading below the base of the levee, and that the strength of the foundation material is not affected by the loading above. None of these assumptions is true. Uniform added base pressure would result only from a uniform depth of fill, unlimited in all directions. There would be no lateral distribution of loading below the base of the levee only if the material had a zero internal friction coefficient. The strength of the foundation material is affected by the superimposed loading because of the added internal resistance resulting from the confinement by such loading.

It is unfortunate that the sudden decision to abandon the Passamaquoddy Project ended the study of foundations for rock-fill dams, as reported by Mr. Hough. The report of the work done is of value for future studies, but the writer cannot agree that it has any value in regard to a solution of the foundation design for the proposed dams. The chief reason is that the later effect of the impervious blanket, a much greater weight than the rock-fill section, was not considered. Allowing the foundation material to re-adjust itself and build up resistance to support the rock-fill and then hope that the later addition of about 80 to 100 ft of impervious blanket fill on one slope of the rock-fill could also be carried by the original adjustment, is somewhat optimistic. References to the work of Carlo Barberis,[58] at Spezia, Italy, as well as to similar works[59] in Valparaiso, Chile, and also by Sakamoto and Takanishi[60] in Kobe, Japan, would certainly have been of assistance. In the foregoing three references, similar dams and wharves have been described built on mud or soft clay foundations by the aid of a "floating foundation" of fine sand pumped into place. The combination of mud and sand has proved a proper, safe, and economical solution of the problem studied by Mr. Hough. It may be of interest to mention that, in 1928, while making some studies for private finance groups for this same project, the writer made some designs and estimates for the dams required in connection with the Passamaquoddy Tidal Power Project, based on the work mentioned in the foregoing references. The results were much more economical than the sections illustrated by Mr. Hough.

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[57] *Journal of Applied Mechanics*, Vol. 4, September, 1937, pp. A97–100.

From time to time, reports of, and papers sponsored by, the Committee of the Society on Earths and Foundations have appeared in *Proceedings*.[11] Except for this Symposium, the reports were optimistically heralding the advent of a new era in foundation engineering and the birth of a new soil science. The humility of approach in the 1937 reports, as so well exemplified by the paper by Professor Terzaghi, is refreshing; and it is well that the change of attitude has taken place. Too many practical and practising engineers were attempting to apply the published material to their work; and they were finding it of little help.

If the paper by Professor Terzaghi is to be taken as a criterion, the subject is in a new phase. Theoretical and laboratory investigations are relegated to second position; field investigations of completed structures must now be stressed so as to collect the necessary facts from which reliable theory may be deduced. This is opposite to the previously announced inductive theories which prophesied facts, such as settlements of unbuilt structures, and which too often required continuous adjustments to meet new data.

In order to be of value, complete descriptions of the structures must be presented with settlement readings. In this respect, the paper by Professor Terzaghi is somewhat remiss. For instance, in the example described with Fig. 45, the length, size, and spacing of the piles are not given. One is inclined to wonder why piles were used in that design. In Fig. 47(*b*), there appears to be a definite discontinuity in all settlement contours at the center division wall. An error in the H_3 reading seems to be one explanation of the impossible condition shown.

In Fig. 48, the maximum settlement is a trifle more than $\frac{3}{4}$ in. For the broken up design, with piles of unknown and varying lengths, such a value is not unexpected.

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[58] XVI International Cong. of Navigation, 1935.

[59] *Bulletin*, Navigation Congresses, January, 1927, p. 33.

[60] XIV International Congress of Navigation, 1926.

[11] Progress Rept. of Special Committee on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 788.

In Table 6, it should be carefully noted that all the soils listed, except that for Building *H*, are not the usual materials on which spread footings are placed. This fact explains the great variations in settlements for points on the same structure. Buildings founded on sand or clay mixtures with loam, with base pressures between 2 and 2.5 tons per sq ft, should show some peculiar settlements.

The writer agrees that no reasonable soil examination can show all the variations in soil structure for a complete foundation design. Where soil conditions are uniform, expected settlements are easily guessed. Where soil conditions are not uniform, it is necessary to design the footings as the sub-grades are exposed by excavation. In one large structure, north of Chicago, Ill., borings indicated that at least three different clays would be encountered at the expected base of the footings. After a careful study of all the clays encountered in that vicinity, and comparable loading tests in pits dug into the three different clays (differentiated by their colors), load-bearing values were determined to give equal settlements. The lack of time prevented load-test studies extending over more than a few weeks. Design tables were prepared for the three assumed load values for the complete range of column loads. All footings could be made symmetrical spread footings for single column loads, which simplified the problem. As each footing pit was dug to sub-grade, inspection was immediately made, the color determined, and design requirements chosen from the tables. No unusual or unequal settlements have been noticed.

Much foundation trouble arises from a common error of placing a structure on the side of a slope, exposed to seepage or drainage effects and lateral movements. The error is evident when the structure is placed on a sloping surface, such as the side of a ravine or river bed. If the gap is filled with rubbish, ashes, or the usual municipal rubbish fill, the error is not visible, but still exists. In many cases, the fill makes the matter worse, because its consolidation causes a lateral flow which further tends to tip or slide the building foundation piers or walls.

Y. L. Chang,[61] Esq. (by letter).[61a]—Engineers in China who are confronted with a similar, but more complex, problem when building regulating dikes on the Yellow River, will be much interested in the paper by Mr. Buchanan. Because of traditional methods of construction and the limited financial support given to such large engineering projects, all dikes along the channel of the river could be built and maintained only by using earth of uniform quality at the site. About one-fourth of the total area of the Yellow River basin consists of loess which is a very fine calcareous loam composed of an admixture of minute but considerably flaky particles of quartz and clay. This material is mostly an impermeable soil. The loess has a porosity exceeding 45%, enabling it to hold considerable moisture, and the fineness of the openings facilitates capillary action. Such material cannot be approved for dike construction; and, hence, the problem is reduced to a question of how to design the cross-section of the dike to obtain a degree of safety consistent with minimum cost. Mr. Buchanan certainly makes such an aim possible by a procedure so simple that every engineer can understand it.

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[61] Shanghai, China.

[61a] Received by the Secretary October 8, 1937.

In the report of the Committee of Experts of the League of Nations,[62] five causes of the failure of the dikes on the Yellow River are given, the most important being substantially as follows:

> "A breach may be produced when the body of the dike is saturated with water, either by direct infiltration from the water face or by indirect infiltration through the more permeable layers at the base. The cohesive resistance of the earth is reduced, while at the same time hydraulic pressure, accompanied by seepage, occurs. Its effect is felt when the base of the inner side of the dike collapses. The dike itself is thus weakened, and a breach may result. This case arises especially when the high-water stage lasts a long time or when the dike is of inadequate construction or composition."

With this statement in mind, one can be assured that a failure of such dikes is mainly attributed to the infiltration of water through the dike itself and through the foundation beneath. The writer wishes to discuss Mr. Buchanan's paper as applied to conditions when the dike is subjected to infiltration through side slopes, and foundation, and as applied to the control of seepage.

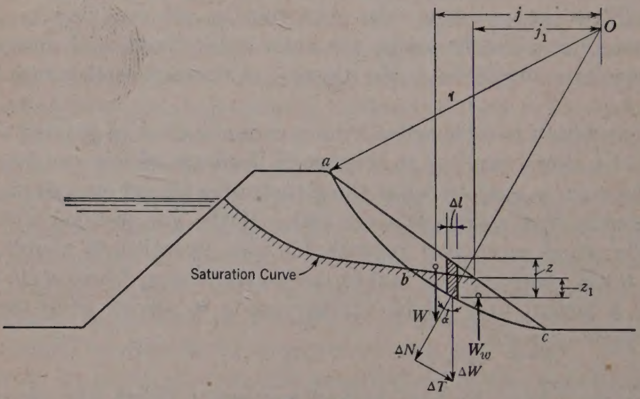

FIG. 63

Side Slopes.—Presumably, Equation (1), which expresses the factor of safety against sliding in terms of the shape of the dike and the frictional and cohesive forces of the material, is for a dry slope. For a dike subject to seepage, such as that shown in Fig. 63, Equation (1) can be written:

$$\text{Factor of safety} = \frac{c\,l + \tan\phi \sum_{0}^{n} \Delta l\,(w\,z - w_w\,z_1)\cos\alpha}{\sum_{0}^{n} \Delta l\,(w\,z - w_w\,z_1)\sin\alpha} \quad \ldots\ldots(20)$$

in which, ϕ, the angle of internal friction, is assumed to be the same for dry or wet material; w_w and w are the unit weights of water and soil, respectively;

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[62] Report by the Committee of Experts on Hydraulic and Road Questions in China, League of Nations, Geneva, 1936.

and l should be the arc length, $a\,b$. If the entire dike is assumed to be saturated, the term $c\,l$ vanishes. A similar equation for the factor of safety against sliding can be expressed in the form of moment instead of force:

$$\text{Factor of safety} = \frac{r\left[c\,l + \tan\phi \sum_{0}^{n} \Delta l\,(w\,z - w_w\,z_1)\cos\alpha\right]}{W\,j - W_w\,j_1} \quad \ldots.\ (21)$$

in which W_w and W are, respectively, the total weights of water and soil within the arc of sliding, $a\,b\,c$. The smaller value determined by Equations (20) and (21) gives the safer criterion for stability.

Mr. Buchanan's treatment of the hydrodynamic effect of seepage on stability of slopes is especially creditable. In testing a model embankment of loess the writer has observed a slide near the toe despite the fact that the factor of safety, as computed by Equation (21), exceeded 1.78 for $c = 0$. The writer believes that the hydrodynamic force in a saturated slope plays an important part in starting a slide.

Foundations.—A breach may occur in the dike when its foundation is so saturated that the underlying material settles or is displaced. In this state, the foundation could be considered more plastic than elastic. Studies of such failures due to over-stressing the foundation have been presented by Mr. Hough. Attention should be called to the fact that there is a horizontal, as well as a vertical, load on the foundation when the dike is resisting the pressure of high water. For earth or rock-fill embankments this pressure can be transmitted to the foundation only in the form of a shearing force. In investigating the stability of dike foundations this horizontal hydrostatic load requires as careful study as the vertical dead load.

Control of Seepage.—Mr. Buchanan has studied the problem of the seepage through a dike with the aid of Professor Gilboy's formulas as given by Equations (2) and (3). The writer has compared this method with that suggested by Pavlovsky,[63] in the case of model dikes of Chinese loess, and has found that the latter coincides more nearly with experimental results. The latter also affords a simple procedure for designing the best section. A study of this method reveals that the permeability factor, K, cannot be constant in a newly built dike in which the seepage rate may be quite high.

The essential points demonstrated by this method are:

(1) A dike subjected to considerable infiltration over a long period of time is at its most dangerous stage; its stability is undoubtedly reduced by the hydrodynamic force of the water seeping through it.

(2) In estimating the shearing failure of the foundation of a dike the horizontal transmission of water pressure should not be neglected.

(3) Theoretically, the best method of reducing infiltration and lowering the saturation line of a dike is to increase the flatness of the wet surface.

(4) The orientation of flaky particles of a material by capillary flow probably plays quite an important part in the variation of permeability with time.

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[63] 1er Congrès des Grand Barrages—Scandinavie, Vol. II, 1933; see, also, "The Percolation of Water Through Earth Dams," by N. N. Pavlovsky; tr. by Andreas Luksch; U. S. Bureau of Reclamation, Denver, Colo.

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DISCUSSIONS

THE DESIGN OF ROCK-FILL DAMS

Discussion

By Harold K. Fox, M. Am. Soc. C. E.

Harold K. Fox,[6] M. Am. Soc. C. E. (by letter).[6a]—Core walls in the center of the fill are inadvisable, as Mr. Galloway states, the principal reason being their inaccessibility.

The writer questions the advisability of using too much so-called fines in the fill because there is a hazard of obstructing drainage and also because of the possibility of the fines acting as a lubricant of the larger material and thereby facilitating slides. He has had the experience of sinking a shaft 140 ft from the top of an old rock-fill dam through the fill to bed-rock, and bases his belief on that experience.

The writer does not agree that gates cannot be used in the spillway because there are cases in which it can be done with safety, due to the character of the run-off and the possibility of maintaining an attendant to operate the gates. Gate control will often add materially to the usefulness of the project.

In regard to the stability of a rock-fill dam, the writer cannot overlook an experience with a small one about 50 ft high which was built by a logging crew. Rock was obtained from the surface at the abutments, and the facing was of lumber. Cross-sections indicated that the rock had been piled up as steeply as possible instead of allowing it to take its angle of repose. The lumber facing had been projected to form about 3 ft of flash-boards above the crest, and there is plenty of evidence of overtopping. Of course, no engineer would recommend any such conditions; however, there is value in knowing that a dam of the rock-fill type has stood under such conditions for more than forty years. It leaves little doubt as to the stability of the modern well built structures.

Note.—The paper by J. D. Galloway, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1937, by Messrs. Cecil E. Pearce, and H. B. Muckleston.

[6] Constr. Engr., San Joaquin Light and Power Corp., Bakersfield, Calif.

[6a] Received by the Secretary December 14, 1937.